Vibration behavior of simply supported inclined single-walled carbon nanotubes conveying viscous fluids flow using nonlocal Rayleigh beam model

Keivan Kiani *

Department of Civil Engineering, Islamic Azad University, Chalous Branch, Chalous, Mazandaran, Iran

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A mathematical model is proposed to investigate the dynamic response of an inclined single-walled carbon nanotube (SWCNT) subjected to a viscous fluid flow. The tangential interaction of the inside fluid flow with the equivalent continuum structure (ECS) of the SWCNT is taken into account via a slip boundary condition. The dimensionless equations of motion describing longitudinal and lateral vibrations of the fluid-conveying ECS are obtained in the context of nonlocal elasticity theory of Eringen. The unknown displacement fields are expressed in terms of admissible mode shapes associated with the ECS under simply supported conditions with immovable ends. Using Galerkin method, the discrete form of the equations of motion is derived. The time history plots of the normalized longitudinal and transverse displacements as well as the nonlocal axial force and bending moment of the midspan point of the SWCNT are provided for different levels of the fluid flow speed, small-scale parameter, and inclination angle of the SWCNT. The effects of small-scale parameter, inclination angle, speed and density of the fluid flow on the maximum dynamic amplitude factors of longitudinal and transverse displacements as well as those of nonlocal axial force and bending moment of the SWCNT are then studied in some detail.

1. Introduction

Since the discovery of carbon nanotubes (CNTs) in 1991, they have attracted attention of scientific and industrial communities of various disciplines because of their tremendous properties. To date, no preceding material has exhibited such exceptional physical and chemical properties. It has been validated both experimentally and theoretically that CNTs have extremely high strength and stiffness [1,2] as well as extraordinarily high thermal and electrical conductivities [3–5]. These unprecedented properties provide CNTs as perfect materials for a wide range of applications [6–8]. CNTs could be efficiently utilized as nanopipes for transporting of nanoparticles [9], nanoconveyers for passing on the flow of fluids [10–12], or nanocarriers for delivering of drugs [13,14] to specific areas within the body with of cells’ dimensions. The later application would be of great importance in medical sciences because it would result in a reduction of the general side effects of drugs and allowance of more efficient use of drugs [15]. It is worth mentioning that the flow of water molecules inside CNTs has been modeled via molecular dynamics (MD) [16,17]. The obtained results offered them as small molecule transporters. On the other hand, simulations [11,10,18] and experimentally observed data [12,19] of mass flow inside CNTs show high transport rates with respect to equivalent channels. This is mainly related to frictionless nature of inner surface of CNTs reported by...
In a related work, Yoon et al. [35] examined the influence of moving fluid on free vibration and flow-structure interaction (FSI) model for the flow of fluids inside CNTs. It was concluded that in a fluid-CNT system, the motion of the CNT plays a crucial role in the fluid flow. However, dynamic displacements and the generated forces within the movement of the fluids inside CNTs, in which will be introduced to the readers, focused on the overall vibration induced structural instability of SWCNTs using the classical Euler–Bernoulli beam model. It was shown that the fluids flow instability would occur and vibration magnifies with amplitude of the SWCNT. Lee and Chang [37] analyzed the influence of water flow velocity, chirality of the SWCNT, diameter and aspect ratio of the SWCNT, stiffness of the surrounding elastic medium, small-scale parameter, and end conditions on the natural frequencies were then examined. In order to discrete the resulting equations of motion in the spatial domain, finite element method was exploited. The effects of water flow velocity on the flexural frequencies and the corresponding mode shapes of vibration of fluid-conveying SWCNTs with clamped ends. For this purpose, the Euler–Bernoulli beam theory in the context of nonlocal continuum mechanics of Eringen was used to study transverse vibration of the nanotube structure under fluids flow. In another work, Lin and Qiao [38] investigated vibration and instability of fluid-conveying SWCNTs using Euler–Bernoulli beam model. To this end, the differential quadrature method (DQM) was implemented to discretize the equation of motion in the spatial domain. By solving a generalized eigenvalue problem, the resonant frequencies were then calculated for SWCNTs with clamped and simply supported ends. The obtained results showed the possibility of occurrence of coupled-mode flutter at higher levels of fluid flow velocity. Ghavanloo et al. [39] studied vibration and instability of SWCNTs conveying fluid embedded in a viscoelastic medium under different boundary conditions. By exploiting the Euler–Bernoulli beam theory, the problem was investigated in the context of classical continuum theory via finite element method. The effects of damping coefficient of the surrounding media and the fluid flow velocity on the resonant frequencies of the nanostructure were explored in some detail. Lee and Chang [40] inspected the influences of the nonlocal effect, viscosity effect, aspect ratio, and elastic medium constant on the fundamental frequency of a viscous-fluid-conveying SWCNT embedded in an elastic medium. The results indicated that the effect of the small-scale parameter on the frequency of the SWCNT becomes more obvious for lower levels of the fluid flow velocity. Additionally, the effect of fluid flow viscosity on the frequency of SWCNT would be significant as the fluid flow velocity increases. Recently, Hashemnia et al. [41] explored vibration of SWCNTs conveying water by taking into account water-carbon bond potential energy and nonlocal effect. In that work, water was considered as a discrete medium. The interaction between carbon atoms and water molecules was modeled by employing the carbon-water bond potential energy. The equations of motion of the SWCNT conveying water were then constructed on the basis of the nonlocal continuum mechanics of Eringen. In order to discrete the resulting equations of motion in the spatial domain, finite element method was exploited. The effects of water flow velocity, chirality of the SWCNT, diameter and aspect ratio of the SWCNT, stiffness of the surrounding elastic medium, small-scale parameter, and end conditions on the natural frequencies were then examined. Combination effects of compressive axial force and internal moving fluid flow on the nonlinear vibration and instability of SWCNTs embedded in an elastic medium were studied by Rasekh and Khadem [42]. Using the multiple-scale perturbation technique, the relations between the nonlinear amplitudes and frequencies of SWCNTs conveying internal fluid flow are
established for a SWCNT with pinned-pinned and immovable-movable ends. In a recent work, Wang [43] proposed a model for assessing surface effects on the free vibration of fluid-conveying CNTs. Using Euler–Bernoulli beam theory, the surface effects were incorporated into the formulations of the problem which were established in the framework of the nonlocal continuum mechanics of Eringen. The results showed that the natural frequency and critical flow velocity of CNTs generally increase as the surface effect with positive elastic constant or positive residual surface tension becomes highlighted. Wang [44] also scrutinized wave propagation within SWCNTs conveying fluid flow on the bases of strain and stress gradient elasticity theories. Both theories were then applied to the equivalent continuum structure associated with the SWCNT under moving fluid flow, which is modeled based on the Euler–Bernoulli and the Timoshenko beam theories. The obtained results revealed that the effect of internal fluid flow on the phase velocity of upstream-traveling wave would be significant for moderately low levels of the wave number. In another study, Wang [45] suggested a modified nonlocal beam model to examine instability of SWCNTs conveying fluid flow. In the newly developed model, the higher-order nonlocal terms were incorporated into the axial stress within the nanotube. In contrast to the partial nonlocal model, for a given speed of the fluid flow, the new model explained that both the natural frequencies and the critical fluid speed would magnify with the small-scale parameter.

