



انتشار امواج در محیط متخلخل

مقدمه

معادلات خاک اشباع

انتشار امواج در خاک اشباع

انتشار امواج در خاک غیر اشباع

مقدمه:

کاربرد انتشار امواج لرزه‌ای: ژئوتکنیک، مهندسی زلزله، ژئوفیزیک و لرزه شناسی

امواج لرزه ای ممکن است به وسیله یک

رویداد لرزه ای،

فروریزش یک حجم در پوسته زمین،

یک انفجار شیمیایی یا هسته ای و یا یک منبع ضربه سطحی تولید شوند

در گسترش امواج لرزه ای بخشی از انرژی تلف می شود و بخشی در سنگ و خاک پخش می شود

خصوصیات مکانیکی لایه های خاک از جمله اشباع بودن یا نبوده بر این امواج تاثیر می گذارند.

مسائل انتشار امواج در محیط متخلخل اشباع اولین بار توسط بیوت (1956 Biot) و سپس توسط سایر محققین (Berryman و همکاران ۱۹۸۸ و سپس Yang 1999) برای محیط اشباع مطالعه شده است.

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معادلات خاک اشباع

معادلات حاکم

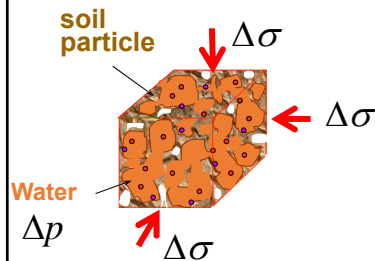
بقای جرم آب و دانه‌های جامد
 بقای ممتوم آب و دانه‌های جامد
 معادلات ساختاری و سازگاری

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معادلات خاک اشباع

Undrained compression of a porous medium



$$\sigma = \sigma' + \alpha p$$

$$\sigma' = \frac{-1}{C_m} \varepsilon$$

$$d\varepsilon = -C_m d\sigma'$$

$$\Delta V_f = -nC_f \Delta p V$$

$$\Delta V_s = -(1-n)C_s \Delta p V$$

C_f compressibility of the pore fluid

C_s compressibility of the solid

v fluid velocity

w Solid velocity

α Biot's coefficient

C_m compressibility of the porous medium

$$S_p = nC_f + (\alpha - n)C_s \quad \text{storativity}$$

$$\rho = n\rho_f + (1-n)\rho_s \quad \text{total density}$$

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معادلات خاک اشباع

Conservation of mass

pore fluid

$$\frac{\partial(n\rho_f)}{\partial t} + \frac{\partial(n\rho_f v)}{\partial x} = 0$$

فرض $\frac{d\rho_f}{dp} = \rho_f C_f$

$$\rho_f \frac{\partial n}{\partial t} + n \frac{\partial \rho_f}{\partial p} \frac{\partial p}{\partial t} + \rho_f \frac{\partial(nv)}{\partial x} + v \frac{\partial(n\rho_f)}{\partial x} \approx 0$$

$$\frac{\partial n}{\partial t} + n C_f \frac{\partial p}{\partial t} + \frac{\partial(nv)}{\partial x} = 0$$

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معادلات خاک اشباع

Conservation of mass

solid

$$\frac{\partial[(1-n)\rho_s]}{\partial t} + \frac{\partial[(1-n)\rho_s w]}{\partial x} = 0$$

فرض $\frac{\partial \rho_s}{\partial t} = \frac{\rho_s C_s}{1-n} \left(\frac{\partial \sigma}{\partial t} - n \frac{\partial p}{\partial t} \right)$

$$-\frac{\partial n}{\partial t} + C_s \left(\frac{\partial \sigma}{\partial t} - n \frac{\partial p}{\partial t} \right) + \frac{\partial(w)}{\partial x} - \frac{\partial(nw)}{\partial x} = 0$$

حذف $\frac{\partial n}{\partial t}$ در بقای جرمها

$$n(C_f - C_s) \frac{\partial p}{\partial t} + C_s \frac{\partial \sigma}{\partial t} + \frac{\partial w}{\partial x} + \frac{\partial(n(v-w))}{\partial x} = 0$$

or $S_p \frac{\partial p}{\partial t} + \alpha \frac{\partial w}{\partial x} = S_p \frac{\partial p}{\partial t} + \alpha \frac{\partial \varepsilon}{\partial x} = -\frac{\partial(n(v-w))}{\partial x}$

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معادلات خاك اشباع

Conservation of momentum

$$\text{fluid} \quad n\rho_f \frac{\partial v}{\partial t} + \tau n\rho_f \frac{\partial(v-w)}{\partial t} = -n \frac{\partial p}{\partial x} - \frac{n^2 \mu}{\kappa} (v-w)$$

τ tortuosity factor,

κ Intrinsic permeability of the porous medium m^2

$$\text{specific discharge} \quad q = n(v-w)$$

$$\text{In the absence of acceleration terms} \quad q = -\frac{\kappa}{\mu} \frac{\partial p}{\partial x}$$

Darcy's law

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معادلات خاك اشباع

Conservation of momentum

$$\text{fluid plus particles} \quad n\rho_f \frac{\partial v}{\partial t} + (1-n)\rho_s \frac{\partial w}{\partial t} = -\frac{\partial \sigma}{\partial x}$$

$$\sigma = \sigma' + \alpha p \quad n\rho_f \frac{\partial v}{\partial t} + (1-n)\rho_s \frac{\partial w}{\partial t} = -\frac{\partial \sigma'}{\partial x} - \alpha \frac{\partial p}{\partial x}$$

particles

$$(1-n)\rho_s \frac{\partial w}{\partial t} - \tau n\rho_f \frac{\partial(v-w)}{\partial t} = -\frac{\partial \sigma'}{\partial x} - (\alpha-n) \frac{\partial p}{\partial x} - \frac{n^2 \mu}{\kappa} (v-w)$$

