Vibration and instability of a single-walled carbon nanotube in a three-dimensional magnetic field

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A B S T R A C T
The vibration and instability of a single-walled carbon nanotube (SWCNT) under a general magnetic field are of particular interest to researchers. Using nonlocal Rayleigh beam theory and Maxwell’s equations, the dimensionless governing equations pertinent to the free vibration of a SWCNT due to a general magnetic field were derived. The effects of the longitudinal and transverse magnetic fields on the longitudinal and flexural frequencies as well as their corresponding phase velocities were addressed and are discussed below. The critical transverse magnetic field (CTMF) associated with the lateral buckling of the SWCNT was obtained. The obtained results reveal that the CTMF increases with the longitudinally induced magnetic field. Further, its value decreases as the effect of the small-scale parameter increases.

1. Introduction

Since the discovery of multi-walled carbon nanotubes (MWCNTs) and single-walled carbon nanotubes (SWCNTs), they have received attention from various scientific communities. This is mainly related to the extraordinary electrical, mechanical, thermal, physical, and chemical properties of carbon nanotubes (CNTs) (either SWCNTs or MWCNTs) [1–7]. Since the Young’s modulus of CNTs is tremendously high compared [14,15], and magnetically aligned nanotubes in composites to ordinary materials, it is possible to achieve vibration frequencies in the range of megahertz or even terahertz [8–10]. This fact indicates that longitudinal and transverse sound waves could propagate in CNTs with a high speed. Conversely, applying electro-magnetic fields to CNTs leads to the exertion of Lorentz’s forces on CNTs, which inherently changes their vibration behavior. As a result, if the frequencies of the propagated waves within CNTs could be controllably altered by the applied magnetic fields, it would be the allow such technology to be utilized in practical applications. To this end, suitable mathematical models for CNT vibration characteristics are required. Such models are pivotal for designing CNTs for use in nanosensors [11–13], nanooscillators [14,15], and magnetically aligned nanotubes in composites [16–18].

Continuum mechanics models are usually implemented for studying various aspects of vibration and instability of SWCNTs as well as MWCNTs. In the framework of local and nonlocal continuum theories, free longitudinal vibration [19–21], free transverse vibration [22–27], lateral buckling analysis [28–32], fluid flow-induced vibration [33–35], and nanoparticle movement cause excitation [36,37] of such nanostructures, and these have been concerns of applied mechanics researchers over the past decade. However, in the context of the nonlocal continuum mechanics, the effect of applied magnetic fields on the dynamic characteristics of SWCNTs has not been thoroughly investigated.

In the framework of local continuum mechanics, there are a few studies focusing on the aforementioned subject of interest. Recently, the effects of longitudinal and lateral magnetic fields on the dynamic characteristics of MWCNTs have been studied by some researchers [38–40]. The research was limited to classical continuum mechanics; this implies that the effect of interatomic bonds was not incorporated into the equations of motion. In order to conquer such a deficiency in the classical continuum theory, Narendar et al. [41] proposed a nonlocal Euler-Bernoulli beam model to examine the effect of longitudinal magnetic fields on the longitudinal wave propagation in SWCNTs. By employing nonlocal shear deformable beam theories, Kiani [42] investigated the role of a longitudinal magnetic field on the propagation of lateral waves within elastically supported SWCNTs. The analytical expressions of the frequencies as well as the corresponding phase and group velocities were procured. The influences of the slenderness ratio, the radius of the SWCNT, the small-scale parameter, the longitudinal magnetic field, the lateral and rotational stiffness of the surrounding matrix on the characteristics of the propagated waves were addressed. In other works, the effect of a longitudinal time-varying magnetic field on the dynamic response of perfectly conducting nanowires was investigated by Kiani [43,44]. Using
nonlocal elasticity, the role of the eddy-current loss was also considered in the latter work. The explicit expressions of magneto-thermo-elastic fields within nanowires due to the axially applied magnetic fields were obtained. The role of various influential factors on the maximum values of elasto-dynamic fields were also studied. In all the studied problems, a general three-dimensional magnetic field was never taken into account. In addition, the instability of the CNTs due to applied magnetic fields was not investigated at all. Such scientific gaps inspired the author to examine the effects of axial and lateral magnetic fields on a lengthy SWCNT in the context of nonlocal continuum mechanics of Eringen [45–47] where the interatomic bonds are appropriately incoroporated into the governing equations by a factor called the small-scale parameter.