Regarding nonlinear analysis of fluid-conveying CNTs, Soltani and Farshidianfar [46] explored vibration SWCNTs embedded in an elastic medium using the Pasternak model and the nonlinear nonlocal Euler–Bernoulli beam theory. The nonlinear natural frequencies were obtained using the energy balance method. The effects of fluid-flow velocity, nonlocal parameter, and initially axial force on the resonant frequencies were addressed. Kuang et al. [47] examined dynamic response of DWCNTs conveying fluid accounting for the nonlinearities of the geometric of the DWCNT as well as the van der Waals interaction forces. The obtained results indicated that the effect of geometric nonlinearity on the amplitude-frequency behavior of DWCNTs could be ignored if both of the abovementioned nonlinearities are taken into account. In another work, Jannesari et al. [48] used a nonlinear nonlocal Donnell thin shell model to study viscous and nonlocal effects on the structural stability of SWCNTs. The obtained results displayed that the viscosity has significant effect for SWCNTs of diameters 40 nm, but may be neglected for those with diameters lower than 7 nm. Herein, we restrict our study to small longitudinal displacement and dynamic deflection of inclined SWCNTs. However, further studies are still required to determine the limitations of linear analyses for inclined SWCNTs conveying viscous fluids flow.

A scrutiny of the literature reveals that the dynamic instability of SWCNTs conveying fluid flow has been fairly well studied in the context of various continuum mechanics; however, no inclusive study on the dynamic response of such nanotube structures due to viscous fluids flow has been reported so far. In order to bridge this scientific gap, longitudinal and transverse vibrations of an inclined slender SWCNT acted upon by a viscous fluid flow are explored in this article. To this end, an equivalent continuum structure (ECS) associated with the inclined slender SWCNT is considered. The slender ECS is modeled according to the Rayleigh beam model. By taking into account the interaction forces of the fluid flow on the inclined ECS, the equations of motion of the ECS are obtained in the framework of the nonlocal continuum theory of Eringen. The nondimensional equations of motion are discretized in the space and time domains using Galerkin method and generalized Newmark–β approach, respectively. Admissible mode shape functions associated with the inclined SWCNT with pinned and immovable ends are selected for the displacement fields of the SWCNT. The influences of small-scale parameter, inclination angle, speed and density of the fluid flow on the maximum values of dynamic displacements as well as those of nonlocal axial force and nonlocal bending moment within the inclined slender SWCNT are then scrutinized in some detail.
2. Model formulations

Consider a simply supported SWCNT conveying viscous fluid flow with a small amplitude vibration as shown in Fig. 1. The inclination angle of the revolution’s axis of the SWCNT with respect to the horizon plane is $\alpha$. At the time $t = 0$, the SWCNT is at the rest condition under initially applied tension load, $T_0$, and the fluid starts to flow from the left end of the SWCNT with a constant average speed $c$ under pressure $P$. The viscous fluid flow is assumed to be a uniform flow. The location of the front of the fluid flow is represented by $x_f$ in the considered Cartesian coordinates in which $x_f = ct$ (see Fig. 1). The density, the area, and the second moment of the area of the fluid are denoted by $\rho_f$, $A_f$, and $I_f$, respectively. Due to the mass weight of the viscous fluid flow and its interaction with the SWCNT, the SWCNT starts to vibrate in the longitudinal and transverse directions. The slender SWCNT conveying fluid flow is modeled by an equivalent continuum structure (ECS). The ECS is an elastic solid hollow cylinder whose most of its frequencies are identical to those of the SWCNT. The inner and outer radius of the ECS are also denoted by $r_s$ and $r_m$, where $r_s$ and $r_m$ in order represent the thickness and the mean radius of the ECS. The density, the cross-sectional area, the second moment of area, Young’s modulus, and the length of the ECS are also denoted by $\rho_s$, $A_s$, $I_s$, and $L$, respectively. The length to the diameter ratio of the ECS associated with the SWCNT is also assumed to be large enough that the hypotheses of the Rayleigh beam theory would be valid.

The mechanism of the fluid-nanostructure interaction is represented by the forces exerted by the viscous fluid flow on the inner surface of the ECS. On the fluid-ECS interface, resultant contact forces are resolved into the normal and tangential forces per unit length, respectively denoted by $f_n$ and $f_t$, acting on the inner surface of the ECS (see Figs. 2(a) and 2(b)).

The longitudinal and lateral displacements of the ECS are denoted by $u_s(x, z, t)$ and $u_t(x, z, t)$, respectively. Using the hypotheses of Rayleigh beam theory, the displacements, the only strain ($\varepsilon_{ss}$), and the only stress ($\sigma_{ss}$) of the ECS are,

\[
\begin{align*}
\varepsilon_{ss}(x, z, t) &= u_x(x, t) - z w_x(x, t) + \frac{T_0}{E_b A_b} x, \\
\sigma_{ss}(x, z, t) &= E_b \left( u_x(x, t) - z w_x(x, t) + \frac{1}{2} (w_x(x, t))^2 \right) + \frac{T_0}{A_b}.
\end{align*}
\]

where $u(x, t)$ is the longitudinal displacement of the neutral axis of the ECS, $w(x, t)$ is the transverse displacement field of the ECS, the subscript $ss$ denotes the derivative with respect to $x$. The deflection and rotation of the ECS lead to the transverse displacement and rotation of the fluid element as well (see Fig. 2(b)). Therefore, the inertial effects of the transverse displacement and rotation of the fluid flow should be appropriately incorporated into the formulation. These effects would be more obvious for those SWCNTs undergo larger deflections and convey fluids flow with higher levels of speed. At an arbitrary time $t$, the kinetic energy of the ECS plus to that of the fluid flow due to the rotation of the ECS, $T(t)$, the elastic strain energy of the ECS, $U(t)$, and the done work by the interactional forces exerted on the ECS, $W(t)$, are expressed as follows

\[
\begin{align*}
T(t) &= \frac{1}{2} \int_0^L \left( \rho_b \left( A_b u_t^2 + A_b \dot{w}_x^2 + I_b \ddot{w}_x^2 \right) + \rho_f I_f (w + c w_x)^2 \right) dx, \\
U(t) &= \frac{1}{2} \int_0^L E_b \left( A_b \left( u_x + \frac{1}{2} w_x^2 + \frac{T_0}{E_b A_b} \right) + I_b w_{xx}^2 \right) dx, \\
W(t) &= \int_0^L \left( f_n \cos \theta - f_t \sin \theta \right) u + \left( f_t \sin \theta + f_n \cos \theta \right) w \right) dx,
\end{align*}
\]

where $\theta$ is the rotation of the ECS. Using Hamilton’s principle, $\int_0^L (\delta T - \delta U + \delta W) dt = 0$, the strong form of equations of motion of the ECS is obtained as in the following form

