Column buckling of doubly parallel slender nanowires carrying electric current acted upon by a magnetic field

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Axial buckling of current-carrying double-nanowire-systems immersed in a longitudinal magnetic field is aimed to be explored. Each nanowire is affected by the magnetic forces resulted from the externally exerted magnetic field plus the magnetic field resulted from the passage of electric current through the adjacent nanowire. To study the problem, these forces are appropriately evaluated in terms of transverse displacements. Subsequently, the governing equations of the nanosystem are constructed using Euler-Bernoulli beam theory in conjunction with the surface elasticity theory of Gurtin and Murdoch. Using a meshless technique and assumed mode method, the critical compressive buckling load of the nanosystem is determined. In a special case, the obtained results by these two numerical methods are successfully checked. The roles of the slenderness ratio, electric current, magnetic field strength, and interwire distance on the axial buckling load and stability behavior of the nanosystem are displayed and discussed in some detail.

1. Introduction

To fabricate three-dimensional flexible circuits, ensembles of vertically aligned nanowires are synthesized as the interconnector between two metallic surface layers [1]. Additionally, some researchers [2,3] have developed miniaturized magnetometer for mapping magnetic field that could have many applications in industry, space physics, biomedicine, oceanography, and environmental sciences. For most of these applications, ensembles of elastically supported current-carrying nanowires (CCNWs) are acted upon by a magnetic field, and understanding the load-bearing capacity of such nanosystems is of great importance. To answer this crucial question: what is the mechanical response of a group of vertically aligned CCNWs acted upon by a magnetic field?, initially, we should realize the buckling behavior of a magnetically affected nanosystem that consists of doubly parallel CCNWs.

At the nanoscale, the ratio of the surface to the volume of solid is high enough that the surface strain energy should be appropriately incorporated into the total strain energy. To this end, Gurtin and Murdoch [4,5] proposed an elegant elasticity model for the surface layer. In fact, the surface was introduced as a very thin layer adhered to its adjacent bulk. The equations of motion as well as the constitutive relations of the surface layer were originally developed by Gurtin and Murdoch. For isotropic surface layers, the surface stresses are linked to the surface strains through three constants (i.e., residual surface stress under unconstrained conditions, and two Lame’s constants). Since the surface layer has been tightly attached to the bulk, the displacements and strains of the surface layer are identical to those of the neighboring bulk. Up to now, surface elasticity theory of Gurtin–Murdock has been widely exploited in buckling [6–12], postbuckling [13–15], static analysis [16–18], as well as free and forced vibrations [19–25] of nanoscaled beam-like structures.

Regarding individual CCNWs in the presence of a longitudinal magnetic field, their free dynamic responses [26], forced vibrations [27], and axial buckling behavior [28,29] were investigated. In these studies, the roles of the initial tensile force within the nanowires, surface effect, and shear deformation in the mechanical response of the nanostructure were displayed.

The mechanical response of doubly current-carrying nanowires (DCCNWs) is completely different from that of individual CCNWs. When an electrical current passes through doubly parallel nanowires, the produced magnetic field around each nanowire affects buckling and vibration behavior of its neighboring nanowire. Commonly, the magnitude and direction of the magnetic field are displayed by the Biot–Savart law. Such a magnetic field plus the longitudinally exerted one would result in the resultant magnetic field. Based on Lorentz’s law, the applied magnetic force on each deformed nanowire is the product of the electric current and the resultant magnetic field. This force can be evaluated in terms of displacements of the DCCNWs. As it will be shown, the transverse magnetic force corresponds to the instability of the nanosystem. In...
the present work, the author investigates how such a physical mechanism influences on the axial buckling behavior of the nanosystem. To date, free transverse vibrations of magnetically affected lengthy DCCNWs have been investigated [30,31], and further investigations on their buckling behavior should be carried out.

By using Euler–Bernoulli beam theory (EBT) and considering the surface energy effect, the governing equations pertinent to the elastic buckling of the nanosystem are derived. To solve the resulted set of ordinary differential equations, reproducing kernel particle method (RKPM) is adopted. In brief, the RKPM was developed by Liu et al. [32–34], and has been broadly implemented for mechanical analysis of structures [35–38]. In the case of the nanosystem with simply supported ends, the predicted axial buckling loads by the RKPM are also successfully verified with those obtained from the assumed mode method. The roles of the magnetic field strength, electric current intensity, slenderness ratio, and interwire distance on the critical buckling load of the nanosystem are addressed and discussed. The obtained results from this research work would be very useful for optimal design and fabrication of the upcoming nano-electro-mechanical systems (NEMS) and magnetometers. It is hoped that this study would provide the basic steps for better understanding the axial load transfer mechanisms in ensembles or group of magnetically affected nanowires carrying electric current.