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معادلات خاک اشباع

Constitutive equation

one-dimensional case
$$m_v \frac{\partial \sigma'}{\partial x} = - \frac{\partial w}{\partial t}$$

m_v one-dimensional compressibility of the porous medium

$$m_v = \frac{1}{K + \frac{4}{3}G} \rightarrow \text{confined modulus}$$

K Bulk modulus

G Shear modulus

Creep and irreversible plastic deformations can be considered

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معادلات خاک اشباع

System of Equation

basic equations for the propagation of plane waves in a porous medium

total mass conservation
$$\alpha \frac{\partial w}{\partial x} + S_p \frac{\partial p}{\partial t} = - \frac{\partial(n(v-w))}{\partial x}$$

total momentum
$$n\rho_f \frac{\partial v}{\partial t} + (1-n)\rho_s \frac{\partial w}{\partial t} = - \frac{\partial \sigma'}{\partial x} - \alpha \frac{\partial p}{\partial x}$$

momentum of the pore fluid
generalization of Darcy's law
$$n\rho_f \frac{\partial v}{\partial t} + \tau n\rho_f \frac{\partial(v-w)}{\partial t} = -n \frac{\partial p}{\partial x} - \frac{n^2 \mu}{\kappa} (v-w)$$

stress-strain relation of the soil skeleton
$$m_v \frac{\partial \sigma'}{\partial x} = - \frac{\partial w}{\partial t}$$

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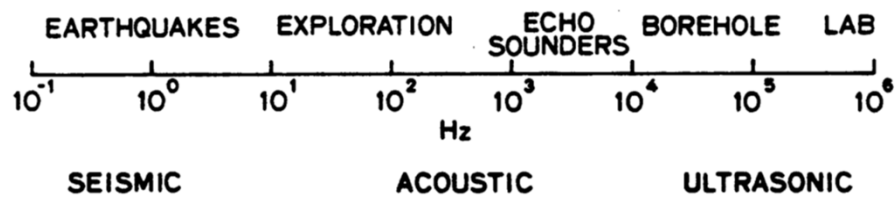
معادلات خاک اشباع
انتشار امواج در خاک اشباع
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انتشار امواج در محیط متخلخل

محدوده تغییرات امواج با توجه به کاربردها



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انتشار امواج در محیط متخلخل اشباع

• روابط تنش - کرنش

$$-\phi p = (Qv_m + Rv_f)$$

$$-\sigma_{ij} - (1-\phi)p\delta_{ij} = 2\mu_m(\varepsilon_{ij} - \frac{1}{3}v_m\delta_{ij}) + (Kv_m + Qv_f)\delta_{ij}$$

$$v_m = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33} \quad v_f = e_{11} + e_{22} + e_{33} \quad \text{انبساط حجمی فازهای اسکلت و سیال}$$

• دلتای کرونگر δ_{ij}

$$K = \frac{(1-\phi)(1-\phi - K_m/K_s)K_s + \phi K_s K_m/K_f}{1-\phi - K_m/K_s + \phi K_s/K_f} \quad \text{ثوابت گداندکن}$$

$$Q = \frac{(1-\phi)(1-\phi - K_m/K_s)\phi K_s}{1-\phi - K_m/K_s + \phi K_s/K_f}$$

$$R = \frac{\phi^2 K_s}{1-\phi - K_m/K_s + \phi K_s/K_f}$$

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انتشار امواج در محیط متخلخل اشباع

• معادله مومنتوم فاز سیال:

$$-\phi \partial_i p = \partial_i (\rho_{12} v_j + \rho_{22} w_j) - b_0 (v_j - w_j)$$

• معادله مومنتوم فاز جامد:

$$-\partial_j \sigma_{ij} - (1-\phi) \partial_i p = \partial_i (\rho_{11} v_j + \rho_{12} w_j) + b_0 (v_j - w_j)$$

$$b_0 = \frac{\eta \phi^2}{k_0}$$

$$\rho_{12} = -\phi \rho_f (\tau - 1)$$

$$\rho_{22} = \phi \rho_f - \rho_{12}$$

$$\rho_{11} = (1-\phi) \rho_s - \rho_{12}$$

$$\tau = 1 - 0.5(1 - 1/\phi)$$

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انتشار امواج در محیط متخلخل اشباع

- از ترکیب روابط تنش - کرنش و معادلات مومنتوم، معادلات موج بر حسب جابه‌جایی فاز جامد و جابه‌جایی فاز سیال بصورت زیر حاصل می‌شود:

$$\rho_{12}\partial_t^2 \mathbf{u}_s + \rho_{22}\partial_t^2 \mathbf{u}_f - b_0\partial_t(\mathbf{u}_s - \mathbf{u}_f) = C\nabla\nabla \cdot \mathbf{u}_s + R\nabla\nabla \cdot \mathbf{u}_f$$

$$\rho_{11}\partial_t^2 \mathbf{u}_s + \rho_{12}\partial_t^2 \mathbf{u}_f + b_0\partial_t(\mathbf{u}_s - \mathbf{u}_f) = F\nabla\nabla \cdot \mathbf{u}_s - G\nabla \times \nabla \times \mathbf{u}_s + Q\nabla\nabla \cdot \mathbf{u}_f$$

$$F = K + 4\mu_m/3$$

$\mathbf{u}_s, \mathbf{u}_f$ جابه‌جایی فازهای اسکلت و سیال

- برای حل بردار جابه‌جایی فاز جامد و فاز سیال را بر حسب پتانسیل موج فشاری و پتانسیل موج برشی تجزیه می‌کنیم. سپس سیستم معادله دیفرانسیل جزئی بدست آمده را با استفاده از تبدیل انتگرالی فوریه به سیستم معادلات دیفرانسیل معمولی در فضای فرکانس - عدد موج تبدیل و در نهایت روابط پراکنش امواج فشاری و برشی را بدست می‌آوریم.