![Fig. 2. (a) An infinitesimal deformed element of the ECS with its applied forces; (b) An infinitesimal fluid element with its acting forces.](image-url)
\[ \rho_b A_b \ddot{u} - N_{b,x} - f_i \cos \theta + f_i \sin \theta = 0, \]  
\[ \rho_b (A_b \dot{w} - I_b \dot{w}_{xx}) - \rho_f I_f (\ddot{w} + 2c \dot{w}_x + c^2 \dot{w}_{xx})_{xx} - (N_b \dot{w}_x)_x - M_{b,xx} - f_i \sin \theta + f_i \cos \theta = 0, \]  
and the boundary conditions involve specifying one quantity of each of the following pairs at \( x = 0 \) and \( x = l_b \).

\[ u \text{ or } N_b = E_b A_b \left( u_x + \frac{1}{2} w_x^2 \right) + T_0, \]  
\[ w \text{ or } Q_b = E_b A_b w_{xxx} - N_b w_x, \]  
\[ w_x \text{ or } M_b = -E_b A_b w_{xx}. \]

where \( N_b, Q_b, \) and \( M_b \) are the resultant axial force, shear force, and bending moment within the ECS, respectively (see Fig. 2(a)). In the context of small deformation, we have \( \sin \theta \approx \theta = w_x \). By substituting this relation into Eqs. (3a) and (3b) and by neglecting the products of derivatives of displacements, Eqs. (3a) and (3b) are now modified to

\[ \rho_b A_b \ddot{u} - N_{b,x} - f_i + f_n w_x = 0, \]  
\[ \rho_b (A_b \dot{w} - I_b \dot{w}_{xx}) - \rho_f I_f (\ddot{w} + 2c \dot{w}_x + c^2 \dot{w}_{xx})_{xx} - T_0 w_{xx} - M_{b,xx} - f_i w_x - f_n = 0, \]

with the following boundary conditions

\[ u \text{ or } N_b = E_b A_b u_x + T_0, \]  
\[ w \text{ or } Q_b = E_b A_b w_{xxx} - T_0 w_x, \]  
\[ w_x \text{ or } M_b = -E_b A_b w_{xx}. \]

By taking an infinitesimal element of the fluid flow with length \( dx \) (see Fig. 2(b)), force balances for the fluid inside ECS yields the governing equations of the fluid flow as follows

\[ (PA_f)_x + f_i - f_n w_x + \rho_f A_f (g_x - g_x w_x + \ddot{u} + 2c \dot{u}_x + c^2 \dot{u}_{xx}) = 0, \]  
\[ (PA_f w_x)_x + f_i w_x + f_n - \rho_f A_f (g_x + g_x w_x - \ddot{w} - 2c \dot{w}_x - c^2 \dot{w}_{xx}) = 0, \]

where \( g_x = g \sin \alpha \), \( g_x = g \cos \alpha \), and \( g \) is the applied gravitational acceleration.

For fluids flow inside nanochannels, the classical Navier–Stokes equations would be no longer valid due to the small-scale effects. Further, the classical boundary conditions (i.e., no-slip boundary conditions) would also fail since the fluid’s molecules could easily move at the vicinity of the wall; it implies that the velocity of the fluid flow is not necessary zero at the its interface with the adjacent wall. Such a phenomenon is explained by the following slip boundary condition [49]:

\[ v_f - v_w = \frac{2 - \sigma_v}{\sigma_v} \frac{Kn}{1 - bKn} \frac{\partial v_f}{\partial r}, \quad r = r_1, \]

where \( r \) is the radial distance of the fluid’s point from the centerline of the ECS, \( v_f \) and \( v_w \) in order are the velocity of the fluid flow and the longitudinal velocity of the ECS, \( \sigma_v \) is the tangential moment accommodation coefficient, \( n \) denotes the normal direction to the inner surface of the ECS, \( b = -1 \), and \( Kn \) is the Knudsen number. Knudsen number is defined as the ratio of the mean free path of the fluid’s molecules, \( l_f \), to the hydraulic diameter of the nanochannel. Recently, Narasimhan [50] showed that the velocity profile of the nonlocal fluid is identical to that of the classical fluid. Thereby, through consideration of the small-scale effect for the fluid flow, the general expression of the velocity profile of the moving fluid inside the lengthly SWCNT would be,

\[ v_f = \frac{p_s}{4 \eta_{fe}} r^2 + C_1 \ln(r) + C_2, \]

where \( \eta_{fe} = \frac{\eta_0}{\tau_0 + 2Kn} \) denotes the effective viscosity of the fluid, \( \eta_{fe} \) is the bulk viscosity of the fluid, \( \eta_f = \frac{2b}{x^3} \tan^{-1}(a_1 Kn^b) \) where \( a_0 = \frac{6b}{3b - 4}, \quad a_1 = 4, \quad B = 0.4 \) [51]. In view of the condition in Eq. (8) and since the fluid flow velocity should be bounded at \( r = 0 \), the unknown coefficients \( C_1 \) and \( C_2 \) in Eq. (9) could be calculated. As a result, the velocity field of the nanofluid flow inside the ECS is derived as:

\[ v_f = \frac{p_s}{4 \eta_{fe}} \left[ r^2 - r_1^2 \left( 1 + \frac{2 - \sigma_v}{\sigma_v} \frac{2Kn}{1 - bKn} \right) + u_w \right], \]

where \( u_w = u_{i,r} \). The average speed of the fluid flow is calculated by

\[ c = \frac{r_1^2 p_s}{8 \eta_{fe}} \left( 1 + \frac{2 - \sigma_v}{\sigma_v} \frac{4Kn}{1 - bKn} \right) + u_s. \]