2. Evaluation of dynamically applied magnetic forces on the CCNWs

Consider doubly parallel nanowires of length $l_b$ at the initial interwire distance $d$ that carry electric current $I_0$ as shown in Fig. 1.

![Fig. 1. A current-carrying double-nanowire-system immersed in a longitudinal magnetic field.](image-url)

The diameter, cross-sectional area, and moment inertia of each nanowire in order are $D_0$, $A_0$, and $I_b$, respectively. The nanosystem has been immersed in a longitudinal magnetic field $B = B_0 \hat{e}_x$. The transverse displacement fields of the ith nanowire along the $y$ and $z$ directions are represented by $u_{yi} = u_{yi}(x, y, z, t)$ and $u_{zi} = u_{zi}(x, y, z, t)$, $i = 1, 2$.

In the context of small deformations, the electric current vector within the nanowires takes the following form:
\[ \mathbf{i}_i = \mathbf{i}_0 \left( \hat{e}_x + \frac{du_{yi}}{dt} \hat{e}_y + \frac{du_{zi}}{dt} \hat{e}_z \right). \]

For each deformed nanowire, the resultant magnetic field is evaluated by adding the externally applied longitudinal magnetic field to that caused by its neighboring current-carrying nanowire. In view of Biot–Savart’s law, such resultant magnetic fields for the nanowires 1 and 2 carrying electric current are formulated as follows:
\[ B_1 = B_0 \hat{e}_x + \frac{\mu_0}{4\pi} \int_{\frac{-l_b}{2}}^{\frac{l_b}{2}} \frac{l_1 \times r_1}{r_1^2} \, dx, \]
\[ B_2 = B_0 \hat{e}_x + \frac{\mu_0}{4\pi} \int_{\frac{-l_b}{2}}^{\frac{l_b}{2}} \frac{l_2 \times r_2}{r_2^2} \, dx, \]

where $\mu_0 = 4\pi \times 10^{-7} \text{T m/A}$ denotes the permeability of free space, and $r_1 = r_2 = (x_2 - x_1) \hat{e}_x + (d + \Delta u_y) \hat{e}_y + \Delta u_z \hat{e}_z$ are the position vectors such that $\Delta u_y = u_{y2} - u_{y1}$ and $\Delta u_z = u_{z2} - u_{z1}$.

Based on the Lorentz’s law, the exerted magnetic force per unit length of the ith nanowire is expressed by $\mathbf{f}_m = \mathbf{i}_i \times \mathbf{B}$. For fairly lengthy nanowires (i.e., $l_b \rightarrow \infty$), the magnetic forces per unit length of the nanowires are obtained as:
\[ f_{m_1} = f_{m_1y} \hat{e}_y + f_{m_1z} \hat{e}_z = \left( B_0 l_0 \frac{du_{y1}}{dx} - \frac{\mu_0 B_0^2}{2\pi d} \left( 1 - \frac{\Delta u_y}{d} \right) \right) \hat{e}_y \]
\[ + \left( - B_0 l_0 \frac{du_{z1}}{dx} + \frac{\mu_0 B_0^2}{2\pi d} \left( \frac{\Delta u_z}{d} \right) \right) \hat{e}_z, \]
\[ f_{m_2} = f_{m_2y} \hat{e}_y + f_{m_2z} \hat{e}_z = \left( B_0 l_0 \frac{du_{y2}}{dx} - \frac{\mu_0 B_0^2}{2\pi d} \left( 1 - \frac{\Delta u_y}{d} \right) \right) \hat{e}_y \]
\[ - \left( B_0 l_0 \frac{du_{z2}}{dx} + \frac{\mu_0 B_0^2}{2\pi d} \left( \frac{\Delta u_z}{d} \right) \right) \hat{e}_z. \]

Excluding the static terms from Eqs. (2a) and (2b), the components of the applied Lorentz forces on the nanowires would be:
\[ f_{m_1y} = B_0 l_0 \frac{du_{y1}}{dx} - \frac{\mu_0 B_0^2}{2\pi d^2} (u_{y2} - u_{y1}), \]
\[ f_{m_2y} = B_0 l_0 \frac{du_{y2}}{dx} - \frac{\mu_0 B_0^2}{2\pi d^2} (u_{y2} - u_{y1}), \]
\[ f_{m_1z} = - B_0 l_0 \frac{du_{z1}}{dx} - \frac{\mu_0 B_0^2}{2\pi d^2} (u_{z2} - u_{z1}), \]
\[ f_{m_2z} = - B_0 l_0 \frac{du_{z2}}{dx} - \frac{\mu_0 B_0^2}{2\pi d^2} (u_{z2} - u_{z1}). \]