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- رابطه پراکنش امواج فشاری مطابق روبرو بدست می‌آید:

$$A_1 k_p^4 + A_2 k_p^2 + A_3 = 0$$

$$A_1 = -Q^2 + FR$$

$$A_2 = (-Ra_{11} + 2Qa_{12} - Fa_{22})\omega^2$$

$$A_3 = (-a_{12}^2 + a_{11}a_{22})\omega^4$$

$$a_{11} = \rho_{11} - i\hat{b}\omega^{-1}$$

$$a_{12} = \rho_{12} + i\hat{b}\omega^{-1}$$

- که اعداد موج فشاری نوع دوم را نتیجه می‌دهد:

$$k_{p1}^2 = \pm \frac{(A_2^2 - 4A_1A_3)^{0.5} + A_2}{2A_1}$$

- که عدد موج برشی زیر را نتیجه می‌دهد:

$$-\mu_m a_{22}\omega^2 k_s^2 - a_{12}^2\omega^4 + a_{11}a_{22}\omega^4 = 0$$

$$k_s^2 = \pm \frac{(a_{11}a_{22} - a_{12}^2)\omega^2}{\mu_m a_{22}}$$

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انتشار امواج در محیط متخلخل اشباع

- با توجه به اعداد موج بدست آمده سرعت و استهلاک ذاتی طبق تعریف بصورت زیر بدست می آید:

$$c_j = \frac{\omega}{\text{Re}(k_j)} \quad \text{سرعت}$$

$$Q_j^{-1} = 2 \frac{\text{Im}(k_j)}{\text{Re}(k_j)} \quad \text{استهلاک ذاتی}$$

- که با قرار دادن $j = p_1$ و $j = p_2$ بر ترتیب سرعت و استهلاک امواج فشاری نوع اول، فشاری نوع دوم و برشی بدست می آیند

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انتشار امواج در محیط متخلخل اشباع

- خصوصیات مصالح مورد استفاده:

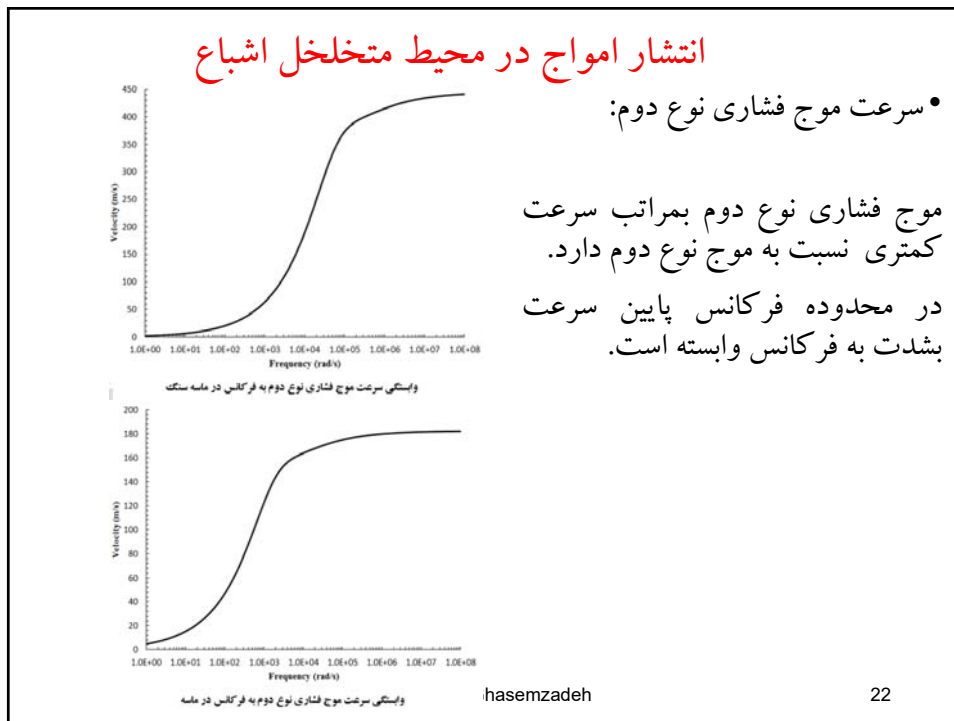
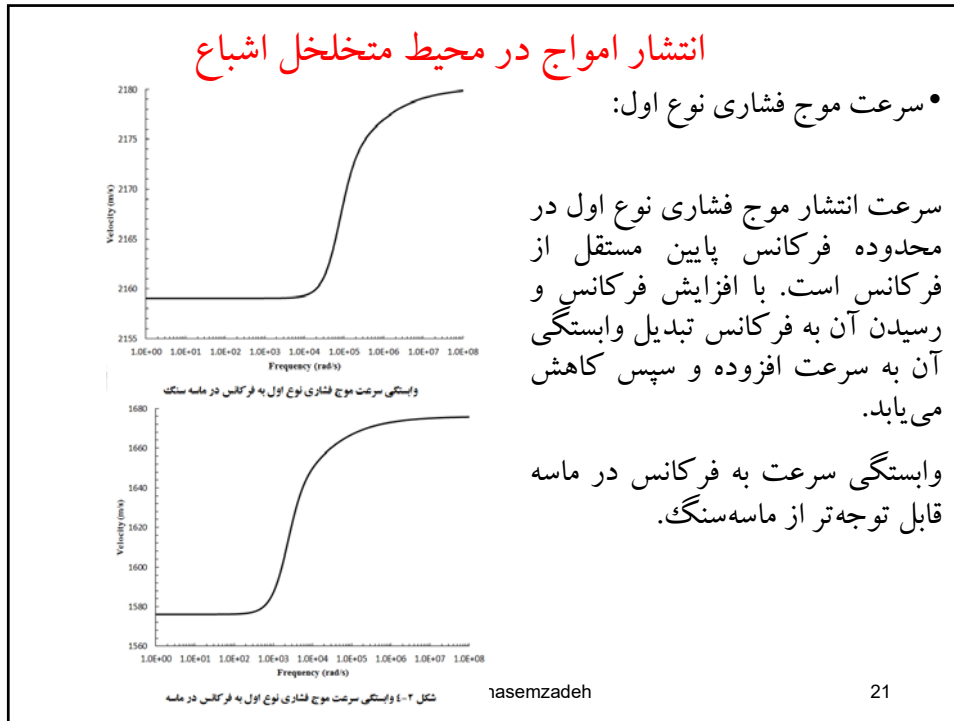
خصوصیات ماسه سنگ و ماسه