The tangential component of the interactional force per unit length, \( f_i \), is determined by \( f_i = \int_{r_1}^{r_2} \tau_n \, dr \) where \( \Gamma_n \) represents the inner surface of the ECS with unit length, and the shear stress is given by \( \tau_n = \eta_{fe} v_{f,r} \). Therefore, the magnitude of \( f_i \) accounting for the slip boundary effect is evaluated as,
By eliminating \( f_i \) from Eqs. \((7a) \) and \((7b) \), and neglecting the products of derivatives of displacements, one could arrive at
\[
(P\alpha)_x = -f_i - \rho \mu \alpha_x (g_x - 2g_2w_x + \ddot{u} + 2c_0u_x + c^2u_{xx}),
\]
\[
(P\alpha)_x w_x = - \left( \frac{f_i + \rho \mu \alpha_x g_x}{w_x} \right) w_x,
\]
by substituting the term, \( f_i = f - n w_x \), from Eq. \((7a) \) into Eq. \((5a) \), then using Eq. \((12a) \), the longitudinal equation of motion of the ECS is obtained
\[
\rho \mu \alpha \ddot{u} - N_{ux} + \rho \mu \alpha g_x w_x = \eta_0 K_l (c - u_x),
\]
by eliminating the term, \( f_i = f - n w_x \), from Eqs. \((5b) \) and \((7b) \), using Eqs. \((13a) \) and \((13b) \), and provoking the product of derivatives of displacements, the lateral equation of motion of the ECS is obtained as
\[
\rho \mu \alpha (\ddot{w} - l_0 \dot{w}_{xx}) - \rho \mu \alpha (\ddot{w} + 2c_0w_x + c^2w_{xx}) = (P\alpha_x - T_0) w_{xx} - M_{b,xx} - \left( 2\rho \mu \alpha g_x + \eta_0 K_l c \right) w_x (1 - H(x - x_l)) + \left( \rho \mu \alpha (\ddot{w} + 2c_0w_x + c^2w_{xx}) \right) (1 - H(x - x_l)) = \rho \mu \alpha g_{cz} (1 - H(x - x_l)),
\]
The frictionless nature of the inside walls of SWCNTs leads to swift movement of fluid’s molecules at the vicinity of the inner surface of the SWCNT. It indicates that the slip length would increase in compare to the macro-scale pipe subjected to a fluid flow, particularly for low levels of the inner diameter of SWCNTs. As the slip length increases, the mean flow velocity increases. As a result, the difference between the minimum and maximum velocities would reduce and the velocity profile becomes more plug-like [52]. Herein it is presumed that the plug-like nanofluid flows from the left end of the SWCNT. In the context of the nonlocal continuum theory of Eringen, Eqs. \((14) \) and \((15) \) are modified to
\[
N_{n}^{nl} = \rho_0 a^2 N_{n,xx}^{nl} = N_0 = T_0 + E_b A_b u_x,
\]
\[
M_{n}^{nl} = (e_o a)^2 M_{n,xx}^{nl} = M_b = -E_b l_b w_{xx},
\]
where \( e_o a \) denotes the small-scale parameter. By combining Eqs. \((16a) \) and \((16b) \) with Eqs. \((17a) \) and \((17b) \), the nonlocal internal forces related to the local axial force and the local bending moment as follows
\[
N_{n}^{nl} = E_b A_b \left\{ T_0 \lambda x^2 + \sum_l (t_l^2) \right\}, \quad \mu = \frac{e_o a}{E_b}, \quad \tau = \frac{1}{l_b} \sqrt{E_b A_b}, \quad \lambda = \frac{l_b}{r_b}, \quad \eta_0 = \frac{\eta_{0,b}}{C_l P A_f}, \quad \Lambda_f = \sqrt{K_l},
\]
Eqs. \((18a) \) and \((18b) \) are substituted into Eqs. \((16a) \) and \((16b) \), and the following dimensionless parameters are introduced to the resulting equations,
\[
\tilde{\lambda} = \frac{\lambda}{r_b}, \quad \tilde{\mu} = \frac{\mu}{\rho_b A_b}, \quad \beta = \frac{c}{C_l}, \quad \tilde{T}_0 = \frac{T_0}{E_b l_b}, \quad \tilde{P} = \frac{P A_f}{E_b l_b}, \quad \tilde{\eta}_0 = \frac{\eta_{0,b}}{C_l P A_f}, \quad \tilde{\gamma}_x = \sqrt{\frac{g_x l_b}{C_l}}, \quad \tilde{\gamma}_z = \sqrt{\frac{g_z l_b}{C_l}}.
\]
where
\[
r_b = \sqrt{\frac{l_b}{A_b}}, \quad r_f = \sqrt{\frac{l_f}{A_f}}, \quad C_l = \sqrt{\frac{E_b}{\rho_b}}.
\]
in Eq. (20), \( \omega_c \) and \( \gamma_t \) denote the gyration radius of the cross-section of the ECS and the fluid flow, respectively. As a result, we can arrive at the nondimensional nonlocal equations of motion of the ECS subjected to a nanofluid flow in terms of displacements as

\[
\begin{align*}
\ddot{\mathbf{\Pi}} - \mu_0^2 \mathbf{\Pi}_{zzzzz} + \mathbf{m}_0 \mathbf{\eta}_0 \mathbf{A}_0^2 \mathbf{J} \left( \mathbf{w} \left( 1 - H(\xi - \zeta_f) \right) - \mu_0^2 \mathbf{\Pi} \left( 1 - H(\xi - \zeta_f) \right) \right) + \mathbf{m}_0 \mathbf{J}^2 \mathbf{z}^2 \left( \mathbf{w} \left( 1 - H(\xi - \zeta_f) \right) \right) - \mu_0^2 \mathbf{\Pi} \left( 1 - H(\xi - \zeta_f) \right),
\end{align*}
\]

\[
\begin{align*}
\mathbf{w}_{tt} - \mu_0^2 \mathbf{w}_{zzzzz} - \lambda^2 \left( \mathbf{w}_{zzzzz} - \mu_0^2 \mathbf{w}_{zzzzz} \right) + \mathbf{m}_0 \left( \mathbf{w}_{zzzz} + 2 \beta_0^2 \mathbf{w}_{zzzz} \left( 1 - H(\xi - \zeta_f) \right) \right) - \mu_0^2 \mathbf{w} \left( 1 - H(\xi - \zeta_f) \right),
\end{align*}
\]

(21a)

\[
\begin{align*}
\mathbf{w}_{tt} + \mu_0^2 \mathbf{w}_{zzzzz} - \lambda^2 \left( \mathbf{w}_{zzzzz} - \mu_0^2 \mathbf{w}_{zzzzz} \right) + \mathbf{m}_0 \left( \mathbf{w}_{zzzz} + 2 \beta_0^2 \mathbf{w}_{zzzz} \left( 1 - H(\xi - \zeta_f) \right) \right) - \mu_0^2 \mathbf{w} \left( 1 - H(\xi - \zeta_f) \right),
\end{align*}
\]

(21b)

If the values of the small-scale parameter as well as the Knudsen number approach zero, the classical form of the governing equations of an inclined macro-scale tubular structure conveying a fluid flow are recovered. In the present work, the generated nonlocal forces within the conveying-fluid SWCNT are also of interest. Therefore, the nonlocal axial force and bending moment of the ECS subjected to a nanofluid flow in terms of the dimensionless displacement components are expressed by

\[
\begin{align*}
N_{\Pi} = E_o A_o \left[ \frac{\mathbf{d} \mathbf{\Pi}}{\mathbf{\Pi}} + \frac{\gamma^2_0 \mathbf{\Pi}}{\gamma^2_0} \right] \left( 1 - H(\xi - \zeta_f) \right),
\end{align*}
\]

(22a)

\[
\begin{align*}
M_{\Pi} = E_o A_o \left[ \frac{\mathbf{d} \mathbf{\Pi}}{\mathbf{\Pi}} + \frac{\gamma^2_0 \mathbf{\Pi}}{\gamma^2_0} \right] \left( 1 - H(\xi - \zeta_f) \right),
\end{align*}
\]

(22b)

In the case of a simply supported SWCNT with immovable ends, the boundary conditions read [29]

\[
\begin{align*}
\mathbf{\Pi}(\xi = 0, \tau) = 0, & \quad \mathbf{\Pi}(\xi = 1, \tau) = 0, \quad \mathbf{\Pi}(\xi = 0, \tau) = 0, & \quad \mathbf{\Pi}(\xi = 1, \tau) = 0,
\end{align*}
\]