3. Formulations of the problem using surface elasticity theory

In this part, the continuum-based governing equations based on the EBT are given and then will be solved via an efficient numerical scheme. Generally, the suggested model can be employed for buckling analysis of slender or very slender magnetically
affected nanowires carrying electric current. For these nanostructures, the share of the shear strain energy to the total elastic energy is small enough that the role of shear deformation in the mechanical behavior could be safely neglected. To study axial buckling of stocky or very stocky nanosystems, appropriate shear deformable beams should be exploited. The application limit of the EBT as well as the Rayleigh beam theory in predicting transverse vibration of magnetically affected nanotubes was examined carefully [39,40]; however, more accurate determination of limitations of the proposed model for the nanosystem at hand needs more explorations that could be considered for future works.

3.1. Displacement–strain–stress relations of the bulk and the surface layer

3.1.1. Strain–displacement relations

Both surface layer and bulk zone undergo the same transverse displacements. Based on the EBT, the displacement fields of the constitutive CCNWs of the nanosystem are expressed by:

\[ u_i(x, y, z) = - \left( z \frac{d^2w_i(x)}{dx^2} + y \frac{d^2v_i(x)}{dx^2} \right); \quad i = 1, 2, \]

\[ u_i(x, y, z) = v_i(x), \quad u_i(x, y, z) = w_i(x), \]

The only nonzero strain field of the bulk zone reads:

\[ \sigma_{xx} = - \left( \frac{d^2w_i(x)}{dx^2} + \frac{d^2v_i(x)}{dx^2} \right) \]

(4)

3.1.2. Stress–displacement relations

According to the Gurtin–Murdoch surface elasticity model [4,5], the stress state within the surface layer is stated by:

\[ \tau_{xx} = \tau_0 + (\lambda_0 + 2\mu_0) \frac{du_i}{dx} = \tau_0 - (\lambda_0 + 2\mu_0) \left( \frac{d^2w_i}{dx^2} + \frac{d^2v_i}{dx^2} \right), \]

\[ \tau_{xy} = n_y \frac{dv_i}{dx}, \quad \tau_{xz} = n_z \frac{dw_i}{dx}, \]

(6)

where \( \tau_{xx} \) is the surface normal stress, \( \tau_{xy} \) and \( \tau_{xz} \) are the surface shear stresses, \( \tau_0 \) is the residual surface stress under unconstrained conditions, \( \lambda_0 \) and \( \mu_0 \) are the surface Lamé’s constants, and \( n = n_x \hat{e}_x + n_z \hat{e}_z \) is the unit normal vector of the surface layer.

By assuming a linear variation of the bulk normal stresses along the \( y \)- and \( z \)-axes between their surface values, one can arrive at:

\[ \sigma_{xx} = E_b \varepsilon_{xx} + \nu_b (\sigma_{yy} + \sigma_{zz}), \]

\[ \sigma_{xy} = - \frac{1}{\sqrt{2}} \left( E_b - \frac{2\lambda_0 + \mu_0}{D_0} \right) \frac{d^2w_i}{dx^2} - \frac{1}{\sqrt{2}} \left( E_b - \frac{2\lambda_0 + \mu_0}{D_0} \right) \frac{d^2v_i}{dx^2} \]

where \( E_b \) and \( \nu_b \) are Young’s modulus and Poisson’s ratio of the bulk material, respectively. The shear stresses of the bulk are zero since their corresponding strains are zero on the basis of the EBT.

