Response of reinforced concrete structures to macrocell corrosion of reinforcements. Part II: After propagation of microcracks via a numerical approach

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Investigation of response of reinforced concrete (RC) structures due to axisymmetric macrocell corrosion of rebars is of concern after propagation of microcracks within the concrete medium. The geometry, boundary and interfaces conditions of the present problem are identical to those stated in part I. As seen in the companion paper, the exact solution to the boundary value problem corresponding to the uncracked steel–rust–concrete composite was possible. After appearance of the microcracks the concrete behavior becomes nonlinear anisotropic with post-cracking softening, and the associated problem is analytically intractable. Therefore, it is proposed to employ a novel meshless method, namely gradient reproducing kernel particle (GRKPM), in the cylindrical coordinates. The analytical and numerical solutions pertinent to the uncracked concrete are in good agreement. Subsequently, the effects of the parameters associated with the mechanical behavior of concrete and properties of rust on the time of surface cracking, the maximum values of consumed rebar per unit area of anode and crack width openings at the time of surface cracking, and the maximum value of radial stress at the rust–concrete interface are scrutinized in some detail.

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1. Introduction

Design of civil structures is generally carried out to fulfill safety, serviceability, and durability. For safety, the designed structure must efficiently perform its load-bearing function and carry the applied loads. For serviceability, the structure must be a safe place for its occupants. A structure that fails serviceability has exceeded a defined limit for one of the following factors: excessive deflection, vibration, local deformation, or sum of crack width openings. For durability, the structure must withstand against deterioration due to environmental influences during its service life. Corrosion of reinforcements in reinforced concrete (RC) structures is one of the rare factors, which gradually endangers all of the above-mentioned design criteria. This issue might become a serious problem for RC structures of nuclear power plants with a higher service life than the allowable one since the design life of such structures is commonly affected by economic or even antitrust considerations (Naus, 2009). It implies that age-related degradation concerns in nuclear power plants, including corrosion of steel reinforcement in water intake structures, corrosion of post-tensioning tendon wires, and leakage of corrosion inhibitors from tendon sheaths should be seriously paid attention to by the corresponding scientific communities. For this purpose, understanding the true nature of the rebar corrosion and its effects on the surrounding concrete would be of great importance. This issue also guides us to propose feasible ways for efficient protection of RC structures against such a destructive phenomenon.

Based on the proposed model by Tuutti (1982), the corrosion period of RC structures is generally idealized as two main stages: the initiation and propagation phases. In the first phase, aggressive ions diffuse through the concrete cover and their concentrations increase around the reinforcing steel. This stage would be over while the magnitudes of concentrations pass the threshold value, resulting in breakage of the passive film over the reinforcement. In the second phase, the electrochemical reactions of corrosion are commenced due to the configuration of anodes and cathodes on the surface of the corroding rebar. The location and distribution of anodes and cathodes highly depend on the concentration profile of the aggressive ions around the reinforcing...
Steel. Thereby, the accumulation of corrosion products around the corroding rebar exerts pressure on the interface of rust and concrete. Subsequently, this fact causes the formation of microcracks in the concrete medium where the tensile resistance of concrete is reached. The development of corrosion products across the surface of the corroding rebar and the associated openings of longitudinal microcracks on the concrete surface reduce the strength of the attacked RC member due to a number of reasons such as the reduction in the rebar/concrete bond, weakening the steel properties (loss of its cross-section and reduction of its fatigue strength), and lessening the mechanical properties of concrete due to propagation of microcracks. Such effects would be the serious threats to the service life of the RC structures. As the rebar corrosion keeps on, the tensile strength of the entire concrete cover is reached. Many developed models defined this point as the end of service life of the RC structures (Yokozeki et al., 1997; Liu and Weyers, 1998).

Modeling corrosion-induced damage in RC structures is a complicated problem due to the some uncertainties involved, such as the mechanical behavior of rust, the chemical composition of corrosion products, the amount of the penetrated rust into the microcracks, the rate of rust growth, and the geometry of rust accumulation around the corroding steel. On the other hand, the development of rational analytical models with closed-form solutions is generally restricted to some special cases. It is mainly due to some reasons, such as the asymmetrical geometry of many RC structures, complexity in analyzing the damage zone in concrete, and the existence of some nonlinearity in the model. To bridge this gap, the authors were inspired to perform an analytical study to investigate the influential factors on the time to the onset of microcracking in the case of axisymmetric macrocell corrosion of rebars (Kiani and Shodja, 2011b); however, the proposed solution was valid up to the stage of appearance of first microcracks.