This paper focuses on the effects of longitudinally and laterally induced magnetic fields on the natural longitudinal and transverse frequencies as well as how they impact buckling of SWCNTs. An equivalent continuum structure (ECS) pertinent to the SWCNT is considered and modeled by employing nonlocal Rayleigh beam theory (NRBT). The characteristic equation of the SWCNT under a general magnetic field is obtained. The influences of small-scale parameter and magnetic fields on the frequencies and phase velocities of the propagated waves as well as the critical transverse magnetic fields of SWCNTs are addressed and discussed.

2. Formulations

Consider an ECS pertinent to the SWCNT as shown in Fig. 1. The ECS is a hollow elastic cylinder of length $l_b$, mean radius $r_m$, thickness $t_b$, cross-sectional area $A_b$, moment inertia $I_b$, density $\rho_b$, and elasticity modulus $E_b$. The ECS is subjected to a uniform static magnetic field $H_0 = H_{0x} + i H_{0y} + k H_{0z}$ where $i$, $j$, and $k$ represent the unit vectors associated with the x, y, and z axes, respectively. Based on the electrodynamics equations which were originated by Maxwell [48–51], the electromagnetic quantities are governed by the following four equations:

$$\nabla \times \mathbf{H} = \mathbf{J},$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{D} = \rho_b,$$

$$\nabla \cdot \mathbf{B} = 0,$$

where $\mathbf{E}$, $\mathbf{H}$, $\mathbf{J}$, $\mathbf{B}$, and $\mathbf{D}$ denote the electric field intensity, magnetic field, current density vector, magnetic field density, and the displacement current density, respectively. Additionally, the over-dot, $\nabla$, and $\nabla \times$ represent the differentiation with respect to the time parameter, the divergence, and the curl operators, respectively. The generalized Ohm’s law and the constitutive relations (E-D and B-H relations) are as follows,

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \times \mathbf{B}),$$

$$\mathbf{D} = \varepsilon \mathbf{E},$$

$$\mathbf{B} = \varphi \mathbf{H},$$

where $\sigma$, $\varepsilon$, and $\varphi$ are the electrical conductivity, permittivity, and permeability of the SWCNT, respectively, and $\mathbf{v} = \mathbf{v}(x, y, z, t)$ is the velocity field vector of the ECS. By neglecting the displacement current density and its derivative with respect to time for the problem at hand, was possible to represent the electromagnetic field quantities as follows:

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{e},$$

$$\mathbf{H} = \mathbf{H}_0 + \mathbf{h},$$

where $\mathbf{e} = \mathbf{e}(x, y, z, t)$ and $\mathbf{h} = \mathbf{h}(x, y, z, t)$ are the small disturbances (i.e., perturbations) of the initially applied electromagnetic field quantities, $\mathbf{E}_0$ and $\mathbf{H}_0$, respectively. For the problem under study, $\mathbf{E}_0 = \mathbf{0}$; thus, $\mathbf{E} = \mathbf{e}$. With good accuracy, the SWCNT could be assumed to be a highly conducted medium, indicating $\sigma$ had a large value, and Eq. (2a) leads to:

$$\mathbf{e} = -\mathbf{u} \times \mathbf{B},$$

where $\mathbf{v} = \mathbf{u}$ and $\mathbf{u}$ represents the displacement field of the ECS. On the other hand, by neglecting the small disturbance magnetic field compared to the initially induced magnetic field (i.e., $\mathbf{h} = \mathbf{H}_0$), and taking Eq. (2c), into account, Eq. (4) can be rewritten as:

$$\mathbf{e} = -\mathbf{u} \times \mathbf{H}_0.$$  

Eq. (6) shows that a small displacement of the ECS may lead to a small disturbance in the magnetic field for highly conducting media. It is emphasized herein that the application of Eq. (6) is limited to low velocity levels of the perfectly conducting media such that the assumption $\mathbf{h} = \mathbf{H}_0$ is not violated. The inserted electromagnetic forces per unit volume of the medium, $f_m$, are called Lorentzian forces and calculated by:

$$f_m = \mathbf{e} \times \mathbf{v} = \mathbf{u} \times \mathbf{H}_0,$$

Then, by introducing Eqs. (1a) and (2c) to Eq. (7) and recalling that $\mathbf{h} = \mathbf{H}_0$, the Lorentzian forces are obtained in terms of the displacement field of the ECS and the initially applied magnetic field as follows:

$$f_m = \mathbf{e} \times \mathbf{v} = \mathbf{u} \times \mathbf{H}_0.$$  

The resultant Lorentz forces, $R_m = (R_{mx}), \mathbf{1} + (R_{my}), \mathbf{j} + (R_{mz}), \mathbf{k}$, and the corresponding bending moments, $M_m = (M_{mx}), \mathbf{i} + (M_{my}), \mathbf{j} + (M_{mz}), \mathbf{k}$, within the ECS caused by the magnetic field are evaluated from:

$$\langle (R_{mx}), (R_{my}), (R_{mz}) \rangle = \int_{A_b} \langle f_{m_1}, f_{m_2}, f_{m_3} \rangle \, dA,$$

$$\langle (M_{mx}), (M_{my}), (M_{mz}) \rangle = \int_{A_b} \langle 0, z(f_{m_3}), y(f_{m_3}) \rangle \, dA,$$

where $\langle f_{m_i} \rangle$; $i = x, y, z$ are the components of the Lorentzian forces, and $dA$ denotes an infinitesimal element of the cross-sectional area. According to the hypotheses of the RBT, the displacement field vector of the ECS in the framework of small displacements would be:

$$\mathbf{u}(x, y, z, t) = (u(x, t) - 2W_{0x}(x, t) - y\nu_1(x, t))\mathbf{i} + v(x, t)\mathbf{j} + w(x, t)\mathbf{k},$$

![Fig. 1. Schematic representation of a SWCNT subjected to a three-dimensional magnetic field.](image-url)
where $u(x, t)$, $v(x, t)$, and $w(x, t)$ denote the displacement fields of the neutral axis of the ECS along the $x$, $y$, and $z$ axes, respectively. By introducing Eq. (10) to Eq. (8) and utilizing Eq. (9), the components of the resultant force per unit length of the ECS in terms of derivatives of the displacements are obtained as:

$$(R_m)_x = -\varphi_0 B_0 (H_0^2 + H_0^2) u_{xx} - H_0 H_0 v_{xx} - H_0 H_2 w_{xx},$$

$$(R_m)_y = -\varphi_0 B_0 (H_0^2 - H_0^2) v_{xx} - H_0 H_0 u_{xx} + H_0 H_2 w_{xx},$$

$$(R_m)_z = \varphi_0 A_0 (H_0^2 - H_0^2) w_{xx} - H_0 H_0 u_{xx} + H_0 H_2 v_{xx},$$

or

$$(M_{nl})_x = -\varphi_0 B_0 (H_0^2 + H_0^2) w_{xxx},$$

$$(M_{nl})_y = -\varphi_0 B_0 (H_0^2 + H_0^2) w_{xxx}.$$

Similarly, the induced bending moments within the ECS due to the applied magnetic field are derived as:

$$(M_{nl})_x = -\varphi_0 B_0 (H_0^2 + H_0^2) w_{xxx},$$

$$(M_{nl})_y = -\varphi_0 B_0 (H_0^2 + H_0^2) w_{xxx}.$$

Based on the nonlocal Rayleigh beam theory, the equations of motion of an ECS when taking into account the induced forces because of the applied magnetic fields are expressed by:

$$\rho_0 A_0 \ddot{u} - (N_{nl})_x = (R_m)_x,$$

$$\rho_0 A_0 \ddot{v} - \rho_0 B_0 v_{xx} - M_{nl, xx} = (R_m)_y,$$

$$\rho_0 A_0 \ddot{w} - \rho_0 B_0 w_{xx} - M_{nl, xx} = (R_m)_z,$$

in Eq. (13), the nonlocal axial force, $N_{nl}^0$, and the nonlocal bending moments, $M_{nl, xx}$ and $M_{nl, xx}$, are related to the derivatives of displacements in the context of the nonlocal continuum theory of Eringen as follows:

$$N_{nl}^0 = -\varphi_0 A_0 \ddot{u} - \rho_0 B_0 (H_0^2 + H_0^2) u_{xx} - H_0 H_0 v_{xx} - (\kappa_0 \omega^2 + \varphi_0 A_0 (H_0^2 + H_0^2) u_{xxx} + \rho_0 B_0 H_0 H_0 (v_{xx} - (\kappa_0 \omega^2) w_{xxx}) + H_0 H_0 (v_{xx} - (\kappa_0 \omega^2) w_{xxx})), $$