3.2. Equations of motion based on the EBT

Using Newton’s second law for both surface layer and bulk, the transverse equations of motion of the constitutive CCNWs of the nanosystem in the presence of a longitudinal magnetic field are obtained as follows:

\[ -\frac{dQ_{b_{y1}}}{dx} + \int_S y \frac{d\sigma_{xx1}}{dx} dS - Q_{b_{y1}} = 0, \]

\[ -\frac{dQ_{b_{z1}}}{dx} + \int_S \frac{d\tau_{xz}}{dx} dS - \frac{\mu_0 B_{0}^2}{2\pi d^2} \int_S (u_{y2} - u_{y1}) dS = 0, \]

\[ \frac{dM_{b_{y1}}}{dx} + \int_S y \frac{d\sigma_{xx1}}{dx} dS - Q_{b_{y1}} = 0, \]

\[ \frac{dM_{b_{z1}}}{dx} + \int_S \frac{d\tau_{xz}}{dx} dS - \frac{\mu_0 B_{0}^2}{2\pi d^2} \int_S (u_{y2} - u_{y1}) dS = 0, \]

\[ -\frac{dM_{b_{y2}}}{dx} + \int_S y \frac{d\sigma_{xx2}}{dx} dS - Q_{b_{y2}} = 0, \]

\[ -\frac{dM_{b_{z2}}}{dx} + \int_S \frac{d\tau_{xz}}{dx} dS - \frac{\mu_0 B_{0}^2}{2\pi d^2} \int_S (u_{y2} - u_{y1}) dS = 0, \]

where \( Q_{b_{y1}}/Q_{b_{y2}} \) and \( M_{b_{y1}}/M_{b_{y2}} \) in order are, respectively, the shear and bending moments of the ith CCNW along the \( y/z \)-axes, and \( dS \) is the length of an element of the cross-sectional perimeter. By taking the first derivative of Eqs. (8a), (8c), and (8e), (8g), and then, combining the resulting relations with Eqs. (8b), (8d), (8f), and (8h), it is obtainable:

\[ -\frac{d^2M_{b_{y1}}}{dx^2} + \int_S \left( y \frac{d^2\sigma_{xx1}}{dx^2} + \frac{d\tau_{xz}}{dx} \right) dS + N \frac{d^2u_{y1}}{dx^2} - B_{0}B_{0} \frac{d\tau_{xz}}{dx} = 0, \]

\[ \frac{dM_{b_{z1}}}{dx} + \int_S \frac{d\tau_{xz}}{dx} dS + N \frac{d^2u_{z1}}{dx^2} + B_{0}B_{0} \frac{d\tau_{xz}}{dx} = 0, \]

\[ -\frac{dM_{b_{y2}}}{dx} + \int_S \left( y \frac{d^2\sigma_{xx2}}{dx^2} + \frac{d\tau_{xz}}{dx} \right) dS + N \frac{d^2u_{y2}}{dx^2} - B_{0}B_{0} \frac{d\tau_{xz}}{dx} = 0, \]

\[ \frac{dM_{b_{z2}}}{dx} + \int_S \frac{d\tau_{xz}}{dx} dS + N \frac{d^2u_{z2}}{dx^2} - B_{0}B_{0} \frac{d\tau_{xz}}{dx} = 0, \]
where the bending moments of the \( i \)-th CCNW based on the EBT are evaluated as:

\[
\begin{align*}
M_{b1i} &= \int_{\alpha} z\sigma_{xi} \, dA = - \left( E_{bi} - \frac{2\tau_{0i} + b_0}{D_{0i}} \right) \frac{d^2w_i}{dx^2} + \frac{\mu_{0i}^2}{2\pi^2} (u_{2i} - u_{1i}), \\
M_{b0i} &= \int_{\alpha} y\sigma_{xi} \, dA = - \left( E_{bi} - \frac{2\tau_{0i} + b_0}{D_{0i}} \right) \frac{d^3v_i}{dx^3} + \frac{\mu_{0i}^2}{2\pi^2} (v_{1i} - v_{2i}).
\end{align*}
\]

By substituting Eqs. (4), (6), (10a) and (10b) into Eqs. (9a)–(9d), the governing equations of CCNWs subjected to a longitudinal magnetic field on the basis of the EBT accounting for the surface effect are derived as:

\[
\begin{align*}
\left[ E_{bi} \left( \lambda_0 + 2\mu_0 \right) \right] \frac{d^2w_i}{dx^2} - \frac{\lambda_0 + 2\mu_0}{D_{0i}} \frac{d^3v_i}{dx^3} + (N - \tau_{0i}S) \frac{d^2v_i}{dx^2} - B_{0i} \frac{d^2w_i}{dx^2} + \frac{\mu_{0i}^2}{2\pi^2} (v_{1i} - v_{2i}) &= 0, \\
\left[ E_{bi} \left( \lambda_0 + 2\mu_0 \right) \right] \frac{d^2v_i}{dx^2} - \frac{\lambda_0 + 2\mu_0}{D_{0i}} \frac{d^3v_i}{dx^3} + (N - \tau_{0i}S) \frac{d^2v_i}{dx^2} - B_{0i} \frac{d^2w_i}{dx^2} - \frac{\mu_{0i}^2}{2\pi^2} (v_{1i} - v_{2i}) &= 0, \\
\left[ E_{bi} \left( \lambda_0 + 2\mu_0 \right) \right] \frac{d^2v_i}{dx^2} - \frac{\lambda_0 + 2\mu_0}{D_{0i}} \frac{d^3v_i}{dx^3} + (N - \tau_{0i}S) \frac{d^2v_i}{dx^2} - B_{0i} \frac{d^2w_i}{dx^2} + \frac{\mu_{0i}^2}{2\pi^2} (w_{2i} - w_{1i}) &= 0, \\
\left[ E_{bi} \left( \lambda_0 + 2\mu_0 \right) \right] \frac{d^2w_i}{dx^2} - \frac{\lambda_0 + 2\mu_0}{D_{0i}} \frac{d^3v_i}{dx^3} + (N - \tau_{0i}S) \frac{d^2v_i}{dx^2} - B_{0i} \frac{d^2w_i}{dx^2} + \frac{\mu_{0i}^2}{2\pi^2} (w_{2i} - w_{1i}) &= 0.
\end{align*}
\]

To investigate buckling behavior of the nanosystem in a more general framework, the following dimensionless parameters are introduced:

\[
\begin{align*}
\xi &= \frac{v_i}{l_b}, \quad w_i = \frac{w_i}{l_b}, \quad \lambda = \frac{b_i}{l_b}, \quad \lambda_0 = \frac{b_0}{l_b}, \quad \sqrt{N} = \sqrt{\frac{\lambda_0^2}{E_{bi}l_b}}, \quad T_0 = \frac{B_{0i}l_b^2}{E_{bi}l_b}, \\
\tau_0 &= \frac{\mu_{0i}^2}{2\pi^2} (u_{2i} - u_{1i}), \quad \tau_0x = \frac{\lambda_0 + 2\mu_0}{D_{0i}} \frac{d^3v_i}{dx^3} - \frac{\lambda_0 + 2\mu_0}{D_{0i}} \frac{d^3v_i}{dx^3}, \quad \tau_0 = \frac{\tau_{0i}S}{D_{0i}l_b^2}.
\end{align*}
\]

hence, the dimensionless governing equations of magnetically affected DCCNWs take the following form:

\[
\begin{align*}
(1 + x_0) \frac{d^4v_i}{dx^4} + (N - \lambda_0x_0) \frac{d^2v_i}{dx^2} - \tau_0 \frac{d^3w_i}{dx^3} + T_0 (v_{2i} - v_{1i}) &= 0, \\
(1 + x_0) \frac{d^4v_i}{dx^4} + (N - \lambda_0x_0) \frac{d^2v_i}{dx^2} - \tau_0 \frac{d^3w_i}{dx^3} - T_0 (w_{1i} - w_{2i}) &= 0, \\
(1 + x_0) \frac{d^4w_i}{dx^4} + (N - \lambda_0x_0) \frac{d^2w_i}{dx^2} - \tau_0 \frac{d^3v_i}{dx^3} - T_0 (v_{2i} - v_{1i}) &= 0, \\
(1 + x_0) \frac{d^4w_i}{dx^4} + (N - \lambda_0x_0) \frac{d^2w_i}{dx^2} + \tau_0 \frac{d^3v_i}{dx^3} + T_0 (w_{2i} - w_{1i}) &= 0.
\end{align*}
\]

### Table 1

<table>
<thead>
<tr>
<th>( d_0 )</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{d} )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The verification of the predicted buckling loads of DCCNWs via RKPM and those obtained by the AMM for different levels of the interwire distance, electric current, and magnetic field strength (\( d_0 = 30, \tau_0 = 5 \) nm; \( b_0 = \frac{\sqrt{\lambda_0 b_0}}{\sqrt{\lambda_0 S}} \)).