To overcome the limitations of the analytical solutions in solving the problem, some useful numerical models have been developed to investigate the problem in a more general form. In this regard, Dagher and Kulendran (1992) devised a two-dimensional finite element (FE) model to predict the damage in concrete bridge decks resulting from production of rust around the corroding reinforcements. The suggested model was based on the smeared crack approach with a number of options for modeling of crack formation and propagation. Volume expansion due to production of corrosion products was then modeled by enforcing displacements on the nodes located at the interface of reinforcement and concrete. Since only examination of damage due to rebar corrosion was of concern, only the concrete cover was considered in the proposed model. The performed analyses showed that a uniformly radial expansion equal to 0.008 mm can cause a plane of delamination in RC structures with rebars of diameter 20 mm which are placed 152 mm apart. It was also observed that the likelihood of cover delamination decreases with the spacing between rebars. Molina et al. (1993) proposed a numerical model based on standard FE for the simulation of cracking in concrete specimens due to rebar corrosion. A smeared crack approach is used to model the behavior of the concrete due to the applied load resulted from the formation of rust. Due to lack of information, they assumed that the mechanical behavior of rust nearly resembles that of water which is one of the main constituents of rust. The proposed model is then applied to four specimens, which were simultaneously tested in the lab and reported by Andrade et al. (1993). The effect of formation of a particular volume of the rust on the rate of crack width is also quantitatively assessed. Pantazopoulos and Papoulias (2001) developed a numerical model to investigate how corrosion products cause concrete cover cracking. In the proposed model, the corrosion products are allowed to be deposited in the vacant spaces of microcracks. However, they assumed that the rebar and the corrosion products behave like a rigid material. Furthermore, the formation of rust is simply modeled by imposing radial displacement on the inner surface of concrete. Using finite difference method, the problem was then solved to examine the effects of cover thickness, material properties of concrete, and the rate of rust accumulation on the time of concrete cover cracking. Castorena et al. (2008) developed a numerical model to predict the time to cover cracking due to localized reinforcement corrosion using FE and statistical analyses. The thickness of concrete cover, the length of corroded rebar, the diameter of rebar, and the mechanical properties of the concrete were considered in the modeling of the problem. The microcrack propagation within the concrete medium was also modeled based on the smeared crack model. The parametric sensitivity of the proposed model was then performed with reference to experimentally observed data of other researchers. A FE-based model was developed by Chen and Mahadevan (2008) to simulate the full degradation process of RC structures from chloride penetration to reinforcement rust expansion to concrete cover cracking. The rebar corrosion model was based on Faraday’s law, and the rust expansion was incorporated into the model by a uniform time-varying radial displacement. Finite element analysis (FEA) with a smeared cracking approach was then implemented to model the rust expansion as well as the concrete cracking process. It is worth mentioning that only concrete medium under rebar corrosion was analyzed. In other words, the composite action of steel–rust–concrete as well as rational behavior of corrosion products was not incorporated into the formulations of the proposed model. Faiz Uddin Ahmed et al. (2007) proposed a FE model for simulating of the corrosion-induced cracking of RC beams. The smeared cracking approach was used to model the cracking of ordinary concrete, ductile fiber-reinforced cementitious composites (DFRCC), and engineered cementitious composites (ECC). The strains obtained from the FE models are compared with those of the accelerated corrosion tests. It was shown that the delamination of the cover of RC beams containing ECC or DFRCC materials will occur at a higher level of steel loss compared with that of an ordinary concrete beam. In another work, Du et al. (2006) investigated the effects of corroding reinforcements on the surrounding concrete via FEA. The three-dimensional physical specimens were idealized as two-dimensional models under the plane strain assumption. In order to analyze the problem, the effect of corrosion of reinforcement on the concrete medium was only modeled as an internal pressure or a radial expansion which exerts on the wall of the holes cast in concrete. In other words, the continua of steel and rust were not taken into account in the modeling of the problem. However, further investigations of Kiani (2002) indicated that the rational consideration of mechanical behavior of rust can strongly affect the predicted results. Sanchez et al. (2010) developed a mesoscopic model to simulate the mechanical behavior of reinforced concrete members affected by reinforcement corrosion. In order to assess the deterioration of the concrete medium, a cohesive model based on the continuum strong discontinuity approach was employed. Steel rebars were modeled using an elasto-plastic constitutive relation. The interface of rebar and concrete was also simulated by contact-friction elements. To this end, the friction degradation was expressed in terms of the degree of rebar corrosion. The capability of the suggested model was then examined through various comparisons of the obtained results with those of experiments, and a reasonably good agreement was reported. Vu and Stewart (2005) established a two-dimensional spatial time-dependent reliability model to predict the amount of rebar corrosion induced cracking. The spatial variability of concrete cover, concrete compressive strength, water–cement ratio, corrosion rate, chloride diffusion coefficient, and surface chloride concentration were taken into account in the proposed stochastic model. The suggested model could predict not only the probability that a given percentage of a
concrete surface has cracked, but also the proportion of an area of concrete experiences severe cracking. The results were obtained for a typical reinforced concrete bridge deck. Subsequently, the effects of concrete cover, concrete quality, limit crack width, and environmental conditions on the predicted results were inspected. Coronelli and Gamb (2004) carefully studied evolution of the structural behavior of RC beams under reinforcement corrosion. To this end, nonlinear FEA was adopted and the validity of the proposed numerical procedure was then checked by comparison of the obtained results with those of experiments. Realizing of some aspects of the progressive damage in an existing structure, and the evaluation of the actual safety level were the major objectives pursued by the authors in their work. Additionally, the service and ultimate limit states of beamlike RC structures were investigated. During corrosion of longitudinal reinforcements of the RC beams, some important phenomena such as decay of stiffness due to impared tension stiffening, deterioration of strength in bending and shear, and occurrence of bond failure along the span and/or at the ends of the RC beam, were also highlighted. Shodja et al. (2010a) proposed a theory for concrete deterioration due to rebar corrosion and introduced a parameter, \( \beta \), defined as the ratio of the volume of the rust penetrated into the microcracks to the total volume of the microcracks. When exploring the effects of uniform corrosion of rebar on the surrounding concrete was of concern, an analytical approach as well as an innovative meshless method was exploited for solving the problem. It was shown that the volume fraction, \( \beta \), has an important role in the determination of the time to cover cracking and the corresponding amount of the consumed rebar. The obtained results also pointed out that the amount of rust penetrated into the microcracks has a considerable effect on the evolution of the radial stress at the rust–concrete interface and the radii of microcracks’ fronts of the damage zone. Hansen and Saouma (1999a, b) simulated the process of reinforcement corrosion and concrete cover cracking through the use of the commercial FE code ABAQUS. They used the mass transfer capabilities of ABAQUS to simulate the electrochemical process of rebar corrosion in the proposed model. A nonlinear FEA using a mixed model discrete crack propagation based on Hillerborg’s model (Hillerborg et al., 1976) was then performed to investigate concrete cracking due to rust build-up. The obtained results indicated that discrete cracks can propagate from the corroding rebar in different directions, and their extents mostly rest on the reinforcement spacing and the concrete cover thickness.

Generally, modeling of crack propagation within cementitious materials is carried out via discrete crack model or smeared crack approach. In the first model, a discontinuity (i.e., a discontinuous line for two-dimensional or a discontinuous plane for three-dimensional problem) in displacement fields would result because of the formation of macrocracks. An appropriate constitutive model pertinent to the material and an effective approach for treatment of displacement discontinuities should be also employed for discrete crack based models. In the second model, the nucleation and propagation of microcracks are incorporated into the formulation through an appropriate modeling of the degradation of the material strength based on the strain field. A fairly comprehensive comparison of discrete crack models with smeared crack models has been investigated by Mosler and Meschke (2004).

In meshless methods, approximation of an unknown function is constructed from individual particles using higher order interpolants. This issue brings about some advantageous properties for meshless methods in comparing to other common techniques. The major features are: (i) no mesh distortion is observed when the medium experiences large deformations and (ii) problems with moving discontinuity such as discrete crack propagation or moving sharp boundaries can be handled more effortlessly. A detailed assessment of meshless methods has been also given by Nguyen et al. (2008). Discrete crack modeling of solids via meshless methods was initiated by Belytschko et al. (1995a, b). Several works concerning special treatments of displacement discontinuity in discrete crack models were then carried out. In this regard, an innovative technique for growing cracks via a meshless method was developed by Rabczuk and Belytschko (2004). The crack could be arbitrarily oriented; however, its growth was characterized individually by activation of crack surfaces at particles. At each cracked particle, a discontinuity along a line in 2D or a plane in 3D was established. It was shown that the newly proposed method could capture reasonably well the experimental results. Rabczuk and Belytschko (2007) introduced a new method for modeling discrete cracks in three dimension based on a meshless method. The crack was modeled by a local enrichment of the test and trial functions with a sign function. The proposed method was then formulated for large deformations as well as arbitrary nonlinear and rate dependent materials. Several problems were analyzed by the suggested model and compared with the available experimental data. In another work, Rabczuk et al. (2008) proposed a geometrically and materially nonlinear three-dimensional model to study fracture behavior of RC structures. To this end, the cracked concrete was analyzed through the extended element-free Galerkin method using a cohesive model. The set of embedded reinforcements was modeled with finite element method using standard \( J_2 \)-plasticity with isotropic hardening. The coupling between the reinforcement and the concrete media was also considered through an appropriate bond model. The proposed methodology was then exploited for modeling the fracture of several RC structures and compared with the experimentally observed data.