$$M_{nl, xx} = -\varphi_0 A_0 \ddot{v} - \rho_0 B_0 v_{xx} - M_{nl, xx} = -\varphi_0 A_0 \ddot{w} - \rho_0 B_0 w_{xx} - M_{nl, xx} = 0,$$

in which $\omega_0$ is the small-scale parameter. By combining Eqs. (13) and (14) by using Eqs. (11) and (12), the nonlocal equations of motion of a SWCNT subjected to a three-dimensional magnetic field as a function of the displacements of the ECS are obtained with:

$$\rho_0 A_0 \ddot{u} - (\kappa_0 \omega^2) \ddot{u}_{xx} - \rho_0 A_0 (H_0^2 + H_0^2) u_{xx} + \rho_0 B_0 H_0 H_0 (v_{xx} - (\kappa_0 \omega^2) w_{xxx}) + H_0 H_0 (v_{xx} - (\kappa_0 \omega^2) w_{xxx}) = 0,$$

$$\rho_0 A_0 \ddot{v} - \rho_0 B_0 (H_0^2 + H_0^2) v_{xx} - (\kappa_0 \omega^2) w_{xxx} + \rho_0 B_0 H_0 H_0 (u_{xx} - (\kappa_0 \omega^2) u_{xxx}) = 0,$$

$$\rho_0 A_0 \ddot{w} - (\kappa_0 \omega^2) w_{xx} - (\kappa_0 \omega^2) w_{xxx} + \rho_0 B_0 H_0 H_0 (u_{xx} - (\kappa_0 \omega^2) u_{xxx}) = 0,$$

in the interested of linearity, following dimensionless parameters are defined:

$$\xi = \frac{x}{l_0}, \ \eta = \frac{u}{b_0}, \ \gamma = \frac{v}{b_0}, \ \zeta = \frac{w}{b_0}, \ \tau = \frac{t}{l_0}, \ \lambda = \frac{\rho_0 b_0}{\varphi_0 A_0},$$

$$\lambda = \frac{b_0}{l_0}, \ \mu = \frac{\omega_0}{b_0}, \ \Gamma = \frac{b_0}{l_0} \gamma, \ m = x, y, \text{ or } z,$$

where $b_0 = \sqrt{l_0 / \lambda}$ denotes the gyration radius of the cross-section of the ECS, $\lambda$ is the slenderness ratio, and finally $\xi$ and $\tau$ represent the dimensionless longitudinal coordinate and the dimensionless time parameter, respectively. By introducing Eq. (16) to Eq. (15), the dimensionless equations of motion of a SWCNT due to an arbitrary magnetic field based on the NRBT are as follows:

$$\begin{align*}
\vec{\nabla}_{\tau x} \cdot \mu^2 \vec{\nabla}_{\tau} - \lambda^2 &+ \vec{\nabla}_{\tau x} \cdot \mu^2 \vec{\nabla}_{\tau} - \lambda^2 (\vec{\nabla}_{\tau x} \cdot \mu^2 \vec{\nabla}_{\tau x} + \vec{\nabla}_{\tau x} \cdot \mu^2 \vec{\nabla}_{\tau x}) = 0,
\end{align*}$$

$$\begin{align*}
\vec{\nabla}_{\tau y} \cdot \mu^2 \vec{\nabla}_{\tau} - \lambda^2 (\vec{\nabla}_{\tau y} \cdot \mu^2 \vec{\nabla}_{\tau y} + \vec{\nabla}_{\tau y} \cdot \mu^2 \vec{\nabla}_{\tau y}) = 0,
\end{align*}$$

$$\begin{align*}
\vec{\nabla}_{\tau z} \cdot \mu^2 \vec{\nabla}_{\tau} - \lambda^2 (\vec{\nabla}_{\tau z} \cdot \mu^2 \vec{\nabla}_{\tau z} + \vec{\nabla}_{\tau z} \cdot \mu^2 \vec{\nabla}_{\tau z}) = 0,
\end{align*}$$