3.3. Buckling analysis via RKPM

Let

\[
\begin{align*}
\xi_i &= \sum_{i=1}^{N_{p_i}} \phi_i^\xi (\xi_i) \, v_i, \quad w_i = \sum_{i=1}^{N_{p_i}} \phi_i^{w} (\xi_i) \, w_i; \quad i = 1, 2,
\end{align*}
\]

where \( N_{p_i} \) is the number of RKPM’s particles used for discretization of the unknown fields of the \( i \)-th CCNW, \( \phi_i^\xi (\xi_i) \) and \( \phi_i^{w} (\xi_i) \) are the shape functions associated with the \( i \)-th particle of the dimensionless fields \( \xi \) and \( w \), respectively, \( \xi_i \) and \( w_i \) are their corresponding unknown parameters. Now, both sides of Eqs. (13a)–(13d) in order are premultiplied by \( \delta \xi_i, \delta v_i, \delta w_i, \) and \( \delta w_i \) where \( \delta \) denotes the variational sign. By taking the integral of the resulting expressions over [0,1] and applying integration by parts in view of Eq. (14), the following set of equations is obtained:

\[
\begin{align*}
\begin{bmatrix}
|K_0|_{\xi_1} & |K_0|_{\xi_2}
|K_0|_{\xi_2} & |K_0|_{\xi_3}
|K_0|_{\xi_3} & |K_0|_{\xi_4}
|K_0|_{\xi_4} & |K_0|_{\xi_5}
\end{bmatrix}
\begin{bmatrix}
v_1
v_2
w_1
w_2
\end{bmatrix}
= \begin{bmatrix}
|K_0|_{\xi_1} & |K_0|_{\xi_2}
|K_0|_{\xi_2} & |K_0|_{\xi_3}
|K_0|_{\xi_3} & |K_0|_{\xi_4}
|K_0|_{\xi_4} & |K_0|_{\xi_5}
\end{bmatrix}
\begin{bmatrix}
0
0
0
0
\end{bmatrix}.
\end{align*}
\]
where the nonzero dimensionless matrices are provided by:

\[
\begin{bmatrix}
\int_{0}^{1} d\phi_i^{(1)} d\phi_j^{(1)} - \tilde{R} \phi_i^{(1)} \phi_j^{(1)}
\end{bmatrix}
\]

(16a)

\[
\begin{bmatrix}
\int_{0}^{1} d\phi_i^{(1)} d\phi_j^{(1)}
\end{bmatrix}
\]

(16b)

\[
\begin{bmatrix}
\int_{0}^{1} \tilde{R} \phi_i^{(1)} \phi_j^{(1)}
\end{bmatrix}
\]

(16c)

\[
\begin{bmatrix}
\int_{0}^{1} d\phi_i^{(n)} d\phi_j^{(n)}
\end{bmatrix}
\]

(16d)

\[
\begin{bmatrix}
\int_{0}^{1} d\phi_i^{(n)} d\phi_j^{(n)} - \tilde{R} \phi_i^{(n)} \phi_j^{(n)}
\end{bmatrix}
\]

(16e)

\[
\begin{bmatrix}
\int_{0}^{1} \tilde{R} \phi_i^{(n)} \phi_j^{(n)}
\end{bmatrix}
\]

(16f)

where \(i, j = 1, 2\). In the case of simply supported DCCNWs, the following conditions should be satisfied:

\[
\begin{align*}
\mathbf{w}(0) &= \mathbf{w}(1) = 0; & \bar{M}_{00}(0) &= \bar{M}_{00}(1) = 0; & \bar{M}_{02}(0) &= \bar{M}_{02}(1) = 0, \\
\bar{\tau}(0) &= \bar{\tau}(1) = 0; & \bar{M}_{22}(0) &= \bar{M}_{22}(1) = 0,
\end{align*}
\]

(17)

where \(\bar{M}_{00} = \frac{f_{00} b}{x_1} \) and \(\bar{M}_{02} = \frac{f_{02} b}{x_1}\). To satisfy the essential boundary conditions, the corrected collocation method [41] is employed. If the determinant of the coefficient matrix pertinent to the nodal parameter values of free degree-of-freedoms is set equal to zero, the critical buckling load of the magnetically affected nanosystem would be readily determined.
4. Results and discussion