In this paper, a nonlinear mathematical model is proposed to determine the displacement and stress fields within steel–rust–concrete composite media due to macrocell corrosion of rebars. The rational modeling of the mechanical behavior of rust and the penetration of the corrosion products into the microcracks of concrete, results in the nonlinearity of the governing boundary value problem (BVP). The problem is solved via an innovative meshless technique, called gradient reproducing kernel particle method (GRKPM). Application of the GRKPM to the problems of beam–columns and plates shows remarkable results (Shodja and Hashemian, 2007; Hashemian and Shodja, 2008a). Furthermore, the high performance and accurate resolution of GRKPM when dealing with the Burgers’ and Buckley–Leverett equations exhibiting evolutionary high gradients are well established (Hashemian and Shodja, 2008b; Shodja and Hashemian, 2008). Subsequently, Behzadan et al. (2010) derived the formulation of GRKPM in a more general context. Furthermore, error estimates for the interpolants of GRKPM are also obtained and discussed. Through generalizing the concept of corrected collocation method, it was shown that in the case of application of GRKPM to a BVP, the essential boundary conditions involving the function as well as its derivatives are satisfied exactly at particles located on the boundary. In another work, Shodja et al. (2010b) applied reproducing kernel particle method (RKPM) to the problems involve material discontinuities through the imposition of the essential boundary conditions via augmented corrected collocation method. It was shown that the proposed methodology is a capable one in the determination of elastic fields within the composite and fractured media, particularly the regions close to the discontinuous boundaries. To date, penalty method has been implemented for enforcing essential boundary conditions of various one-dimensional problems using RKPM (Kiani et al., 2009a; Kiani, 2010) or generalized moving least square method (GMLSM) (Kiani et al., 2009b, 2010). Herein, penalty method is employed for imposing the boundary conditions of the proposed model. The resulting set of nonlinear governing equations is then solved via Newton’s method at each time step. The performance of GRKPM is then investigated through comparison of the GRKPM results.
with those of the proposed analytical approach in the companion work (Kiani and Shodja, 2011b). A parametric study is also carried out to examine the effects of the parameters associated with the mechanical behavior of rust and concrete on the variation of crucial parameters pertinent to the RC structure under macrocell corrosion for different levels of the rust penetration into microcracks.

2. Problem definition

Consider a circular cylindrical RC member with a coaxial reinforcement analogous to that shown in Fig. 1 of the companion paper. The reinforcement experiences axisymmetric macrocell corrosion (see Fig. 1(a)). At the beginning of the corrosion process, exertion of pressure to the inner wall of concrete increases with volume of the corrosion products. When the hoop stress of concrete at its interface with rust exceeds the tensile strength of concrete, microcracks are formed along the radial directions. Therefore, it is expected that the microcracks are propagated axisymmetrically through the concrete cover. The main objective of this study is to determine the displacement and stress fields within the steel–rust–concrete composite media, particularly after propagation of microcracks within the concrete. For this purpose, an effective meshless method is employed to solve the problem.

In the next part, the two-dimensional GRKPJM’s shape functions and their first derivatives are introduced. Subsequently, the implication of this novel method to the under study problem with the boundary conditions explained in the companion paper, is provided in some detail.

3. Numerical solution of the problem via GRKPM

3.1. An introduction to two-dimensional GRKPM

In the two-dimensional GRKPM, the reproducing function is defined in terms of the function and its first derivatives as follows

\[ u^k(x) = \sum_{k=0}^{2} \int_{\Omega} \tilde{\varphi}^k_a(x; x - y) u^a(y) \, d\Omega, \quad k = 0, 1, 2, \tag{1} \]

where \( x \) and \( y \) are the coordinates of two points in a two-dimensional domain such that \( x = (x_1, x_2) \) and \( y = (y_1, y_2) \), \( d\Omega \) denotes an infinitesimal area, \( u^0(y) = u(y) \), and \( u^k(y) = \partial^k u(y) / \partial y_k \) are, respectively, the two-dimensional field function and its first derivatives. \( \tilde{\varphi}^k_a \) is the modified kernel function associated with \( u^k(y) \) defined as

\[ \tilde{\varphi}^k_a(x; x - y) = \left[ b_{00}(x) + \sum_{j=1}^{2} b_{aj}(x)(x_j - y_j) \right] \prod_{j=1}^{2} \frac{1}{a_j} \phi \left( \frac{x_j - y_j}{a_j} \right), \quad k = 0, 1, 2, \tag{2} \]

in which \( a_j \) is the dilation parameter pertinent to the \( j \)th direction, \( \phi \) is a window function, and \( b_{aj}; k, j = 0, 1, 2 \) are the unknown moving coefficients determined through satisfaction of the completeness conditions. For this purpose, \( u(y) \) is approximated by the Taylor series expansion about \( x \) up to the second-order terms

\[ u(y) = u(x) + \sum_{i=1}^{2} (y_i - x_i) u^{(i)}(x) + \frac{1}{2} \sum_{i=1}^{2} \sum_{j=1}^{2} (x_i - y_i)(x_j - y_j) u^{(ij)}(x), \tag{3} \]

where \( u^{(ij)}(x) = \partial^2 u(x) / \partial x_i \partial x_j \). Substitution of Eq. (3) into Eq. (1) leads to

\[ u^k(x) = \sum_{i=0}^{2} u^{(i)}(x) \mathcal{Y}_i^k(x) + \sum_{j=0}^{2} u^{(ij)}(x) \mathcal{Y}_j^k(x), \tag{4} \]

where the functions \( \mathcal{Y}_i^k(x); i = 0, 1, 2 \) and \( \mathcal{Y}_j^k(x); j = 1, 2 \) are readily calculated as follows

\[ \mathcal{Y}_i^k(x) = b_{00}(x)m_{00}(x) + b_{10}(x)m_{10}(x) + b_{20}(x)m_{02}(x) - b_{01}(x)m_{01}(x) - b_{11}(x)m_{11}(x) - b_{21}(x)m_{01}(x) - b_{12}(x)m_{10}(x), \tag{5} \]

\[ \mathcal{Y}_j^k(x) = b_{00}(x)m_{00}(x) + b_{10}(x)m_{10}(x) + b_{20}(x)m_{20}(x) - b_{21}(x)m_{10}(x) - b_{12}(x)m_{01}(x) - b_{02}(x)m_{02}(x) - b_{11}(x)m_{11}(x) - b_{22}(x)m_{02}(x), \]

where \( m_{ij} \) is called moment matrix, and is defined as

\[ m_{ij} = \int_{\Omega} (x_1 - 1)^i (x_2 - 1)^j \phi_0(x - y) \, d\Omega. \tag{6} \]

In view of Eq. (4), the completeness conditions on \( u^k(x) \) leads to the following six independent conditions

\[ \mathcal{Y}_0^k = 1, \quad \mathcal{Y}_i^0 = 0, \quad \mathcal{Y}_i^j = \mathcal{Y}_j^i = 0, \quad i, j = 1, 2, \tag{7} \]