(23a)

\[
\begin{align*}
\mathbf{\Pi}(\xi = 0, \tau) = 0, & \quad \mathbf{\Pi}(\xi = 1, \tau) = 0, \quad \mathbf{\Pi}(\xi = 0, \tau) = 0, & \quad \mathbf{\Pi}(\xi = 1, \tau) = 0,
\end{align*}
\]

(23b)

\[
\begin{align*}
\mathbf{\Pi}(\xi = 0, \tau) = 0, & \quad \mathbf{\Pi}(\xi = 1, \tau) = 0, \quad \mathbf{\Pi}(\xi = 0, \tau) = 0, & \quad \mathbf{\Pi}(\xi = 1, \tau) = 0,
\end{align*}
\]

(23c)

and the initial conditions of the SWCNT subjected to the internal fluid flow are as,

\[
\begin{align*}
\mathbf{\Pi}(\xi, \tau = 0) = 0, & \quad \mathbf{\Pi}(\xi, \tau = 0) = 0, \quad \mathbf{\Pi}(\xi, \tau = 0) = 0, & \quad \mathbf{\Pi}(\xi, \tau = 0) = 0,
\end{align*}
\]

(24a)

\[
\begin{align*}
\mathbf{\Pi}(\xi, \tau = 0) = 0, & \quad \mathbf{\Pi}(\xi, \tau = 0) = 0, \quad \mathbf{\Pi}(\xi, \tau = 0) = 0, & \quad \mathbf{\Pi}(\xi, \tau = 0) = 0,
\end{align*}
\]

(24b)

3. Numerical solution of the problem

In order to construct the discrete form of the governing equations, both sides of Eqs. (21a) and (21b) are multiplied by \( \delta \mathbf{\Pi} \) and \( \delta \mathbf{w} \), respectively, and then the resulted equations are integrated over the length of the SWCNT. After successful integration by parts, one could arrive at

\[
\begin{align*}
\int_0^1 \left( \delta \mathbf{\Pi}_{zzzz} + \mu_0^2 \delta \mathbf{w}_{zzzzz} + \mathbf{m}_0 \mathbf{\eta}_0 \mathbf{A}_0^2 \mathbf{J} \delta \mathbf{w} \left( 1 - H(\xi - \zeta_f) \right) + \mathbf{m}_0 \mathbf{J}^2 \mathbf{z}^2 \delta \mathbf{w} \left( 1 - H(\xi - \zeta_f) \right) - \mu_0^2 \mathbf{w} \left( 1 - H(\xi - \zeta_f) \right),
\end{align*}
\]

(25)

the unknown dimensionless displacements are now stated in terms of mode shape functions as follows:

\[
\begin{align*}
\mathbf{\Pi}(\xi, \tau) = \sum_{i=1}^{N_m} \phi^n_i(\xi) \mathbf{w}_i(\tau), \quad \mathbf{w}(\xi, \tau) = \sum_{i=1}^{N_m} \phi^n_i(\xi) \mathbf{w}_i(\tau),
\end{align*}
\]

(26)

where \( \phi^n_i(\xi) \) and \( \phi^n_i(\zeta) \) are, respectively, the ith mode shape functions associated with the longitudinal and transverse displacements, \( \mathbf{w}_i(\tau) \) and \( \mathbf{w}_i(\tau) \) are the unknown parameters pertinent to the ith mode, and \( N_m \) is the number of vibration modes.
required to obtain fairly accurate displacements for the ECS. Substitution of Eq. (26) into Eq. (25) would result in the following discrete form of the governing equations

\[
\begin{bmatrix} M_{b1}^{\text{ww}} & M_{b2}^{\text{ww}} \\ M_{b1}^{\text{ww}} & M_{b2}^{\text{ww}} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{1r} \\ \mathbf{u}_{2r} \end{bmatrix} + \begin{bmatrix} C_{b1}^{\text{ww}} & C_{b2}^{\text{ww}} \\ C_{b1}^{\text{ww}} & C_{b2}^{\text{ww}} \end{bmatrix} \begin{bmatrix} \mathbf{w}_{1r} \\ \mathbf{w}_{2r} \end{bmatrix} + \begin{bmatrix} K_{b1}^{\text{ww}} & K_{b2}^{\text{ww}} \\ K_{b1}^{\text{ww}} & K_{b2}^{\text{ww}} \end{bmatrix} \begin{bmatrix} \mathbf{u}_{1r} \\ \mathbf{u}_{2r} \end{bmatrix} = \begin{bmatrix} \mathbf{f}_{b1}^{\text{u}} \\ \mathbf{f}_{b2}^{\text{w}} \end{bmatrix},
\]

where the nonzero dimensionless submatrices are provided in Appendix A. For a SWCNT with simply supported boundary conditions and immovable ends, the following admissible normalized mode shape functions are taken into account

\[
\phi_i^w(\zeta) = \sqrt{2} \sin(i\pi \zeta), \quad \phi_i^w(\zeta) = \sqrt{2} \sin(i\pi \zeta),
\]

by substituting Eq. (28) into Eqs. (A.1)–(A.9) and evaluating the integrals yield the simpler statements for the submatrices in Eqs. (A.1)–(A.9). In order to solve the set of ordinary differential equations of Eq. (27) in the time domain, the generalized Newmark-\(\beta\) approach is exploited (see Ref. [53]).

4. Results and discussion

4.1. Validity of the proposed model

In order to ensure about the accuracy of calculations, we compare the obtained frequencies of transverse vibration by the proposed model for an SWCNT conveying fluid flow with those of other researchers. For this purpose, the obtained flexural frequencies of the model are verified with those of Yoon et al. [35]. Consider two horizontal SWCNTs conveying fluid flow; the geometry data of the ECS associated with the first SWCNT, denoted by SWCNT1, are \(r_2 = 50\) (nm), \(t_b = 10\) (nm) and those of the ECS corresponding to the second SWCNT, represented by SWCNT2, are \(r_2 = 40\) (nm), \(t_b = 20\) (nm). The other data of the system of SWCNT under moving water flow are as those mentioned in [35]. It is also assumed that the influence of the small-scale parameter would be negligible. In Figs. 3(a)–(c), the plots of the first and the second flexural frequencies (i.e., \(f_i = \sqrt{EJ_b/(\rho_2 A_b)}\omega_i/(2\pi)^2\); \(i = 1, 2\) where \(\omega_i\) denotes the \(i\)th circular flexural frequency of the fluid-conveying SWCNT) of the two above-mentioned SWCNTs conveying fluid flow as a function of the speed of fluid flow are demonstrated. The results are provided for three levels of the aspect ratio, \(l_b/r_2 = 40, 200,\) and 1000 in Figs. 3(a), (b), and (c), respectively. As it is seen in Figs. 3(a)–(c), the predicted frequencies by the proposed model and those of Yoon et al. [35] are in line. Additionally, the discrepancy of frequencies of the proposed model and those of the model of Yoon et al. [35] is generally lower than 8%. Therefore, there is a reasonably good agreement between the predicted results of the proposed model and those of Yoon et al. [35]. As it is obvious from Figs. 3(a)–(c), the frequencies of the SWCNT decrease with the speed of the fluid flow. As the fluid flow speed increases up to a certain value, called critical speed, the frequency of the first vibration mode reduces to zero and the system of SWCNT conveying fluid flow becomes unstable. If the speed of the fluid flow does not alter about the critical level,

![Fig. 3](image-url)
the dynamic transverse displacement of each point of the SWCNT except its two ends would grow with time, and dynamic instability would occur in the system. It is also clear from Figs. 3(a)–(c) that the critical speed of the fluid flow decreases with the aspect ratio (or slenderness ratio) of the SWCNT. Furthermore, the variation of the speed of the fluid flow on the frequency of a SWCNT with a higher aspect ratio is more obvious (see Figs. 3(b) and (c)); this result confirms the previously obtained result by Yoon et al. [35].