where

$$L_{11} = -\kappa_0^2 (1 + \mu^2 \kappa_0^2 + \lambda^2 \kappa_0^2 + \lambda^2 \kappa_0^2) (1 + \mu^2 \kappa_0^2),$$

$$L_{12} = \kappa_0^2 - \kappa_0^2 (1 + \mu^2 \kappa_0^2) (1 + \lambda^2 \kappa_0^2) + \lambda^2 \kappa_0^2 (1 + \mu^2 \kappa_0^2),$$

$$L_{12} = -\kappa_0^2 (1 + \mu^2 \kappa_0^2),$$

$$L_{13} = -\kappa_0^2 (1 + \mu^2 \kappa_0^2),$$

$$L_{14} = \kappa_0^2 (1 + \mu^2 \kappa_0^2).$$

In order to evaluate the dimensionless frequencies of the propagated longitudinal and transverse waves within the SWCNT that was subjected to a three-dimensional magnetic field, the determinant of the set of equations in Eq. (19) is set equal to zero. Therefore, we could arrive at the following equation:

$$c_1 \omega^2 + c_2 \omega^2 + c_1 \omega^2 + c_0 = 0,$$

where

$$c_i, \ 0 \leq i \leq 3$$

are as follows:

$$c_1 = (1 + \mu^2 \kappa_0^2)(1 + \lambda^2 \kappa_0^2),$$

$$c_2 = (1 + \mu^2 \kappa_0^2)(1 + \lambda^2 \kappa_0^2),$$

$$c_3 = (1 + \mu^2 \kappa_0^2)(1 + \lambda^2 \kappa_0^2)(c_{12} + c_{13} + c_{14} + c_{11}),$$

$$c_4 = (1 + \mu^2 \kappa_0^2)(c_{12} - c_{13} - c_{14} + c_{11}),$$

and

$$c_i, \ 1 \leq i \leq 3$$

are as follows:

$$c_1 = \kappa_0^2 (1 + \mu^2 \kappa_0^2),$$

$$c_2 = \kappa_0^2 + \lambda^2 \kappa_0^2 (1 + \mu^2 \kappa_0^2) + \lambda^2 \kappa_0^2 (1 + \mu^2 \kappa_0^2),$$

$$c_3 = \kappa_0^2 (1 + \mu^2 \kappa_0^2) + \lambda^2 \kappa_0^2 (1 + \mu^2 \kappa_0^2),$$

$$c_4 = -\kappa_0^2 (1 + \mu^2 \kappa_0^2),$$

$$c_5 = \kappa_0^2 (1 + \mu^2 \kappa_0^2),$$

$$c_6 = \kappa_0^2 (1 + \mu^2 \kappa_0^2).$$

In the following section, sound wave propagation and buckling of a SWCNT subjected to a general magnetic field will be examined in some specific cases.
3. Results and discussions

Consider an ECS with the following data: $E_b = 1$ TPa, $r_0 = 3$ nm, and $t_0 = 0.34$ nm. The dispersion curves of both longitudinal and transverse waves within the SWCNT exposed to different magnetic fields are explored below. The roles of the small-scale parameter, slenderness ratio, and the strength of the longitudinal and transverse magnetic fields on the lateral buckling magnetic fields of the simply supported SWCNTs are also addressed.

3.1. $\Pi_x \neq 0; \Pi_y = \Pi_z = 0$

In the absence of the transverse magnetic fields applied to the SWCNT, Eq. (21) is simplified to the following equation:

$$(- \sigma^2(1 + \mu^2\kappa^2) + x^2\kappa^2)^2 (1 + \mu^2\kappa^2) + \Pi_x^2\kappa^2 (1 + x^2\kappa^2)^2 = 0,$$

Thus, the dimensionless longitudinal and flexural frequencies are readily calculated as:

$$\sigma_l = \frac{x^2}{1 + \mu^2\kappa^2}, \quad \sigma_f = \frac{k^4}{(1 + x^2\kappa^2)(1 + \mu^2\kappa^2)} + \frac{\Pi_x^2\kappa^2}{1 + x^2\kappa^2}$$  \hspace{1cm} (25)