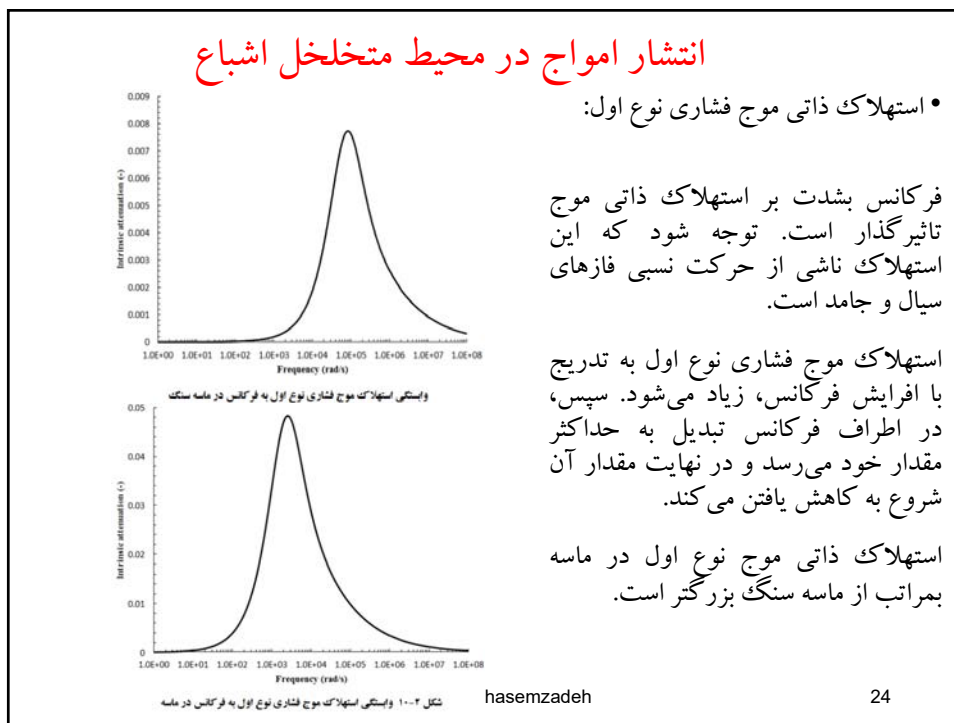
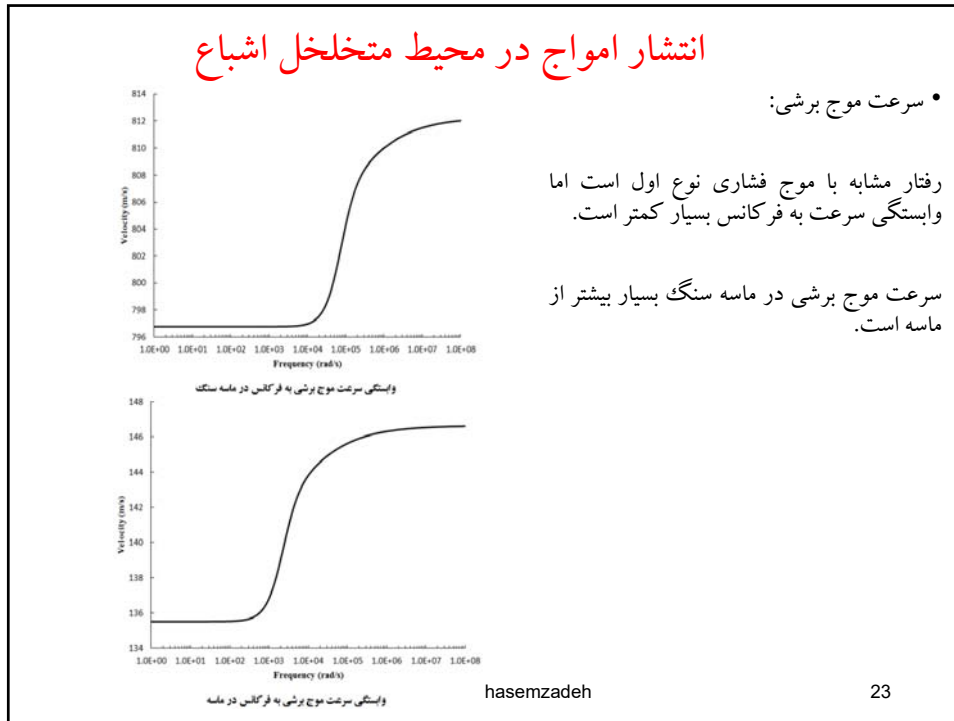
پارامتر	نشانه	واحد	ماسه سنگ	ماسه
تخلخل	ϕ	-	0.23	0.4
چگالی دانه های جامد	ρ_s	kg m^{-3}	2650	2600
مدول بالک اسکلت جامد	K_m	GPa	1.021	70×10^{-3}
مدول برشی اسکلت جامد	μ_m	GPa	1.441	35×10^{-3}
مدول بالک دانه های جامد	K_s	GPa	35	35
نفوذپذیری ذاتی	k_0	m^2	9×10^{-13}	1×10^{-10}