4.1. A comparison study

Consider a magnetically affected nanosystem of doubly parallel silver nanowires whose duty is to carry electric current from one end to another. The geometry and mechanical properties of the bulk and the surface layer of the CCNWs are as $r_0 = 5$ nm, $E_b = 76$ GPa, $\nu = 0.26$, $\rho_b = 10,500$ kg/m$^3$, $\lambda_0 = 1$ N/m, $\mu_0 = 0.11$ N/m, and $\tau_0 = 0.89$ N/m. For RKPM analysis of each nanowire, 11 particles with identical interparticle distance, 10 similar integration cells, linear base function, 6 Gaussian points in each cell, and cubic spline window function have been used, and the dilation parameter is set equal to 3.2 times of the inter-particle distance. To check the accuracy of the suggested RKPM numerical model, the predicted critical buckling loads of the nanosystem are checked with those of the assumed mode method (AMM). In AMM analysis of the problem, the admissible mode shapes pertinent to the supports' conditions of the DCCNWs are exploited. For instance, in the case of simply supported DCCNWs, the mode shapes, $\phi_1(\xi) = \sin(\pi \xi)$; $I = 1, 2, ..., 11$, are used in buckling analysis of the nanosystem. To this end, these admissible mode shapes should be substituted into the shape functions of RKPM in Eqs. (16a)–(16h). Through solving the resulting eigenvalue equations for buckling loads, the smallest one which represents the critical compressive buckling load is readily obtained. In Table 1, the predicted axial buckling load of the magnetically affected DCCNWs based on the RKPM and AMM is provided for various levels of the electric current, magnetic field strength, and interwire distance. As it is obvious, there exists a reasonably good agreement between the predicted results by the RKPM and those of the AMM in all cases. By an increase of the electric current or magnetic field strength, the relative discrepancies between the results based on the RKPM and those obtained by the AMM would somewhat increase. Such a fact is more apparent for those nanosystems whose interwire distances are lower. The maximum above-mentioned discrepancy is reported to be about 3.85 percent in the case of $\lambda = \frac{d}{r_0} = 5$, $B = 80$, and $I = 2$.

5. Roles of crucial factors on buckling behavior of the nanosystem

Consider a current-carrying double-silver nanowire system in the presence of a longitudinal magnetic field with the given mechanical properties in the previous part. In this part, it is going to be investigated that how the slenderness ratio, electric current, magnetic field, nanowire’s radius, and interwire distance could influence on the critical buckling load of the nanosystem. Additionally, the effect of the surface energy of the nanowires on the obtained results is also explained and discussed. In all plotted results, those obtained based on the surface elasticity theory (SET) are presented by the solid lines whereas those obtained on the basis of the classical elasticity theory (CET) are demonstrated by the dashed lines.
5.1. Effect of the slenderness ratio

An important parametric study is conducted to display the influence of the slenderness ratio on the buckling behavior of the nanosystem. In Figs. 2(a)–(c), the plotted results of the critical buckling load of the magnetically affected DCCNWs as a function of the slenderness ratio have been presented for three levels of the magnetic field strength (i.e., $B_0 = 20, 40, \text{ and } 60$) as well as three values of the electric current (i.e., $I_0 = 0.01, 0.05, \text{ and } 1$). Irrespective of the electric current and magnetic field strength, both CET and SET predict that the axial buckling load of the nanosystem would lessen as the slenderness ratio increases. Such a reduction is more obvious for higher levels of the electric current and magnetic field strength. It implies that the variation of the slenderness ratio is more influential on the variation of the critical buckling loads of the nanosystems that carry higher levels of electric current and are subjected to higher magnetic field strength. For a given DCCNWs subjected to a magnetic field, the predicted buckling loads by the SET are generally greater than those obtained based on the CET. This is mainly related to this fact that the surface elastic modulus of the silver nanowires (i.e., $E_0 = \lambda_0 + 2\mu_0$) is positive. Thereby, the surface layer helps the transverse stiffness as well as bending rigidity of the nanowire, and the load-bearing capacity of the nanosystem increases. Because of this fact, the predicted slenderness ratio of the nanosystem corresponds to the zero critical buckling load, the so-called critical slenderness ratio, based on the SET is greater than that obtained by the CET. By increasing of the electric current and the magnetic field strength, the critical buckling load of the nanosystem would decrease.

In the previous part, the interwire distance was kept fixed for all levels of the electric current and magnetic field. We are also interested to examine the role of the slenderness ratio on the stability of the nanosystem for different levels of the interwire distance. To this end, the plotted results of the critical buckling load of the nanosystem in terms of the slenderness ratio have been given in Figs. 3(a)–(c). The demonstrated results are provided for three levels of the magnetic field strength (i.e., $B_0 = 5, 10, \text{ and } 15$) and three levels of the interwire distance (i.e., $d_0/\lambda_0 = 5, 10, \text{ and } 20$) in the case of a constant electric current (i.e., $I_0 = 1.5$). The plotted results based on both SET and CET display that the critical buckling load would reduce as the interwire distance decreases. It is mainly related to the reduction of the lateral stiffness of the nanosystem due to the mechanism of the applied magnetic force on each deformed nanowire because of the passage of the electric current through its adjacent deformed nanowire. Such a fact holds true for all levels of the strength of the externally applied magnetic field. Because of the above-mentioned fact, the critical slenderness ratio of the nanosystem would also reduce as two parallel nanowires become closer. It is indicated that the stability of the nanosystem is endangered as the interwire distance decreases. Regarding the capability of the CET in capturing the predicted results by the SET, the relative discrepancies between the obtained results based on the SET and those of the CET would usually grow as the interwire distance decreases. Such a fact is more apparent for those nanosystems which are subjected to a higher magnetic field strength.