As seen in Eq. (25), the longitudinal magnetic field has no effect on the longitudinal frequency of the SWCNT in the context of a small deflection. The phase velocity is defined as the ratio of the frequency to the wavenumber. The dimensionless longitudinal and transverse phase velocities are now defined by $v_x = \sigma_l / (jk)$, $v_t = \sigma_f / (k)$, where $V_x$ and $V_t$ denote the phase velocities associated with the longitudinal and transverse waves, respectively. The plots of the phase velocities of the longitudinal and transverse waves are provided in Figs. 2(a) and 2(b), respectively, for different levels of axially applied magnetic fields as well as with the small-scale parameter. As seen in Fig. 2, the phase velocities of both longitudinal and transverse waves decrease with the small-scale parameter. The axially applied magnetic field has no effect on the phase velocity of the longitudinal waves; however, the phase velocity of the transverse waves increases as the axially applied magnetic field increases. The main reason for this is that the application of a longitudinal magnetic field on the nanostructure leads to an increase of the structure's transverse stiffness (see statements of $L_{22}$ and $L_{33}$ in Eq. (20)). Thus, the frequencies associated with the transverse magnetic vibration magnify as the strength of the longitudinal magnetic field increases.

3.2. $\Pi_x = 0; \Pi_y \neq 0; \Pi_z \neq 0$

In the absence of a longitudinal magnetic field, the dimensionless longitudinal and flexural frequencies are obtained by solving Eq. (21) as follows:

$$\sigma_l = \frac{x^2}{1 + \mu^2\kappa^2} + \Pi_y^2 + \Pi_z^2,$$

$$\sigma_f = \frac{c_{22} + c_{33} - \sqrt{(c_{22} + c_{33})^2 - 4(c_{22}c_{33} - c_{23}^2)}}{2(1 + \mu^2\kappa^2)}$$

$$\sigma_f = \frac{c_{22} + c_{33} + \sqrt{(c_{22} + c_{33})^2 - 4(c_{22}c_{33} - c_{23}^2)}}{2(1 + \mu^2\kappa^2)}$$  \hspace{1cm} (26)

The phase velocities of the longitudinal and transverse waves are presented in Figs. 2(a)–(c). The plotted results are provided for three levels of the small-scale parameter as well as three levels of the transversely applied magnetic fields. As seen in Fig. 3, the phase velocity of the longitudinal waves increases as the laterally applied magnetic field increases in strength. For lower wavenumbers (i.e., $k < 20$), the phase velocity of the flexural waves decreases as the applied transverse magnetic field increases in strength. However, for higher wavenumber (i.e., $k > 20$), the phase velocity of the flexural waves increases as the transverse magnetic increases in strength. As the wavenumber increases, the influence of the transverse magnetic field on the phase velocity of the flexural waves becomes more apparent. Additionally, the classical continuum theory overestimates the magnitudes of the phase velocities of the propagated waves. The phase velocities of both longitudinal and transverse waves decrease as the effect of the small-scale parameter becomes highlighted. This is more obvious for the longitudinal waves than for the transverse waves. Such reductions are mainly attributed to the incorporation of the small-scale parameter into the inertial term of the nanostructure (see expressions of $L_{11}$, $L_{22}$, and $L_{33}$ in Eq. (20)). The inertial effects of...
the nanostructure generally magnify as the small-scale parameter increases. Therefore, the frequencies of the nanostructure decrease as the small-scale parameter increases, and the phase velocities decrease as well.

Under special circumstances, \( \nu_1 \) could be equal to zero according to Eq. (26) and this instability would originate within the SWCNT. The requirement of such a condition is:

\[
c_{zz} c_{33} \leq c_{22}^2.
\]

By doing some manipulations, Eq. (27) leads to:

\[
\overline{H}_y^2 + \overline{H}_z^2 \geq \frac{k^2}{(1 + \mu^2 x^2)(1 - \lambda^2 - x^2)}.
\]

For a SWCNT with simply supported and immovable ends, the geometrical and force boundary conditions in order are:

\[
\Pi_i(\zeta_i, \tau) = \overline{\Pi_i}(\zeta_i, \tau) = \overline{\overline{\Pi_i}}(\zeta_i, \tau) = 0 \quad \text{and} \quad \Pi_{0i}(\zeta_i, \tau) = M_{0i}(\zeta_i, \tau) = 0; \quad i = 1, 2,
\]

where \( \zeta_1 = 0 \) and \( \zeta_2 = 1 \). In such a case, the dimensionless wave number associated with the nth modes of both longitudinal and transverse vibrations would be \( \pi \). Thus, \( \overline{K} = \pi \) is the corresponding wavenumber of the first vibration mode. In such a case, the instability condition is as follows:

\[
\overline{H}_y^2 + \overline{H}_z^2 \geq \frac{\pi^2}{(1 + \mu^2 x^2)(1 - \lambda^2 - x^2)}.
\]