خصوصیات آب

پارامتر	نشانه	واحد	مقدار
ویسکوزیته	η	Pa.s	1×10^{-3}
چگالی سیال	ρ_f	kg m^{-3}	1000
مدول بالک	K_f	GPa	2.25

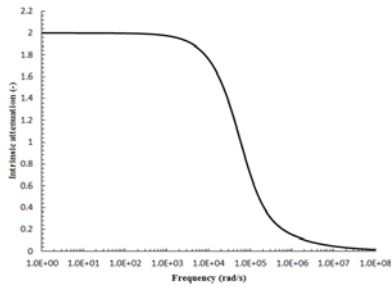
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انتشار امواج در محیط متخلخل اشباع

• استهلاك ذاتی موج فشاری نوع دوم:



وابستگی استهلاك موج فشاری نوع دوم به فرکانس در ماسه سنگ

بیشترین مقدار استهلاك موج فشاری نوع دوم در محدوده فرکانس پایین رخ می دهد.

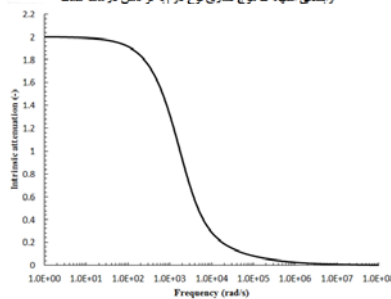
با افزایش فرکانس، استهلاك این نوع موج کاهش می یابد.

علت عمده بالا بودن استهلاك آن غیرهم فاز بودن قسمت های سیال و جامد است.

in a saturated porous medium two compressive waves can be generated,

- 1- the particles and the fluid move together
- 2- they move in opposite directions.

The second wave is strongly damped, because of the friction between the soil particles and the fluid in the small pores.



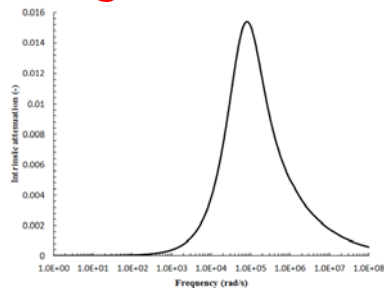
وابستگی استهلاك موج فشاری نوع دوم به فرکانس در ماسه

asam Chasemzadeh

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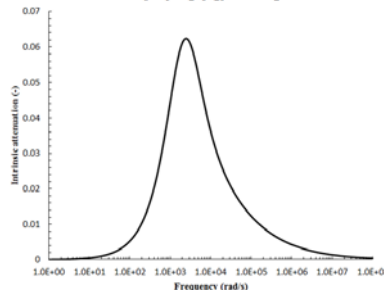
انتشار امواج در محیط متخلخل اشباع

• استهلاك ذاتی موج برشی:



وابستگی استهلاك موج برشی به فرکانس در ماسه سنگ

رفتار استهلاكی موج برشی مشابه با موج فشاری نوع اول می باشد. استهلاك موج برشی نیز در ماسه بزرگتر از ماسه سنگ است.



وابستگی استهلاك موج برشی به فرکانس در ماسه

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Ghasemzadeh H., Abounouri A.A., 2012, Effect of subsurface hydrological properties on velocity and attenuation of compressional and shear wave in fluid-saturated viscoelastic porous media, Journal of Hydrology, 460-461 (2012) 110-116.



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Effect of subsurface hydrological properties on velocity and attenuation of compressional and shear wave in fluid-saturated viscoelastic porous media
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 Velocity
 Porosity
 Permeability
 Borehole