5.2. Effect of the interwire distance

An interesting study is carried out to determine the role of the interwire distance on the buckling behavior of the current-
carrying double-nanowire-system in the presence of a longitudinal magnetic field. In Figs. 4(a)–(c), the plots of the critical buckling load as a function of the interwire distance have been provided for three levels of the magnetic field strength (i.e., $B_0 = 20, 50,$ and $80$) and three levels of the electric current (i.e., $I_0 = 0.50, 1.50,$ and $8.0$). For given values of magnetic field strength and electric current, the critical buckling load of the nanosystem would grow as the interwire distance increases. It is because of this fact that the influence of the interactional magnetic forces between the nanowires on their vibration would reduce. As a result, the transverse stiffness of each nanowire would increase and the critical buckling load of the nanosystem would enhance. For large enough interwire distance and low electric currents (for example, $d = 10 r_0,$ and $I_0 = 0.5$), the above-mentioned interactions between double nanowires can be safely ignored (see the demonstrated results with square markers in Figs. 4(a)–(c)). The rate of increase of the critical buckling load in terms of the interwire distance is more obvious for lower levels of the interwire distance and magnetic field strength. In other words, for higher levels of the magnetic field strength, variation of the interwire distance has a less influence on the variation of the critical buckling load of the magnetically affected DCCNWs. Furthermore, rate of growth of the critical buckling load as a function of interwire distance would magnify as the electric current within the nanowires increases. Concerning the role of the surface energy on buckling behavior of the nanosystem, a close scrutiny of the presented results in Figs. 4(a)–(b) shows that the relative discrepancies between the predicted results by the SEC and those of the CET would generally reduce as the interwire distance increases. Such a rate of reduction is more obvious for magnetically affected nanowires that carry higher electric currents. Additionally, by increasing of the magnetic field strength, the role of the surface energy on buckling behavior of the nanosystem becomes highlighted.

5.3. Effect of the nanowire’s radius

Another crucial geometry factor that could influence on the buckling behavior of the nanosystem is the radius of the nanowire. In Figs. 5(a)–(c), the plots of the critical buckling load of the nanosystem as a function of the nanowire’s radius have been presented for three levels of the magnetic field strength (i.e., $B_0 = 20, 100,$ and $200$) and three levels of the electric current (i.e., $I_0 = 0.5, 1,$ and $1.5$) for DCCNWs with $l_0=60$ nm and $d=40$ nm. As it is obvious from these figures, the axial load-bearing capacity of the nanosystem would increase as the nanowire’s radius increases. The rate of growth of the critical buckling load in terms of the nanowire’s radius is also more apparent for higher levels of the nanowire’s radius, electric current, and magnetic field strength. Generally, the predicted critical buckling loads by the CET are lower than those of the SET. Additionally, by an increase of the nanowire’s radius, the relative discrepancies between the results of the CET and those of the SET would reduce. This fact holds true for all levels of the magnetic field strength and electric current. Further, the above-mentioned rate of reduction is more obvious.
for DCCNWs with higher levels of magnetic field strength as well as electric current. Irrespective of the nanowire’s radius, the discrepancies between the predicted critical buckling loads of the nanosystem based on the CET and those of the SET would increase with the electric current. Such a fact is also more obvious for those nanosystems which are subjected to higher magnetic field strength.

6. Conclusions

Axial buckling behavior of magnetically affected slender DCCNWs is studied using Euler–Bernoulli beam theory accounting for the surface energy effect. By appropriate evaluation of the applied magnetic forces on each nanowire in terms of transverse displacements, the governing equations associated with the elastic buckling of such a nanosystem are obtained in the context of the surface elasticity theory of Gurtin–Murdock. Subsequently, RKPM is adopted to solve the four coupled equations for the critical buckling load. To ensure regarding the carried out calculations by the RKPM, the predicted buckling loads by this methodology are verified with those obtained via the AMM and a remarkable result is reported. The influences of the slenderness ratio, electric current, magnetic field strength, interwire distance, and nanowire’s radius on the critical buckling loads of the nanosystem are carefully addressed. In each study, the role of the surface energy on the obtained results is also explained.

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References