Eq. (29) displays the coordinates of the points in the \( \Pi_y - \Pi_z \) plane that yield the instability of a simply supported SWCNT (i.e., instable zone). These points are placed on the pyramid and outside of a circle with center (0, 0) and radius \( \sqrt{(1 + \mu^2 x^2)(1 - \lambda^2 - x^2)} \). For a given dimensionless transversely applied magnetic field along the y direction, \( \overline{\Pi}_y \), the critical transverse magnetic field, \( \overline{\Pi}_{zz,c} \), can be calculated as:

\[
\overline{\Pi}_{zz,c} = \sqrt{\frac{\pi^2}{(1 + \mu^2 x^2)(1 - \lambda^2 - x^2)}} - \overline{H}_y^2.
\]

As seen in this equation, consideration of the small-scale parameter leads to instability of the SWCNT for lower levels of the transverse magnetic fields. Furthermore, a slender SWCNT would more likely buckle at lower levels of transverse magnetic fields relative to a stocky SWCNT. It is also worth mentioning that the term \( \lambda^2 \) in Eq. (30) is related to the rotary inertia effect of the SWCNT which is incorporated into the formulations by the Rayleigh beam model. In the absence of this term (i.e., hypotheses of the Euler-Bernoulli beam theory), the lateral magnetic field corresponding to the instability of the SWCNT is underestimated.

3.3. \( \Pi_x \neq 0; \Pi_y = 0; \Pi_z \neq 0 \)

In such a case, the roots of Eq. (21)—a third-order algebraic equation for \( \nu^2 \)—can be evaluated numerically and the dimensionless natural frequencies of a SWCNT subjected to a general magnetic field can be determined. The variations of the dimensionless longitudinal and transverse frequencies of the simply supported SWCNT in terms of dimensionless longitudinal and transverse magnetic fields are demonstrated in Figs. 4(a)–(c). As shown in clear from Fig. 4(a), the dimensionless longitudinal frequency increases as the transversely applied magnetic field increases in strength. However, variation of the longitudinal magnetic field has no effect on the variation of the frequency of the longitudinal wave. Both transverse frequencies of the SWCNT increase as the strength of longitudinally applied magnetic field increases. The first flexural frequency drastically decreases as the strength of the transverse magnetic field increases, however, the second one slightly increases under the same conditions. In Fig. 4(b), the points associated with \( \sigma_1 = 0 \) denote the buckled zone. As seen in this figure, the transverse magnetic field corresponding to the SWCNT lateral buckling, called the critical transverse magnetic field (CTMF), increases as the longitudinal magnetic field increases. The effects of both the small- scale parameter and longitudinal magnetic field on the CTMF are other interesting subjects that will be explored later in the paper.

In Figs. 5(a)–(c), the plots of the longitudinal and flexural frequencies as functions of the components of the transverse magnetic field are shown for a SWCNT with \( \lambda = 30 \) and \( \sigma_e a = 2 \) nm experiences with \( \Pi_x = 3 \). As is clear shown in Figs. 5(a)–(c), both the longitudinal and the second flexural frequencies increase as their transverse magnetic fields increase in strength. The first flexural frequency obviously decreases under the same conditions. Furthermore, the variations of the lateral magnetic fields have less effect on the variation of the second flexural frequency in compare to other ones.

For a simply supported SWCNT, the requirement for initiation of flexural instability (i.e., lateral buckling) is \( \nu = 0 \) for \( \overline{K} = \pi \). Using
Eq. (31), we arrive at the instability condition:

\[
\begin{align*}
\alpha_1 & = \alpha_1^2 + \alpha_1^2 (\gamma_1^2 + \gamma_2^2), \\
\alpha_2 & = \alpha_2^2 + \alpha_2^2 (\gamma_1^2 - \gamma_2^2) + \alpha_2^2 (\gamma_1^2 + \gamma_2^2), \\
\alpha_3 & = \alpha_3^2 + \alpha_3^2 (\gamma_1^2 - \gamma_2^2) + \alpha_3^2 (\gamma_1^2 + \gamma_2^2), \\
\alpha_4 & = -\alpha_4^2 (\gamma_1^2 - \gamma_2^2) - \alpha_4^2 (\gamma_1^2 + \gamma_2^2), \\
\end{align*}
\]