Eq. (7) could also be written in the matrix form as follows

\[ \mathbf{M}(\mathbf{x}) \mathbf{b} = \mathbf{h}, \tag{8} \]

where

\[
\begin{align*}
M_{11} = m_{00}, & \quad M_{12} = m_{10}, & \quad M_{13} = m_{01}, & \quad M_{21} = m_{10}, \\
M_{22} = m_{20} - m_{00}, & \quad M_{23} = m_{11}, & \quad M_{31} = m_{02}, & \quad M_{32} = m_{11}, \\
M_{33} = m_{02} - m_{00}, & \quad M_{35} = -m_{10}, & \quad M_{44} = -m_{02} - m_{20}, \\
M_{36} = -m_{01}, & \quad M_{41} = m_{20}, & \quad M_{42} = m_{30} - 2m_{10}, & \quad M_{43} = m_{21}, \\
M_{44} = -2m_{20}, & \quad M_{45} = -2m_{10}, & \quad M_{51} = m_{11}, & \quad M_{52} = m_{21} - m_{01}, \\
M_{53} = m_{21} - m_{10}, & \quad M_{54} = m_{12}, & \quad M_{55} = -m_{10} - m_{20}, \\
M_{56} = -m_{11}, & \quad M_{61} = m_{02}, & \quad M_{62} = m_{12}, & \quad M_{63} = m_{03} - 2m_{01}, \\
M_{65} = -2m_{11}, & \quad M_{66} = -2m_{02}, & \quad h = [1, 0, 0, 0, 0, 0]^{T},
\end{align*}
\]

by solving the set of linear equations in Eq. (8), the unknown elements of the vector \( \mathbf{b} \) are calculated. Moreover, by taking once and twice differentiation of both sides of Eq. (8) with respect to \( x \), the first and second derivatives of \( \mathbf{b} \) are readily obtained

\[
\begin{align*}
b^{(i)}(x) &= -M^{-1}(x)M^{(i)}(x)b(x), \\
b^{(ij)}(x) &= -M^{-1}(x)M^{(ij)}(x)M^{-1}(x)M^{(ij)}(x)b(x) \\
&\quad + M^{-1}(x)M^{(i)}(x)M^{-1}(x)M^{(ij)}(x)b(x) \\
&\quad - M^{-1}(x)M^{(j)}(x)M^{-1}(x)M^{(ij)}(x)b(x), \quad i, j = 1, 2.
\end{align*}
\tag{10}
\]

In order to evaluate the unknown elements of \( \mathbf{b} \) and its derivatives, the moments matrices should be calculated via an appropriate numerical scheme. In this article, trapezoidal method is exploited to discrete the integrals of the moments matrices. As a result, the reproduced function in Eq. (1) could be rewritten in the following discrete form

\[ u^k(x) = \sum_{k=0}^{NP} \sum_{i=1}^{2} \mathcal{Y}_i^k(x) u^{(i)}(x), \tag{11} \]
where $NP$ denotes the number of particles, $u^{(k)}_i$ are the independent degree of freedoms (DOFs), and $\psi^{(k)}_i(x)$ is the shape function of the $k$ th kind associated with the $i$ th particle at point $x$

$$\psi^{(k)}_i(x) = \left[ b_{i0}(x) + \sum_{j=1}^{2} b_{ij}(x)(x_j - y^j_i) \right] \prod_{j=1}^{2} \frac{1}{a_j} \phi \left( \frac{x_j - y^j_i}{a_j} \right) \, \, d\Omega_j,$$

$k = 0, 1, 2.$

(12)

where $y^j_i$ is the coordinates of the $i$ th particle, and $d\Omega_j$ represents a portion area of the spatial domain associated with the $i$ th particle. In contrast to the RKPM, which introduces only one shape function and one DOF for each particle regardless of the dimension of the spatial domain, the two-dimensional GRKPM introduces three kinds of shape functions as well as three DOFs for each particle with a field function. The first derivatives of GRKPM shape functions are also evaluated from

$$\psi^{(a)}_i(x) = \left[ b^{(a)}_{i0}(x) + b_{i0}(x) + \sum_{j=1}^{2} b^{(a)}_{ij}(x)(x_j - y^j_i) \right]$$

$$\times \prod_{j=1}^{2} \frac{1}{a_j} \phi \left( \frac{x_j - y^j_i}{a_j} \right) \, \, d\Omega_j$$

$$+ b_{i0}(x) + \sum_{j=1}^{2} b_{ij}(x)(x_j - y^j_i)$$

$$\times \prod_{j=1}^{2} \frac{1}{a_j} \phi \left( \frac{x_j - y^j_i}{a_j} \right) \, \, d\Omega_j, \quad n = 1, 2.$$  

(13)

3.2. Application of the GRKPM to the present problem

Let $\Gamma_{con}$ be the steel and concrete interfaces with the corrosion products; see Fig. 1. The conditions on this boundary reads (Kiani and Shodja, 2011b)

$$u^{con}(r, z) - u^{st}(r, z) = \delta(z)$$

$$\sigma^{con}_{zz}(r, z) = \mu z (u^{con}_{zz}(r, z) - u^{st}_{zz}(r, z))$$

on $\Gamma_{corr}$,  

$$\text{on } \Gamma_{corr},$$

where $u^{con}$ and $u^{st}$ are the radial/longitudinal displacements associated with the concrete and steel media, respectively. Since the problem is axisymmetric with respect to the axis of revolution of the steel–rust–concrete composite media, the field quantities are only expressed in terms of $r$ and $z$. Therefore, the displacement fields within the steel and concrete could be discretized as

$$u^{stT}(r, z) = \left[ u^{st}_{rr}(r, z), u^{st}_{rz}(r, z) \right] = \sum_{j=1}^{NPst} \hat{\Psi}^{st}_j (r, z) [u^{st}_{rj}, u^{st}_{zj}],$$

$$\text{on } \Gamma_{corr},$$

$$\text{on } \Gamma_{corr},$$

(14)

where

$$\hat{\Psi}^{st}_j = [\psi^{st}_j]^{[1]} \psi^{st}_j]^{[2]} \hat{\Psi}^{con}_j = [u^{con}_j]^{[1]} \hat{\Psi}^{con}_j = [u^{con}_{rj}, u^{con}_{zj}]^{[1]} \hat{\Psi}^{con}_j = [u^{con}_{rj}, u^{con}_{zj}]^{[2]}$$

(15)

$$\text{on } \Gamma_{corr},$$

$$\text{on } \Gamma_{corr},$$

(16)

where $u^{st}(r, z)$ and $u^{con}(r, z)$ represent the displacement field vectors associated with the steel and concrete, respectively. $NPst$ and $NPcon$ in order are the number of GRKPM particles within the steel...
and concrete media. Eq. (15) could also be rewritten in a more
compact form as follows

\[
\mathbf{u}^{st}(r, z) = \sum_{j=1}^{N_{PS}} \mathbf{u}^{j}_{S}(r, z) \mathbf{u}^{st}_{j},
\]

\[
\mathbf{u}^{con}(r, z) = \sum_{j=1}^{N_{PS}} \mathbf{u}^{con}_{j}(r, z) \mathbf{u}^{con}_{j},
\]

where

\[
\mathbf{u}^{st}_{j} = \begin{bmatrix}
    [\psi_{j}^{[0]} & \psi_{j}^{[1]} & \psi_{j}^{[2]} & 0 & 0 & 0 ]^{T} \\
    0 & 0 & 0 & \psi_{j}^{[0]} & \psi_{j}^{[1]} & \psi_{j}^{[2]} ]^{T}
\end{bmatrix},
\]

\[
\mathbf{u}^{con}_{j} = \begin{bmatrix}
    [u_{j}^{(0)} & u_{j}^{(1)} & u_{j}^{(2)} & u_{j}^{(0)} & u_{j}^{(1)} & u_{j}^{(2)} ]^{T}
\end{bmatrix},
\]