4.2. Numerical results

In this part, the effects of crucial parameters on dynamic longitudinal and transverse displacements as well as nonlocal axial force and bending moment of inclined SWCNTs conveying viscous fluid flow are studied. Consider an ECS associated with the SWCNT with the following data: $E_b = 10^{12} \text{Pa}$, $\rho_b = 2500 \text{ kg/m}^3$, $v = 0.2$, $t_b = 0.34 \text{ nm}$, $r_m = 0.7 \text{ nm}$, and $\lambda = 40$. The properties of the fluid flow are as: $\rho_f = 1000 \text{ kg/m}^3$, $\nu_f = 1.002 \times 10^{-3} \text{ Pa.s}$, $\sigma_y = 0.8$, $I_f = 0.3 \text{ nm}$, $A_f = \pi r_f^2$, and $l_f = \pi r_f^4/4$. For the sake of convenience, the following normalized parameters are introduced: $u_N(\xi, \tau) = 8\mu(\xi, \tau)/(m_f \eta_f A_f^2 \beta)$, $w_N(\xi, \tau) = 384 \mu(\xi, \tau)E_b t_b/(5\rho_f A_f g l_b^3)$, $N_{SN}(\xi, \tau) = 2N_b(\xi, \tau)/(E_b A_b m_f \eta_f A_f^2 \beta)$, $M_{SN}(\xi, \tau) = 8M_b(\xi, \tau)/(\rho_f A_f g l_b^5)$, and $c_N = c A_f/(\pi \sqrt{E_b/\rho_b})$. The maximum values of dynamic amplitude factors (MDAFs) of normalized displacements, nonlocal axial force and nonlocal bending moment of the SWCNT conveying fluid flow are given by $\text{MDAF}_{\max} = \max\{|u_N(\xi, \tau)|\}$ in which $|\ | = u$ or $w$ and MDAF$_{\max} = \max\{|N_{SN}(\xi, \tau)|\}$ where $|\ | = N$ or $M$.

Commonly, the magnitude of small-scale parameter is determined by comparing the predicted dispersion curves by the nonlocal continuum mechanics with those of an atomistic-based model. The carried out works reveal that the chosen value of small-scale parameter only depends on the boundary conditions of the SWCNT. By choosing $e_0$ magnifies with the aspect ratio: $e_0$ varies in the range of 3-19 for $8<\frac{h}{r_m}<15$. Arash and Ansari [55] studied vibration characteristics of initially strained SWCNTs with different boundary conditions via a nonlocal shell model as well as molecular dynamics. The small-scale parameter was calibrated for a wide range of aspect ratios of SWCNTs by comparing the predicted resonant frequencies of the nonlocal model with those of the MD. The predicted results showed that the small-scale parameter only depends on the boundary conditions of the SWCNT. By choosing $e_0a = 1.7$ and 2 nm for fully clamped and cantilevered boundary conditions, respectively, a good agreement between the predicted results by the nonlocal shell model and those of MD simulations was obtained. In the present work, $N_m = 5$, and $e_0a$ is considered in the range of 0–2 nm.

In Figs. 4(a)–(c), time history plots of normalized dynamic displacements, nonlocal axial force, and nonlocal bending moment at the midspan point of the SWCNT have been provided for different values of the fluid flow speed and small-scale

**Fig. 4.** Plots of normalized dynamic displacements, nonlocal axial force, and nonlocal bending moment at the midspan point of the SWCNT for various values of the fluid flow speed as well as small scale effect parameter: (a) $c_s = 0.1$, (b) $c_s = 0.2$, (c) $c_s = 0.3$; (•••) $e_0a = 0 \text{ nm}$, (--) $e_0a = 1 \text{ nm}$, (--) $e_0a = 2 \text{ nm}$; $\alpha = 0$. 
The obtained results are demonstrated for three levels of the speed of fluid flow (i.e., $c_N = 0.1, 0.2,$ and $0.3$) as well as three values of the small-scale parameter (i.e., $e_0a = 0.1,$ and $2$ nm) in the case of $\alpha = 0$. As it is seen in Figs. 4(a)–(c), variation of the speed of the fluid flow has no distinguished influence on the time history plots of normalized longitudinal displacement during the first and the second courses of vibration (i.e., when ECS is partially acted upon by the viscous fluid flow ($\tau < 1$) and when the whole of ECS is acted upon by the viscous fluid flow ($\tau > 1$)). In all studied cases, the maximum nonlocal axial force generally occurs when the front of the fluid flow traverses the midspan point of the SWCNT. Furthermore, no obvious amplitudes are observed for both dynamic longitudinal displacement and nonlocal axial force in the second phase of vibration. As the effect of the small-scale parameter becomes highlighted, the maximum values of normalized longitudinal displacement as well as nonlocal axial force within the SWCNT commonly magnify. For an assumed level of the speed of fluid flow, variation of the small-scale parameter has a few effects on the variation of $u_N$ and $N_BN$ with respect to $w_N$ and $M_BN$. The effect of the small-scale parameter on the variation of normalized dynamic deflection and nonlocal bending moment is more obvious in the second course of vibration in comparison to the first course of vibration. Moreover, the maximum values of normalized transverse displacement and nonlocal bending moment of the SWCNT increase with the speed of fluid flow, and generally their peaks occur in the second phase of excitation. Another interesting issue should be addressed is the sudden jump down of the nonlocal bending moment as the midspan point of the SWCNT when it is traversed by the front of the fluid flow (see Figs. 4(a)–(c)). This matter is related to a combinational effects of the following issues: (i) movement of all molecules of the front of the fluid flow simultaneously reach to the midspan point when they are initially at rest; (ii) some approximations are entered in the proposed nonlocal beam model. For example, the kernel function associated with the nonlocal stress tensor of an infinite one-dimensional solid, which was established in a pioneering work by Eringen [56], is employed herein. Such facts lead to the appearance of the statement $\mu^2(1 - H(\xi - \xi_0))$ in the formulation of nonlocal bending moment of the SWCNT (see Eq. (22b)). As it is also clear from Figs. 4(a)–(c), the amount of jump of $M_BN$ intensifies with the small-scale parameter.