SUMMARY

Over the past years there has been a growing attention in the use of seismic waves to gain a rapid information about the Earth's subsurface. Despite all achievements, the problem of accurately logging field measurements of seismic attributes to subsurface hydrological properties such as porosity and intrinsic permeability is still ambiguous. The goal of the present paper is to provide a comprehensive study on the effect of subsurface hydrological properties on seismic attributes such as wave attenuation and velocity. This is achieved by studying dispersion relations obtained from equations of wave motion that are derived from Biot's theory of poroelasticity. Since the attenuation predicted from Biot's theory is only due to relative motion of the solid and fluid phases, experimentally obtained data are normalized to consider the attenuation caused by grain-to-grain contact. The dispersion relations for body waves including fast waves, slow waves and shear waves are solved in both low and high frequency ranges where viscous and thermoelastic effects are dominant, respectively. Numerical simulations are performed on sand samples over a wide range of frequencies (1 Hz to 1 MHz) for samples with constant porosity, but different intrinsic permeabilities. It is demonstrated that the velocity of slow waves is higher for the more permeable sample over the full frequency range. The velocity of fast waves and shear waves is higher for the more permeable sample, but the difference is significant only at intermediate frequencies (10–100,000 Hz). However, the corresponding peak velocity and attenuation of each of the wave modes are almost equal for different intrinsic permeability values and, therefore, independent of intrinsic permeability. Another series of numerical simulations are carried out on sand samples with different porosity values. It is shown that the most permeable sand has higher slow wave and shear wave velocity, but lower fast wave velocity. Also, the peak attenuation of fast waves and shear waves gets larger as sand porosity increases, but slow wave behavior is opposite. Numerical results show that all wave modes become more dispersive when porosity increases. Thus, neglecting the dependence of wave velocity on frequency can lead to significant misinterpretation of wave velocity recorded samples with high porosity values.

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1. Introduction

Seismic methods are often used in hydrogeological problems with wide variety of applications, see, for example, Biot (1955), Weisberg (2005), Geller and Gurdal (2008) and McClintock et al. (2011). Subsurface hydrological properties of sands, such as porosity and intrinsic permeability are of great importance in problems encountered in groundwater monitoring, contaminant transport, aquifer characterization, recharge patterns, identification of permeable zones to treat water wells and design of subsurface waste disposal systems. However, applying seismic methods to these hydrogeological problems demand an accurate assessment of permeability and porosity influence on wave velocity and attenuation.

For the first time, Biot (1955) formulated wave motion in fluid-saturated porous media using the Biot's theory of poroelasticity and predicted the existence of two independent compressional waves, namely the fast (compressional) and the slow (shear) waves (compressional waves and slow shear waves). Solid and fluid displacements are out-of-phase for the slow wave and they are in-phase for the fast wave. Many researchers have failed to report the detection of slow wave due to its highly attenuative nature until Paoletti (1980) who observed slow wave in laboratory by using water saturated cement paste leads to his experiments. Kucir and Savello (1987) were first to report ultrasonic detection of slow wave in water saturated sandstone under atmospheric conditions.

Regarding theoretical studies in the field of hydrology, Sills et al. (2004) obtained low-frequency equations of elastic wave

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حل معادله موج در محیط اشباع
با فرضیات ساده کننده

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Undrained compression of a porous medium

soil particle

Water Δp

$$\Delta V_f = -nC_f \Delta p V$$

$$\Delta V_s = -(1-n)C_s \Delta p V$$

C_f compressibility of the pore fluid

C_s compressibility of the solid

v fluid velocity

w Solid velocity

α Biot's coefficient

C_m compressibility of the porous medium

$S_p = nC_f + (\alpha - n)C_s$ storativity

$\rho = n\rho_f + (1-n)\rho_s$ total density

$$\sigma = \sigma' + \alpha p$$

$$\sigma' = \frac{-1}{C_m} \varepsilon$$

$$d\varepsilon = -C_m d\sigma'$$

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معادلات خاك اشباع

System of Equation

basic equations for the propagation of plane waves in a porous medium

total mass conservation

total momentum

momentum of the pore fluid

generalization of Darcy's law

stress-strain relation of the soil skeleton

$$\alpha \frac{\partial w}{\partial x} + S_p \frac{\partial p}{\partial t} = -\frac{\partial(n(v-w))}{\partial x}$$

$$n\rho_f \frac{\partial v}{\partial t} + (1-n)\rho_s \frac{\partial w}{\partial t} = -\frac{\partial \sigma'}{\partial x} - \alpha \frac{\partial p}{\partial x}$$

$$n\rho_f \frac{\partial v}{\partial t} + \tau n\rho_f \frac{\partial(v-w)}{\partial t} = -n \frac{\partial p}{\partial x} - \frac{n^2 \mu}{\kappa} (v-w)$$

$$m_v \frac{\partial \sigma'}{\partial x} = -\frac{\partial w}{\partial t}$$

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Special case: Undrained waves

permeability is very small: fluid and the solids move together $v = w$

System of Equation

$$p = -\frac{\alpha m_v}{\alpha^2 m_v + S_p} \sigma$$

$$\sigma' = -\frac{S_p}{\alpha^2 m_v + S_p} \sigma$$

soft saturated soil $C_f, C_s \Rightarrow S_p \ll m_v, \alpha = 1$

$$(K_u + \frac{4}{3}G) \frac{\partial w}{\partial x} = -\frac{\partial \sigma}{\partial t}$$

$$\rho \frac{\partial w}{\partial t} = -\frac{\partial \sigma}{\partial x}$$

undrained
compression
modulus

$$K_u = K + \frac{\alpha^2}{S_p} = K + \frac{\alpha^2}{nC_f + (\alpha - n)C_s}$$

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compression modulus of the dry soil

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Special case: Undrained waves

$$\sigma - \frac{K_u + \frac{4}{3}G}{c} \omega = f_1(x + ct)$$

$$\sigma + \frac{K_u + \frac{4}{3}G}{c} \omega = f_2(x - ct)$$

wave velocity c

$$c = \sqrt{\frac{K_u + \frac{4}{3}G}{\rho}} = \sqrt{\frac{1}{\rho m_v} + \frac{\alpha^2}{\rho(nC_f + (\alpha - n)C_s)}}$$