\[
\mathbf{u}^{j}_{S}(r, z) = \begin{bmatrix}
    [u_{j}^{(0)} & u_{j}^{(1)} & u_{j}^{(2)} & u_{j}^{(0)} & u_{j}^{(1)} & u_{j}^{(2)} ]^{T}
\end{bmatrix},
\]

where \(\mathbf{u}^{st}\) and \(\mathbf{u}^{con}\) denote the nodal parameter values of GRKPM shape functions associated with the steel and concrete media, respectively. The displacement conditions according to the location(s) of anode(s) as well as the distribution of corrosion current density, symmetry or asymmetry w.r.t. the plane \(z=0\), take the following form (see Eq. 1(b) and (c))

\[
\mathbf{u}^{st}_{j}(0, z) = 0, -L \leq z \leq L
\]

\[
\mathbf{u}^{con}_{j}(0, z) = \mathbf{u}^{con}_{j}(0, 0), 0 \leq z \leq R_{c},
\]

\[
\mathbf{u}^{con}_{j}(r, 0) = \mathbf{u}^{con}_{j}(0, 0), 0 \leq r \leq R_{c},
\]

in which \(\Gamma_{u}\) denotes the boundary associated with the prescribed displacements. The strain–displacement relations for a cylindrical material of revolution read the following

\[
\varepsilon_{r} = \frac{\partial u_{r}}{\partial r}, \quad \varepsilon_{\theta} = \frac{u_{r}}{r}, \quad \varepsilon_{z} = \frac{\partial u_{z}}{\partial z}, \quad \gamma_{rz} = \frac{\partial u_{r}}{\partial z} + \frac{\partial u_{z}}{\partial r},
\]

where \(\varepsilon_{r}, \varepsilon_{\theta}, \varepsilon_{z}, \) and \(\gamma_{rz}\) are the strain components. Using Eqs. (17) and (20), the strain fields within the steel and concrete are obtained in terms of nodal parameter values as

\[
\varepsilon^{st} = \sum_{j=1}^{N_{PS}} \mathbf{B}^{st}_{j} \mathbf{u}^{st}_{j},
\]

\[
\varepsilon^{con} = \sum_{j=1}^{N_{PS}} \mathbf{B}^{con}_{j} \mathbf{u}^{con}_{j},
\]

where

\[
\varepsilon^{st} = \begin{bmatrix}
    \varepsilon_{r}^{st} & \varepsilon_{\theta}^{st} & \varepsilon_{z}^{st} & \gamma_{rz}^{st}
\end{bmatrix},
\]

\[
\varepsilon^{con} = \begin{bmatrix}
    \varepsilon_{r}^{con} & \varepsilon_{\theta}^{con} & \varepsilon_{z}^{con} & \gamma_{rz}^{con}
\end{bmatrix},
\]

and the stress–strain relations for the steel and concrete media are expressed as

\[
\sigma^{st} = \mathbf{D}^{st} \varepsilon^{st},
\]

\[
\sigma^{con} = \mathbf{D}^{con} \varepsilon^{con},
\]

where

\[
\mathbf{D}^{st} = \begin{bmatrix}
    D^{st}_{rr} & D^{st}_{r\theta} & D^{st}_{rz} \\
    D^{st}_{r\theta} & D^{st}_{\theta\theta} & D^{st}_{r\theta} \\
    D^{st}_{rz} & D^{st}_{r\theta} & D^{st}_{zz}
\end{bmatrix},
\]

\[
\mathbf{D}^{con} = \begin{bmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    0 & 0 & 0
\end{bmatrix},
\]

and the steel medium is assumed to be homogeneous and isotropic,

\[
\sigma^{st} = \varepsilon^{st} \sigma^{st}, \quad \sigma^{con} = \varepsilon^{con} \sigma^{con},
\]

the values of \(E_{r}, E_{\theta}, \) and \(E_{z}\) are determined based on the mechanical behavior of the cracked concrete. Smeared crack approach is assumed for propagation of microcracks. For the under study problem, the principal stresses within the concrete are readily calculated as follows

\[
\sigma_{1} = \frac{\sigma_{rr} + \sigma_{zz}}{2} + \sqrt{\frac{\sigma_{rr}^{2} + (\sigma_{rr} - \sigma_{zz})^{2}}{4}},
\]

\[
\sigma_{2} = \frac{\sigma_{rr} + \sigma_{zz}}{2} - \sqrt{\frac{\sigma_{rr}^{2} + (\sigma_{rr} - \sigma_{zz})^{2}}{4}},
\]

\[
\sigma_{3} = \sigma_{\theta\theta},
\]

as is seen in Eq. (28) the hoop stress is one of the principal stresses, which is due to the axisymmetric conditions of the problem. Thereby, the radial smeared cracks are expected to be configured in the radial directions as the hoop stresses exceed the tensile capacity of the concrete. Additionally, the stress–strain relation of concrete under only compressive loading is commonly given by Park and Paulay (1975)

\[
\sigma_{c} = \left(2 \left( \frac{\varepsilon_{c}}{\varepsilon_{0}} \right) - \left( \frac{\varepsilon_{c}}{\varepsilon_{0}} \right)^{2} \right) E_{c}, \quad \varepsilon_{c} = \varepsilon_{0},
\]

where \(\varepsilon_{0} = 2 f_{c}/E_{0}, f_{c}\) is the uniaxial compressive strength of concrete in kg/cm² and \(E_{0}\) is the initial elastic modulus of concrete in kg/cm²; for a normal concrete, \(E_{0} = 15, 100 \sqrt{f_{c}}\) (Kainya, 1998). In order to simulate the mechanical behavior of concrete in tension,
model with bilinear softening process is employed (CEB-FIP; Bazant and Oh, 1983)

\[
\sigma_i = \begin{cases} 
\frac{f_{ct} - \epsilon_i}{\epsilon_{cr}}, & 0 \leq \epsilon_i < \epsilon_{cr} \\
\frac{f_{ct} - f_{ct, res} - \epsilon_i}{\epsilon_{cr} - \epsilon_{1}}, & \epsilon_{cr} \leq \epsilon_i < \epsilon_{1} \\
f_{ct, res} + \frac{f_{ct, res} - \epsilon_i}{\epsilon_{1} - \epsilon_u}(\epsilon_{1} - \epsilon_u), & \epsilon_{1} \leq \epsilon_i \leq \epsilon_u \\
0, & \epsilon_i \geq \epsilon_u 
\end{cases},
\]

(30)
in which \(\epsilon_i/\sigma_i\) are the corresponding uniaxial tensile strain/stress within the concrete medium and the parameters \(\epsilon_{cr}, \epsilon_{1}, \) and \(\epsilon_u\) are the tensile strains associated with the tensile strength \(f_{ct}\), the residual tensile strength \(f_{ct, res}\), and the ultimate tensile strength \(f_{ct, u} = 0\) of the concrete material, respectively. The values of \(\epsilon_i\) and \(\epsilon_u\) are determined based on the maximum aggregate size of the concrete mix and fracture energy of the concrete (CEB-FIP, 1990; Bazant and Oh, 1983). Furthermore, the average values of \(f_{ct}\) and \(f_{ct, res}\) are usually taken as \(1.725 \sqrt{f_{ct}}\) (Kaynia, 1998) and \(0.15f_{ct}\) (CEB-FIP, 1990) for a normal concrete, respectively. The moduli of concrete in compression and tension are defined, in order, as the secant slopes of the compressive and tensile stress–strain curves.