(31)

where the parameters \(\alpha_i; i = 1, \ldots, 4\) are defined as:

\[
\begin{align*}
\alpha_1 & = \lambda \alpha, \alpha_2 = \kappa \sqrt{1 + \mu^2 \alpha^2}, \alpha_3 = \kappa^2, \alpha_4 = \kappa \times \sqrt{1 + \mu^2 \alpha^2}. \\
\end{align*}
\]

(32)

Eq. (31) can be rewritten in terms of \(\gamma_3\) as follows:

\[
\begin{align*}
\gamma_1^2 & \gamma_4^2 + \gamma_2^2 \gamma_2^2 + \gamma_4^2 \gamma_4^2 + \gamma_0^2 = 0, \\
\end{align*}
\]

(33)

where

\[
\begin{align*}
\gamma_1^2 & = \alpha_1^2 \alpha_1^2 (\gamma_1^2 - \gamma_2^2), \\
\gamma_2^2 & = -\alpha_2^2 (\gamma_1^2 - \gamma_2^2) \gamma_4^2 + \alpha_4^2 \gamma_4^2 (\gamma_1^2 + \gamma_2^2). \\
\end{align*}
\]

(34)

In general, Eq. (34) has six roots which could be evaluated numerically or even analytically. The minimum value of the positive real roots corresponds to the flexural buckling of the SWCNT (i.e., CTMF) and is represented by \(\gamma_3^{cr}\). In Fig. 6, the plots of \(\gamma_3^{cr}\) in terms of \(\gamma_3\) are shown for different levels of the small-scale parameter as well as for different longitudinal magnetic fields. As seen in Fig. 6, for a given value of \(\gamma_3\), the CTMF increases as the longitudinal magnetic field increases, however, the CTMF
increasing the strength of the longitudinal magnetic field as well as for different small-scale parameter values: ((...) εdL = 0 nm,
(−) εdL = 1 nm, (−) εdL = 2 nm; (−) πHx = 0, (−) πHx = 3, (−) πHx = 6, (−) πHx = 9; λ = 10).

decreases as the effect of the small-scale parameter becomes highlighted.

Fig. 7 shows the role of the slenderness ratio of the simply supported and magnetically affected SWCNT on the normalized CTMF, denoted by HxSWCNT. As seen in Fig. 7, by increasing the strength of the longitudinal magnetic field, the strength of the transverse magnetic field corresponding to the buckling of the nanostructure increases. Irrespective of the level of the applied longitudinal magnetic field, the predicted CTMF decreases as the slenderness ratio of the SWCNT increases. Further, for higher longitudinal magnetic field strengths, the slenderness ratio influences the normalized CTMF more significantly.

4. Conclusions

Free vibrations and lateral instability of SWCNTs subjected to three-dimensional magnetic fields were studied. An isotropic homogeneous ECS associated with the SWCNT was considered. In the context of the nonlocal continuum theory of Eringen, the ECS was modeled based on the hypotheses of the Rayleigh beam theory. The arbitrarily induced magnetic field was taken into account in the model through appropriate body forces. The dimensionless governing equations describing free vibration of a SWCNT in both longitudinal and lateral directions were derived.

The corresponding characteristic equation was then obtained. In the case of the ECS with simply supported and immovable ends, the lateral instability was investigated. The effects of the small-scale parameter as well as the longitudinally and transversely induced magnetic fields on the longitudinal frequencies, flexural frequencies and lateral instability of the SWCNT were discussed. The results show that the lateral instability is possible in the presence of only a lateral component of the applied magnetic field. The critical transverse magnetic field (CTMF) that corresponded to the lateral instability of the SWCNT was also determined for different case studies. The results reveal that CTMF increases as the longitudinally applied magnetic field increases in strength. However, the CTMF decreases as the influence of the transverse magnetic field component on the SWCNT becomes highlighted. Further, for a SWCNT subjected to a transverse magnetic field, the possibility of lateral instability magnifies as the small-scale parameter or the slenderness ratio of the SWCNT increases.

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