For a completely saturated soft soil

$$K_u + \frac{4}{3}G \approx \frac{1}{n} C_f$$

$$C_f = 0.5 \times 10^{-9} \text{ m}^2/\text{N}$$

$$n = 0.4, \rho = 2000 \text{ kg/m}^3 \Rightarrow c = 1600 \text{ m/s}$$

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Special case: Rigid solid matrix

very stiff porous rock

$$w = 0$$

disregard the stress-strain relation and the momentum balance of the solid matrix

System of Equation

$$n \frac{\partial v}{\partial x} = S_p \frac{\partial p}{\partial t}$$

$$(1 + \tau) \rho_f \frac{\partial v}{\partial t} = - \frac{\partial p}{\partial x} - \frac{n\mu}{\kappa} v$$

Response: harmonic waves

$$p(x, t) = P e^{i(\lambda(x-ct))}$$

$$v(x, t) = V e^{i(\lambda(x-ct))}$$

λ wave number $\lambda = \omega/c$ may be complex

ω frequency of the wave is real

Substitution and combination

$$\frac{(1 + \tau) \rho_f S_p}{n} \left[1 + i \frac{n\mu}{(1 + \tau) \rho_f \omega \kappa} \right] c^2 = 1$$

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Special case: Rigid solid matrix

Definition

$$B = \frac{n\mu}{(1 + \tau) \rho_f \omega \kappa} = \frac{ng}{(1 + \tau) \omega k} \quad k = \frac{\kappa \rho_f g}{\mu}$$

κ Intrinsic permeability of the porous medium m^2

k hydraulic conductivity of the porous medium m/s

A- For normal soil or rock $k = 10^{-4} \text{ cm/s}$ Imaginary part dominates so

$$\Rightarrow c^2 = -i \frac{\omega \kappa}{\mu S_p} \quad \lambda = \omega/c = -(1+i) \sqrt{\frac{S_p \omega \rho_f g}{2k}}$$

B- For extremely high frequencies

Real part dominates so

$$c^2 = \frac{n}{(1 + \tau) \rho_f S_p} = \frac{1}{(1 + \tau) \rho_f C_f}$$

waves of this type will be strongly damped by the friction with the solids

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Special case: Rigid solid matrix

Example A- A soil completely saturated with water

$$\begin{aligned} \omega &= 1 \text{ s}^{-1} & \lambda &= -(1+i)\sqrt{\frac{S_p \omega \rho_f g}{2k}} \\ k &= 10^{-4} \text{ cm/s} \\ n &= 0.4 & \Rightarrow \text{Re}(\lambda) &= 1 \text{ m}^{-1} \\ S_p &= nC_f = 0.2 \times 10^{-9} \text{ m}^2/\text{N} \end{aligned}$$

the wave will be damped in the immediate vicinity of the source

Example B- A soil completely saturated with water

$$\begin{aligned} C_f &= 0.5 \times 10^{-9} \text{ m}^2/\text{N} \\ \rho_f &= 1000 \text{ kg/m}^3 \\ \tau &= 0 \end{aligned} \quad c = \sqrt{\frac{1}{(1+\tau)\rho_f C_f}} = 1400 \text{ m/s}$$

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System of Equation

total mass conservation

$$\alpha \frac{\partial w}{\partial x} + S_p \frac{\partial p}{\partial t} = -\frac{\partial(n(v-w))}{\partial x}$$

total momentum

$$n\rho_f \frac{\partial v}{\partial t} + (1-n)\rho_s \frac{\partial w}{\partial t} = -\frac{\partial \sigma'}{\partial x} - \alpha \frac{\partial p}{\partial x}$$

momentum of the pore fluid
generalization of Darcy's law

$$n\rho_f \frac{\partial v}{\partial t} + \tau n\rho_f \frac{\partial(v-w)}{\partial t} = -n \frac{\partial p}{\partial x} - \frac{n^2 \mu}{\kappa} (v-w)$$

stress-strain relation of the soil skeleton

$$m_v \frac{\partial \sigma'}{\partial x} = -\frac{\partial w}{\partial t}$$

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حل تحلیلی

general periodic pore pressure at the free end of a very long column



فرض جواب

$$p(x,t) = P e^{i(\lambda x + \omega t)}$$

$$\sigma'(x,t) = S e^{i(\lambda x + \omega t)}$$

$$v(x,t) = V e^{i(\lambda x + \omega t)}$$

$$w(x,t) = W e^{i(\lambda x + \omega t)}$$

ω given frequency
 λ Unknown wave number
 Maybe complex number

Substitution

$$n\lambda V + (\alpha - n)\lambda W = S_p \omega P$$

$$m_v \omega S = -\lambda W$$

$$n\rho_f \omega V + (1 - n)\rho_s \omega W = -\lambda S - \alpha \lambda P$$

$$(1 + \tau)n\rho_f \omega V - \tau n\rho_f \omega W = -n\lambda P + \frac{i n^2 \mu}{\kappa} (V - W)$$

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Dimensionless parameters

$$d_f = \rho_f / \rho_s$$

$$c^2 = \frac{1}{\rho m_v}$$

$$d_s = \rho_s / \rho_s$$

$$a = \frac{n\mu}{\kappa \rho_f \omega} = \frac{n g}{k \omega}$$

$$\gamma = \frac{c \lambda}{\omega}$$

$$b = S_p / m_v$$

$$\lambda^2 = \rho m_v \omega^2 \gamma^2$$

The homogeneous system of equations has a non-zero solution only if

$$A(\gamma)^4 + B(\gamma)^2 + C = 0$$

$$A = n,$$

$$B = -n(1 - n)d_s - [(\alpha - n)^2 + \alpha^2 \tau]d_f - (1 + \tau)d_f b + i a d_f (\alpha^2 + b),$$

$$C = [(1 - n)d_s + \tau]d_f b - i a d_f b.$$

$$\lambda_1 = \pm(q_1 + i r_1)(\omega/c) \quad \lambda_2 = \pm(q_2 + i r_2)(\omega/c)$$