Herein, the penalty method is employed for enforcing the boundary conditions in Eq. (14). As a result, the modified total strain energy associated with the steel–rust–concrete composite medium is outlined as

\[
\pi = \frac{1}{2} \int_{\Omega^{st}} \varepsilon^{st T} \sigma^{st} d\Omega + \frac{1}{2} \int_{\Omega^{con}} \varepsilon^{con T} \sigma^{con} d\Omega \\
+ \frac{1}{2} \int_{\Gamma_{cor}} [\chi_1(u^{con} - u^{st}) - \delta(z)]^2 \ d\Gamma \\
+ \frac{1}{2} \int_{\Gamma_{cor}} \sigma_u (u^{st} - \tilde{u}^{st}) (u^{con} - \tilde{u}^{con}) d\Gamma \\
+ \frac{1}{2} \int_{\Gamma_{con}} \sigma_u (u^{con} - \tilde{u}^{con}) (u^{con} - \tilde{u}^{con}) d\Gamma,
\]

(31)
where \(\Omega^{st}\) and \(\Omega^{con}\) represent in order the steel and concrete domains, \(\Gamma_{cor}\) is the prescribed displacement boundary for the [.] medium such that \(\Gamma_u = \Gamma_{nis} \cup \Gamma_{con} \times \chi\) and \(\sigma_u\) are the penalty factors, and \(\tilde{u}^{st}\) denotes the prescribed displacement associated with the

\[
\mathbf{K}_u \mathbf{u} = \mathbf{f}_{cor},
\]

\[
[\mathbf{K}_u]_{ij} = \begin{bmatrix} 
\int_{\Omega^{st}} \mathbf{B}_i^{st T} \mathbf{D}^{st} \mathbf{B}_j^{st} d\Omega & 0 \\
0 & \int_{\Omega^{con}} \mathbf{B}_i^{con T} \mathbf{D}^{con} \mathbf{B}_j^{con} d\Omega 
\end{bmatrix}
\]

\[+
\int_{\Gamma_{cor}} \mathbf{B}_i^{st} \mathbf{q}^{st T} \psi_j^{st} \ d\Gamma \\
- \int_{\Gamma_{con}} \mathbf{B}_i^{con} \mathbf{q}^{con T} \psi_j^{con} \ d\Gamma
\]
It should be noted that $f_{\text{corr}}$ is a nonlinear vector function in terms of the nodal parameter values of particles. The Newton’s method is employed for solving the nonlinear set of equations in Eq. (32),

$$K(\mathbf{u} - \mathbf{u}_0) = -\mathbf{R}(\mathbf{u}),$$

where $\mathbf{u}$ is the previous value of $\mathbf{u}$ in the iteration process, and other matrices are as:

$$\mathbf{R}(\mathbf{u}) = \mathbf{K}_c - \mathbf{f}_{\text{corr}},$$

$$K_c = K_c - \frac{X_{c_4}}{N_{\text{corr}}/\Psi_1}\int_{\Omega_{\text{corr}}} \mathbf{P} \mathbf{d} \Gamma + \frac{1}{2} \beta \lambda \chi \left(1 + \epsilon_{\text{corr}}\right) Q \mathbf{d} \Gamma,$$

in which

$$P_i = \begin{pmatrix} 0 & -\lambda_i \psi_i \Sigma_i \left(D_{\varepsilon_0} \psi_i \partial \psi_i / \partial x - D_{\varepsilon_0} \psi_i \partial \psi_i / \partial y + D_{\varepsilon_0} \psi_i \partial \psi_i / \partial z \right) \\ 0 & 0 & 0 \end{pmatrix},$$

$$Q_{r_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

$$\lambda = \left(R_i + \frac{1}{\pi} \left(V_i - V_i - \lambda \beta \theta \right) \right)^{-1/2} \left(R_i - R_i \right)^{1/2},$$

the unknown nodal parameter values in Eq. (34) are determined by applying iterative operations.

4. Results and discussion

In all of the GRKPM analyses, as shown in Fig. 2, 8×51 and 15×51 exponentially distributed particles within the steel and concrete media are used, respectively. 3×3 Gaussian points for each computational cell are considered. The pertinent shape functions are calculated using linear base function, third order spline window function, and dilation parameters, $a_1 = a_2 = 2$. In this study, the penalty factors are set equal to 10$^{5}$Est, and $\mathbf{u}^{st} = \mathbf{u}^{\text{corr}} = \mathbf{0}$ on $\Gamma_u$.

In the following parts, the capabilities of the GRKPM in predicting the interesting parameters of the problem under consideration are inspected by comparison of its results with those of the analytical solution in the companion work (Kiani and Shodja, 2011b). Thereafter, using the present numerical model, the effects of various parameters associated with the properties of concrete and rust on the interested factors are scrutinized in some detail.

4.1. Comparison of GRKPM and analytical solutions

Consider a cylindrical reinforced concrete member under a macrocell corrosion with the following data: $R_i = 0.7$ (cm), $R_{\text{corr}} = 6$ (cm), $L = 100$ (cm), and $L_{\text{corr}} = 0.5 L_i$. $\ell_{\text{corr}}(z) = 10 \exp (-8z/L_0)$ ($\mu A/cm^2$). Moreover, the material properties of steel, rust, and concrete of the under study specimen are as follows: $f_c = 250$ (kg/cm²), $E_0 = 15$ (100 $\sqrt{f_c}$), $E_{\text{d}} = 2 \times 10^{6}$ (kg/cm²), $V_{\text{r}} = 0.17$, $V_{\text{i}} = 0.3$, $\eta_{\text{corr}} = 7$, $K_{\text{corr}} = 1.5$, $100 \times 10^4$ (kg/cm²), $\alpha = 20.9$, and $\sigma_{\text{r}} = 0.622$. In Fig. 3(a)–(c), the predicted radial displacement and stress as well as the hoop stress at the interfaces of the steel and concrete with the rust are presented at different times via the proposed numerical and analytical schemes. Due to the existing symmetry of the applied corrosion current density about the plane $z = 0$, the results have been only provided for $0 \leq z \leq L$. As it is seen in Fig. 3(a)–(c), there is a reasonably good agreement between the predicted results by GRKPM and those of analytical solution at different times.