Another interesting study is to explore the influence of inclination angle of the inclined SWCNT on the dynamic displacements and nonlocal forces within the SWCNT due to internal fluid flow for various values of the small-scale effect parameter. In the case of $c_N = 0.3$, the time history of longitudinal and transverse displacements, nonlocal axial force, and nonlocal bending moment of the midspan point of the SWCNT are demonstrated in Figs. 4(a), (b), and (c) for $\alpha = \pi/6$, $\pi/3$, and $\pi/2$, respectively. The dotted, the dashed, and the solid lines in order are associated with $e_0a = 0.1$, and $2$ nm. As it is seen in Figs. 5(a)–(c), variation of $\alpha$ has a trivial effect on the variation of longitudinal displacement and nonlocal axial force of the midspan point of the SWCNT at each time. However, the values of transverse displacement and nonlocal bending moment decrease with $\alpha$. In the case of $\alpha = \pi/2$, as expected, the maximum values of transverse displacement and nonlocal bending moment of the SWCNT is approximately equal to zero. In an upcoming part, the effect of $\alpha$ on the maximum values of

![Fig. 5](image-url). Plots of normalized dynamic displacements, nonlocal axial force, and nonlocal bending moment at the midspan point of the SWCNT for various values of the inclination angle as well as small scale effect parameter: (a) $\alpha = \pi/6$, (b) $\alpha = \pi/3$, (c) $\alpha = \pi/2$; (−−−) $e_0a = 0$ nm, (−−) $e_0a = 1$ nm, (−) $e_0a = 2$ nm; $c_N = 0.3$.


displacements and generated nonlocal forces will be also explained in some detail. As it is seen in Figs. 5(a)–(c), regardless the value of $a$, the transverse displacement grows with the small-scale parameter. Nevertheless, the nonlocal bending moment would lessen as the value of small-scale parameter increases.

In this part, the effect of the small-scale parameter on the maximum dynamic amplitude factors of displacements and nonlocal forces of SWCNTs subjected to inside fluids flow is inspected in some detail. In the case of a horizontal SWCNT (i.e., $a = 0$), the plots of MDAF$_u$, MDAF$_w$, MDAF$_N$, and MDAF$_M$ as a function of $e_0a$ have been provided in Fig. 6. The obtained results are presented for three levels of speed of fluid flow (i.e., $c_N = 0.05, 0.1, 0.15$). As it is seen, the magnitudes of MDAF$_u$ and MDAF$_N$ commonly increase with $e_0a$, regardless of the speed of fluid flow. Furthermore, the value of MDAF$_w$ increases with $e_0a$, and the slope of variation of MDAF$_w$ as a function of $e_0a$ is more obvious for higher levels of speed of fluid flow. The magnitude of MDAF$_M$ generally decreases with $e_0a$, irrespective of the speed of fluid flow. Additionally, the slope of variation of MDAF$_M$ in terms of $e_0a$ is more apparent for lower levels of the fluid flow speed. The effect of the fluid flow velocity on MDAFs of displacements and nonlocal forces is another interesting subject that will be explored in an upcoming part.

Equally important is the examination of the effect of the inclination angle of the SWCNT on the MDAFs of displacements and nonlocal forces of SWCNTs subjected to fluids flow. The predicted values of MDAF$_u$, MDAF$_w$, MDAF$_N$, and MDAF$_M$ in terms of $\alpha$ have been plotted in Fig. 7 for $c_N = 0.15$ and three levels of the small-scale parameter (i.e., $e_0a = 0$ nm, $1$ nm, $2$ nm).
As it is expected, the variation of $\alpha$ has a trivial effect on the variation of $\text{MDAF}_w$ and $\text{MDAF}_N$. However, the magnitudes of $\text{MDAF}_w$ and $\text{MDAF}_N$ decrease with $\alpha$. As it is seen, the maximum values of both of $\text{MDAF}_w$ and $\text{MDAF}_N$ occur in $\alpha = 0$, and the minimum values of them occur in $\alpha = \pi/2$. Regarding the plots of $\text{MDAF}_w$ and $\text{MDAF}_N$ as a function of $\alpha$, the effect of variation of the small-scale parameter on the variation of $\text{MDAF}_w$ and $\text{MDAF}_N$ is more obvious for lower values of $\alpha$. It implies that the effect of the small-scale parameter on $\text{MDAF}_w$ and $\text{MDAF}_N$ vanishes as $\alpha$ increases. In other words, the discrepancies between the predicted $\text{MDAF}_w/\text{MDAF}_N$ by the classical model and those of the nonlocal theory would reduce with the inclination angle.

The effect of fluid flow speed on the $\text{MDAF}_w$s of displacements and nonlocal forces of SWCNTs would be an important issue in design and control of such nanoconveyers. In the case of $\alpha = \pi/3$, the graphs of $\text{MDAF}_w$, $\text{MDAF}_w$, $\text{MDAF}_w$, and $\text{MDAF}_N$ as a function of $c_N$ are provided in Fig. 8. The predicted results are plotted for three levels of the small-scale parameter (i.e., $e_0a = 0.1, 1$, and $2$ nm). As it is clear from Fig. 8, the predicted values of $\text{MDAF}_w$ and $\text{MDAF}_N$ generally increase with $e_0a$, regardless of the speed of fluid flow. The magnitude of $\text{MDAF}_w$ decreases with the speed of the fluid flow up to $c_N = 0.1$, irrespective of the small-scale parameter. For normalized speed of fluid flow greater than such a value, $\text{MDAF}_w$ magnifies with the speed of the fluid flow. Further, the magnitude of $\text{MDAF}_N$ grows with $c_N$. It is also observed that $\text{MDAF}_w$ increases with $e_0a$, irrespective of the speed of fluid flow. For $c_N < 0.16$, $\text{MDAF}_N$ decreases with $e_0a$; however, for $c_N > 0.16$, the magnitude of $\text{MDAF}_N$ increases with $e_0a$. A close scrutiny of the plotted results in Fig. 8 reveal that the discrepancies between the predicted $\text{MDAF}_w/\text{MDAF}_N$ by the local model and those of the nonlocal model also depend on the speed of fluid flow. Such discrepancies commonly magnify with the fluid flow speed. For example, classical continuum theory could not capture the predicted $\text{MDAF}_w$ and $\text{MDAF}_N$ by the nonlocal model ($e_0a = 2$ nm) with relative error lower than 10 percent for $c_N > 0.1$ and $c_N > 0.165$, respectively.