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جواب

$$p = A_p \exp[-(\omega/c)(r_1 - iq_1)x] \exp(i\omega t) + B_p \exp[-(\omega/c)(r_2 - iq_2)x] \exp(i\omega t),$$

$$w = A_w \exp[-(\omega/c)(r_1 + iq_1)x] \exp(i\omega t) + B_w \exp[-(\omega/c)(r_2 - iq_2)x] \exp(i\omega t),$$

$$\sigma' = A_s \exp[-(\omega/c)(r_1 - iq_1)x] \exp(i\omega t) + B_s \exp[-(\omega/c)(r_2 - iq_2)x] \exp(i\omega t),$$

$$v = A_v \exp[-(\omega/c)(r_1 - iq_1)x] \exp(i\omega t) + B_v \exp[-(\omega/c)(r_2 - iq_2)x] \exp(i\omega t).$$

$$\frac{A_v}{A_w} = -\frac{\alpha - n}{n} - \frac{(1 - \alpha d_f - \gamma_1^2)b}{n(d_f b - \alpha \gamma_1^2)}, \quad \frac{A_w}{A_p} = \frac{(d_f b - \alpha \gamma_1^2)cm_v}{(1 - \alpha d_f - \gamma_1^2)\gamma_1}, \quad \frac{A_s}{A_w} = -\frac{\gamma_1}{cm_v},$$

$$\frac{B_v}{B_w} = -\frac{\alpha - n}{n} - \frac{(1 - \alpha d_f - \gamma_2^2)b}{n(d_f b - \alpha \gamma_2^2)}, \quad \frac{B_w}{B_p} = \frac{(d_f b - \alpha \gamma_2^2)cm_v}{(1 - \alpha d_f - \gamma_2^2)\gamma_2}, \quad \frac{B_s}{B_w} = -\frac{\gamma_2}{cm_v},$$

کاهندگی با فاصله

boundary conditions

$$x = 0 : \sigma' = (1 - \alpha)p_0 \exp(i\omega t), \quad \frac{A_p}{p_0} = \frac{[d_f b - \gamma_2^2 + (1 - \alpha)(1 - \alpha d_f)](1 - \alpha d_f - \gamma_1^2)}{(\gamma_1^2 - \gamma_2^2)(\alpha - \alpha^2 d_f - b d_f)},$$

$$x = 0 : p = p_0 \exp(i\omega t), \quad \frac{B_p}{p_0} = -\frac{[d_f b - \gamma_1^2 + (1 - \alpha)(\alpha - \alpha d_f)](1 - \alpha d_f - \gamma_2^2)}{(\gamma_1^2 - \gamma_2^2)(\alpha - \alpha^2 d_f - b d_f)}.$$

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جواب

Approximation of the solution

For real soils $k\omega$ often very small and $a = \frac{ng}{k\omega}$ will be very large

A, B and C may be approximated by

$$A = n \quad B = iad_f(\alpha^2 + b) \quad C = iad_f b$$

Possible solution for $x \geq 0$

$$\Rightarrow \gamma_1 = -\sqrt{\frac{b}{a^2 + b}}, \quad \gamma_2 = -(1 - i)\sqrt{\frac{c^2}{2c_v\omega}}$$

one-dimensional consolidation coefficient of the porous medium $c_v = \frac{k}{\rho_f g(\alpha^2 m_v + S_p)}$

wave is noticeable only for a distance $L = 4\sqrt{2c_v/\omega}$

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جواب

$$c_2 = \left[\frac{1 - \alpha + S_p/m_v}{\alpha - n + \tau + (1 + \tau)S_p/m_v} \right] \frac{n}{S_p \rho_f}$$

Approximation of velocity

$$\text{if } \alpha = 1, \tau = 0, S_p = nC_f$$

$$c_2 = \left[\frac{nC_f/m_v}{1 - n + nC_f/m_v} \right] \frac{1}{C_f \rho_f}$$

Two wave:

$$\begin{aligned} \gamma = \gamma_1 & \quad \frac{v}{w} \approx 1 \\ \gamma = \gamma_2 & \quad \frac{v}{w} \approx - \left[\frac{\alpha - n}{\alpha} + \frac{S_p}{\alpha n m_v} \right] \end{aligned}$$

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Example: soil properties

Symbol	Property	Value
ρ_s	Density of solids (kg/m ³)	2650
ρ_f	Density of fluid (kg/m ³)	1000
k	Permeability (m/s)	0.001
n	Porosity (-)	0.400
τ	Tortuosity (-)	0.000
α	Biot coefficient (-)	1.000
m_v	Compressibility of soil (m ² /MN)	0.0002
C_f	Compressibility of fluid (m ² /MN)	0.0005
C_s	Compressibility of solids (m ² /MN)	0.000
ω	Frequency (1/s)	10

Exact solution

$$\gamma_1 = -0.707106781 + 0.000000004i : A_v/A_w = 0.999999981 - 0.000012500i,$$

$$\gamma_2 = -22.439191923 + 22.394414404i : B_v/B_w = -4.000000019 - 0.000012500i.$$

approximate solution

$$\gamma_1 = -0.7071068 : A_v/A_w = 1,$$

$$\gamma_2 = -22.416793 + 22.416793i : B_v/B_w = -4.$$

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