4.2. Validation of the proposed model

In order to ensure the accuracy of the suggested model, the predicted results are compared with those of the available laboratory data as well as those of other models. In doing so, the experimentally observed data of Torres-Acosta and Sagues (1998) for three cylindrical specimens under uniform corrosion are given in Table 1. These specimens have been exposed to outdoor marine environments for about five years. In Table 1, $C \Omega_{\text{ave}}$ denotes the average crack width openings at the surface of the concrete and $X$ represents the corrosion penetration of the rebar (note: corrosion penetration is defined as the reduction of rebar radius due to the rebar corrosion). In Table 1, the values of these parameters, $C \Omega_{\text{ave}}$, and $X$, have been reported at the end of the experimental period of 59 months. $X_{t_{1}}$ and $X_{t_{2}}$ are the corrosion penetration associated with $\Omega_{\text{ave}} = 0.05$ mm and 0.1 mm, respectively. Based on the linear relation between $X$ and $\Omega_{\text{ave}}$ (Andrade et al., 1993), the computed values of $X_{t_{1}}$ and $X_{t_{2}}$ by Bhargava et al. (2006) are provided in Table 2. Additionally, the predicted results by Bhargava et al. (2006) based on the models of other researchers, and the results of the GRKPM are presented in Table 2. The predicted results by the proposed model are provided in the case of $\beta = 0$ and $\sigma_{\text{r}} = 0.622$. As it is seen in Table 2, the obtained results by Bhargava et al. (2006) based on the model of Liu and Weyers (1998) could not predict the correct values of corrosion penetration for all design specimens. On the other hand, the proposed model by Bhargava et al. (2006) predicts the identical values for both values of corrosion penetration. It is mainly because of this fact that in the proposed model by Bhargava et al. (2006), the time to cover cracking is established based on the exceedance of hoop stress at the concrete surface from the tensile strength of the concrete. As a result, this model could not predict the needed information based on the crack width openings. Moreover, the predicted values by Bhargava et al. (2006) are generally close to the evaluated values of $X_{t_{2}}$ from the experimentally observed data. The predicted values of corrosion penetration according to the model of Torres-Acosta and Sagues (1998) are also given based on the linear elastic fracture mechanics approach (LEFMA) and the volumetric approach (VA). The predicted values of corrosion penetration corresponding to the concrete cover cracking by Torres-Acosta and Sagues (1998) based on the LEFMA are somehow close to the computed values of $X_{t_{2}}$ from experimentally observed data. As it is seen in Table 2, the discrepancies between the predicted values of corrosion penetration by the proposed model and those evaluated from experimental data are commonly lower in compare to other models.

4.3. Parametric studies

In this part, the effects of the parameters associated with the mechanical behavior of both concrete and rust on the time of surface cracking ($T_{s}$), maximum amount of weight loss of rebar per unit area up to the time of surface cracking ($W_s$), and maximum values of both crack width openings and radial stress at the interface of rust and concrete, respectively, denoted by $C \Omega_{\text{ave}}$ and $\sigma_{\text{r}}$, are examined for different values of $\beta$. For this purpose, consider a cylindrical reinforced concrete specimen under an axisymmetrically macrocell corrosion with the given data: $R_i = 0.8$ (cm), $C = 2.7$ (cm), $L = 100$ (cm), $L_{\text{corr}} = 0.5 L_i$, and $\ell_{\text{corr}}(z) = 3.75 \exp (-8z/L_0)$ ($\mu A/cm^2$). In order to investigate the effects of the above-mentioned parameters, the following normalized parameters are introduced: $T_{cr} = T_{cr}/T_{cr}$, $W_{s_{\text{max}}} = W_{s_{\text{max}}}/W_{s_{\text{max}}}$, $\sigma_{\text{r}_{\text{max}}} = \sigma_{\text{r}_{\text{max}}}/\sigma_{\text{r}_{\text{max}}}$, $C \Omega_{\text{cor}} = C \Omega_{\text{cor}}/C \Omega_{\text{cor}}$, $\rho_{\text{d}} = \rho_{\text{d}}/\rho_{\text{d}}$, $\epsilon_{\text{corr}} = \epsilon_{\text{corr}}/\epsilon_{\text{corr}}$, $\lambda_{\text{corr}} = \lambda_{\text{corr}}/\lambda_{\text{corr}}$, and $n_{\text{corr}} = n_{\text{corr}}/n_{\text{corr}}$ in which $\epsilon_{\text{corr}} = 0.573$, $\alpha = 2.09$, $K_{\text{corr}} = 7$ (GPa), and $n_{\text{corr}} = 7$. The parameters $T_{cr}$, $W_{s_{\text{max}}}$, $\sigma_{\text{r}_{\text{max}}}$, and $C \Omega_{\text{cor}}$ are the predicted values of the proposed numerical model for the above-mentioned RC specimen with $\nu_{\text{r}} = 0.17$, $E_0 = 15$, 100 $\sqrt{f_c}$, $f_{\sigma} = 1.725 \sqrt{f_c}$, and $f_{\text{cr}, \text{res}} = 0.15f_{\sigma}$. in the
case of $\beta=0$. The predicted values of these parameters are $T_{cr}=264$ (day), $W_{s,\text{max}}=0.0615$ (mg/mm²), $\sigma_{r,\text{max}}=59.92$ (kg/cm²), and $\text{CWO}_{\text{cr, max}}=0.0721$ (mm). For evaluating $\text{CWO}_{\text{cr}}$, the readers are referred to Kiani and Shodja (2011).

Fig. 4(a)–(d) presents the normalized time to surface cracking ($\tilde{T}_{cr}$) and the normalized maximum amount of consumed rebar per unit area ($\tilde{W}_{s,\text{max}}$) in terms of the normalized parameters associated with the mechanical behavior of concrete such as $\tilde{v}_{fr}$, $\tilde{E}_{fr}$, and $\tilde{f}_{cr,\text{res}}$, respectively. As it is obvious in Fig. 4(a)–(d), variation of the Poisson’s ratio of concrete has the less effect on the variation of the predicted values of both $\tilde{T}_{cr}$ and $\tilde{W}_{s,\text{max}}$. Fig. 4(b)–(d) shows that the predicted values of $\tilde{T}_{cr}$ and $\tilde{W}_{s,\text{max}}$ increase with $\tilde{f}_{cr}$ and $\tilde{f}_{cr,\text{res}}$, and decrease dramatically with $\tilde{E}_{0}$, irrespective of the value of $\beta$. Among all of the considered parameters associated with the mechanical behavior of concrete, $\tilde{f}_{cr}$ and $\tilde{E}_{0}$ are the most influential parameters on $\tilde{T}_{cr}$ and $\tilde{W}_{s,\text{max}}$.

Fig. 5(a)–(d) demonstrates the predicted values of $\tilde{T}_{cr}$ and $\tilde{W}_{s,\text{max}}$ in terms of $\tilde{r}_{m}$, $\tilde{a}$, $\tilde{K}_{\text{corr}}$, and $\tilde{n}_{\text{corr}}$, respectively. As it is seen in Fig. 5(a)–(d), for all values of $\beta$, the predicted values of $\tilde{T}_{cr}$ and $\tilde{W}_{s,\text{max}}$ increase with $\tilde{r}_{m}$ and $\tilde{n}_{\text{corr}}$, however, their values decrease with $\tilde{a}$ and $\tilde{K}_{\text{corr}}$. Among the parameters pertinent to the rust, $\tilde{a}$ and $\tilde{n}_{\text{corr}}$ have more influence on the predicted values of $\tilde{T}_{cr}$ and $\tilde{W}_{s,\text{max}}$ than the others. As it is expected, the predicted values of $\tilde{T}_{cr}$ and $\tilde{W}_{s,\text{max}}$ increase with $\beta$ for all parameters associated with the mechanical behaviors of concrete and rust (see Figs. 4 and 5). It is worth mentioning that the general trends of $\tilde{T}_{cr}$ as a function of $\tilde{E}_{0}, \tilde{f}_{cr,\text{res}}$, and $\tilde{f}_{cr}$ are in line with the works of other researchers (Bhargava et al., 2006; Pantazopoulou and Papoula, 2001) in the case of uniform corrosion of rebars with $\beta=0$.