The effect of fluid flow density on the $\text{MDAF}_w$s of displacements, nonlocal axial force, and nonlocal bending moment of a SWCNT subjected to a fluid flow is another crucial matter should be investigated. In Figs. 9(a)–(d), the obtained results of $\text{MDAF}_w$s of displacements and nonlocal forces as a function of the ratio of fluid density to water density ($\rho_f/\rho_w$) are demonstrated for different levels of the fluid flow speed. The plotted results are given for a horizontal SWCNT conveying a viscous fluid flow with of normalized velocities $c_N = 0.1, 0.15, 0.2$, and $0.25$. The results are also provided for three values of the small-scale parameter (i.e., $e_0a = 0.1, 1$, and $2$ nm). As it is obvious in Figs. 9(a)–(d), the magnitudes of $\text{MDAF}_w$ and $\text{MDAF}_N$ generally increase with the fluid flow density. The main reason of this fact is that the inertial effect of the fluid flow increases with the fluid density. For low levels of the speed of fluid flow (i.e., $c_N \leq 0.15$), $\text{MDAF}_w$ and $\text{MDAF}_N$ vary linearly in terms of fluid flow density. For high levels of the speed of fluid flow (i.e., $c_N \geq 0.2$), $\text{MDAF}_w$ and $\text{MDAF}_N$ do not vary linearly as a function of fluid flow density, particularly for high values of small-scale parameter. Similar phenomenon was previously reported by Kiani et al. [53,57,58] where the effects of a moving mass on vibrations of macro-scale beam structures were of concern. It was shown that the linear relation between dynamic amplitude factor of transverse displacement and mass weight of the moving mass would be no longer valid when the moving mass velocity is greater than a particular value. According to Figs. 9(a)–(d), the slope of variation of $\text{MDAF}_w$ as a function of $\rho_f$ would be more obvious for those SWCNTs which are subjected to the moving fluids with higher levels of speed. Further, variation of the ratio of the fluid density to the water density has a very trivial effect on the variations of both $\text{MDAF}_w$ and $\text{MDAF}_N$.

![Fig. 8. Variation of $\text{MDAF}_w$, $\text{MDAF}_w$, $\text{MDAF}_w$, and $\text{MDAF}_N$ in terms of $c_N$; (--) $e_0a = 0$ nm, (--) $e_0a = 1$ nm, (--) $e_0a = 2$ nm; $\alpha = \pi/3$).](image-url)
5. Conclusions

A nonlocal model is developed to examine the interaction of a viscous fluid flow with a simply supported inclined single-walled carbon nanotube (SWCNT). To this end, an appropriate equivalent continuum structure (ECS) associated with the SWCNT is considered and modeled using nonlocal Rayleigh beam theory. Using an appropriate slip boundary condition, the longitudinally applied force on the inner surface of the ECS is obtained. The dimensionless governing equations are established in the context of nonlocal continuum theory of Eringen. By employing Galerkin method, the discrete form of the equations of motion is obtained in terms of mode shapes, and the resulting set of algebraic equations is then solved in the time domain by employing generalized Newmark-$\beta$ method. The time history plots of displacements as well as the nonlocal axial and bending moment are provided for various values of fluid flow speed and inclination angle of the SWCNT. The effects of the small-scale parameter, inclination angle of the SWCNT, speed and density of the fluid flow on the maximum dynamic amplitude factors of the normalized displacements (i.e., MDAF$_u$ and MDAF$_N$) as well as the nonlocal axial and flexural forces (i.e., MDAF$_N$ and MDAF$_M$) are investigated.

The obtained results reveal that the magnitudes of MDAF$_u$ and MDAF$_N$ intensify slightly with small-scale parameter, regardless of the fluid flow speed and inclination angle of the SWCNT. The magnitude of MDAF$_w$ increases with small-scale parameter, and the slope of variation of MDAF$_w$ in terms of small-scale parameter increases as the fluid flow speed magnifies. The predicted results also indicate that the variation of inclination angle of the SWCNT has a trivial effect on the variation of MDAF$_w$ and MDAF$_N$. Nevertheless, MDAF$_w$ and MDAF$_M$ would decrease with inclination angle. Moreover, MDAF$_M$ would commonly increase with the speed of fluid flow; however, the magnitude of MDAF$_w$ would lessen with the speed of the fluid flow up to a certain level. For fluid flow speed greater than that value, the magnitude of MDAF$_w$ generally increases with the fluid flow speed. The magnitudes of MDAF$_w$ and MDAF$_N$ also increase with the fluid flow density since the inertial effects of the fluid flow intensify as the density of the fluid flow increases. However, variation of the fluid flow density has fairly no effect on the variation of MDAF$_u$ and MDAF$_N$. The predicted results show that the linear relation between the MDAF$_w$ (or MDAF$_M$) and the fluid flow density is violated for high levels of the speed of the fluid flow.

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Appendix A

\[ |\mathbf{M}_{b}|_{ij} = \int_{0}^{1} \left( \phi_i^{u} \phi_j^{u} + \mu^2 \phi_i^{u} \phi_j^{u} \right) d\xi, \]  
\[ |\mathbf{M}_{b}|_{ij}^{ww} = \int_{0}^{1} \left( \phi_i^{w} \phi_j^{w} + \lambda^2 \phi_i^{w} \phi_j^{w} - \mu^2 \phi_i^{w} \phi_j^{w} \right) \left( \phi_i^{w} - \lambda^2 \phi_j^{w} \right) \left( 1 - K(\xi - \xi_f) \right) d\xi, \]  
\[ |\mathbf{C}_{b}|_{ij}^{uu} = \bar{m}_{i} \eta_{f_{0}} \lambda \xi^2 \int_{0}^{1} \left( \phi_i^{u} - \mu^2 \phi_i^{u} \phi_j^{u} \right) \left( 1 - K(\xi - \xi_f) \right) d\xi, \]  
\[ |\mathbf{C}_{b}|_{ij}^{ww} = 2\beta \eta_{f_{0}} \lambda \xi^2 \int_{0}^{1} \left( \phi_i^{w} - \mu^2 \phi_i^{w} \phi_j^{w} \right) \left( \phi_i^{w} - \lambda^2 \phi_j^{w} \right) \left( 1 - K(\xi - \xi_f) \right) d\xi, \]  
\[ |\mathbf{K}_{b}|_{ij}^{uu} = \lambda^2 \int_{0}^{1} \left( \phi_i^{u} + \mu^2 \phi_i^{u} \phi_j^{u} \right) d\xi, \]  
\[ |\mathbf{K}_{b}|_{ij}^{ww} = \lambda^2 \int_{0}^{1} \left( \phi_i^{w} + \mu^2 \phi_i^{w} \phi_j^{w} \right) \left( 1 - K(\xi - \xi_f) \right) d\xi, \]  
\[ |\mathbf{R}_{b}|_{ij}^{uu} = \lambda \int_{0}^{1} \left( \phi_i^{u} + \mu^2 \phi_i^{u} \phi_j^{u} \right) \left( 1 - K(\xi - \xi_f) \right) d\xi, \]  
\[ |\mathbf{R}_{b}|_{ij}^{ww} = \lambda \int_{0}^{1} \left( \phi_i^{w} + \mu^2 \phi_i^{w} \phi_j^{w} \right) \left( 1 - K(\xi - \xi_f) \right) d\xi, \]  
\[ |\mathbf{F}_{b}|_{ij}^{uu} = \lambda \int_{0}^{1} \left( \phi_i^{u} + \mu^2 \phi_i^{u} \phi_j^{u} \right) \left( 1 - K(\xi - \xi_f) \right) d\xi, \]  
\[ |\mathbf{F}_{b}|_{ij}^{ww} = \lambda \int_{0}^{1} \left( \phi_i^{w} + \mu^2 \phi_i^{w} \phi_j^{w} \right) \left( 1 - K(\xi - \xi_f) \right) d\xi. \]

References