Fig. 6(a)–(d) illustrates the normalized parameters of maximum value CWOs at the time of surface cracking, $\text{CWO}_{\text{cr, max}}$, and the maximum value of radial stress at the interface of rust and concrete, $\tilde{\sigma}_{r,\text{max}}$, in terms of $\tilde{v}_{fr}$, $\tilde{E}_{0}, \tilde{f}_{cr}$, and $\tilde{f}_{cr,\text{res}}$. The obtained results are plotted for three levels of $\beta$ (i.e., 0, 0.1, and 0.2). Fig. 6(a) indicates that increasing the normalized Poisson’s ratio of concrete leads to a slight increase and decrease in the predicted values of $\text{CWO}_{\text{cr, max}}$ and $\tilde{\sigma}_{r,\text{max}}$, respectively. Fig. 6(b) shows that the predicted value of $\text{CWO}_{\text{cr, max}}$ decreases with $\tilde{E}_{0}$, whereas the predicted value of

<table>
<thead>
<tr>
<th>Design specimen</th>
<th>$D_{rs}$ (mm)</th>
<th>$C$ (mm)</th>
<th>$f_{cr}$ (MPa)</th>
<th>$\text{CWO}_{\text{cr}}$ (mm)</th>
<th>$X$ (mm)</th>
<th>$i_{\text{corr}}$ (µA/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>10</td>
<td>34</td>
<td>35</td>
<td>0.55</td>
<td>0.337</td>
<td>5.7</td>
</tr>
<tr>
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<td>37</td>
<td>0.55</td>
<td>0.299</td>
<td>2.7</td>
</tr>
<tr>
<td>C₃</td>
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<td>34</td>
<td>37</td>
<td>0.6</td>
<td>0.437</td>
<td>6.18</td>
</tr>
</tbody>
</table>

**Table 1**

Experimentally observed data for cylindrical specimens by Torres-Acosta and Sagues (1998).

<table>
<thead>
<tr>
<th>Design specimen</th>
<th>$X_{r_{1}}$ (mm)</th>
<th>$X_{r_{2}}$ (mm)</th>
<th>$r_{m}$</th>
<th>$r_{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>0.0613</td>
<td>0.0307</td>
<td>0.0037</td>
<td>0.0047</td>
</tr>
<tr>
<td>C₂</td>
<td>0.0544</td>
<td>0.0272</td>
<td>0.0038</td>
<td>0.0048</td>
</tr>
<tr>
<td>C₃</td>
<td>0.0728</td>
<td>0.0364</td>
<td>0.0038</td>
<td>0.0048</td>
</tr>
</tbody>
</table>

**Table 2**

Comparison of the evaluated corrosion penetration corresponding to $\text{CWO}_{\text{cr}}=0.05$ and 0.1 mm with those computed by Bhargava et al. (2006) and the present work.
Fig. 4. Normalized time to concrete cover cracking ($t_{cr}$) and normalized amount of consumed rebar ($W_r$) for an axisymmetric macrocell corrosion in terms of: (a) $t_{cr}$, (b) $E_0$, (c) $f_{ct}$, and (d) $W_{n, max}$ for different values of $\beta$: $\beta = 0, \beta = 0.1, \beta = 0.2$. — plot of $T_{cr}$, and —— plot of $W_r$.

Fig. 5. Normalized time to concrete cover cracking ($t_{cr}$) and normalized amount of consumed rebar ($W_r$) for an axisymmetric macrocell corrosion in terms of: (a) $t_{cr}$, (b) $\tilde{a}$, (c) $\tilde{k}_{corr}$, and (d) $\tilde{h}_{corr}$ for different values of $\beta$: $\beta = 0, \beta = 0.1, \beta = 0.2$. — plot of $T_{cr}$, and —— plot of $W_r$.

$\bar{E}_{r, max}$ increases with $E_0$. Fig. 6(c) and (d) shows that increasing $f_{ct}$ and $f_{ct, est}$ results in an increase in the predicted values of both $CWO_{tr, max}$ and $\bar{E}_{r, max}$. A brief assessment of the slope of the depicted graphs in Fig. 6(a)–(d) reveals that the variation of the parameters $E_0$ and $f_{ct}$ have more effect on the variation of $CWO_{tr, max}$ and $\bar{E}_{r, max}$ in comparison to other considered parameters.

Further studies reveal that the parameters associated the rust properties have very slight effect on the predicted values of $CWO_{tr, max}$ and $\bar{E}_{r, max}$, regardless of the value of $\beta$. Such studies also reveal that the value of $\beta$ has a very trivial effect on the predicted values of $CWO_{tr, max}$ and $\bar{E}_{r, max}$. It is emphasized here that these results are identical to the obtained results by Shodja et al. (2010a), in which study of the effects of general rebar corrosion on RC structures was of concern. As a result, for the sake of conciseness, the plotted results are not demonstrated herein.
5. Conclusions

A nonlinear mathematical model for studying the response of RC structures due to the axisymmetric macrocell corrosion of reinforcements is proposed. The mechanical behaviors of steel, rust, and concrete are presumed to be linear isotropic, power law stress–strain relation, and nonlinear anisotropic with post-cracking softening, respectively. Appropriate boundary conditions at the interfaces of rust with the steel and concrete are imposed. The surface and both ends of the RC member are assumed to be traction free. Before microcrack propagation within the concrete, the displacement and stress fields at the interfaces of rust with the steel and concrete obtained using GRKPM are in reasonable agreements with the corresponding analytical solutions provided in the companion paper. A parametric study is also performed to explore the effects of the parameters associated with the mechanical behavior of concrete and corrosion products on the time of surface cracking ($T_{cr}$), the maximum amount of consumed rebar per unit area at the time of surface cracking ($W_{s,max}$), maximum crack width openings at the interface of rust and concrete at the time of surface cracking ($\text{CWO}_{cr,max}$), and maximum value of radial stress at the interface of rust and concrete ($\sigma_{rr,max}$). The obtained results reveal that among all the considered parameters associated with the mechanical behavior of concrete, the variation of tensile strength and initial elastic modulus of concrete have the most influence on the variations of $T_{cr}$, $W_{s,max}$, $\text{CWO}_{cr,max}$, and $\sigma_{rr,max}$; however, Poisson’s ratio of the concrete has a trivial influence on the predicted values of $T_{cr}$ and $W_{s,max}$. Moreover, $T_{cr}$ and $W_{s,max}$ increase with $r_m$ and $n_{corr}$ irrespective of the value of $\beta$, but decrease as the values of $\alpha$ and $K_{corr}$ increase. Among all of the parameters pertinent to the properties of rust, $\alpha$ and $n_{corr}$ have the most influence on the predicted values of $T_{cr}$ and $W_{s,max}$. The obtained results indicate that increasing the tensile and the residual tensile strength of the concrete leads to an increase in the predicted values of $\text{CWO}_{cr,max}$ and $\sigma_{rr,max}$. Furthermore, the predicted value of $\text{CWO}_{cr,max}$ decreases with $E_0$, whereas the predicted value of $\sigma_{rr,max}$ increases with $E_0$.

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