Designing Fuzzy Utility-Based Double Auctions Using Particle Swarm Optimization Algorithm

Mohammad Amin Rigi*, Shahriar Mohammadi* , Morteza Delgir**
** K. N. ToosiUniversity of Technology, Industrial Department, IT Group. **Ferdowsi University of Mashhad, Engineering Faculty, Computer Department, Iran.*
mohammadi@kntu.ac.ir, amin.rigi@ieee.org, morteza.delgir@gmail.com

Abstract

Auction refers to arbitrary resource allocation problems with self-motivated participants: Auctioneer and bidders. Double auction is a kind of auctions with multiple buyers and sellers who trade multiple goods. In double auction each trader submits his bid (price and quantity) to the auctioneer. Then the auctioneer collects all of the bids and tries to assign some buyers to some sellers according to their preferences in a way that the whole of the market reaches its maximum utility, this is called a mechanism. In a traditional double auction market each bid is a linear utility function, but in real world, behavior of buyers and sellers is not linear, therefore we propose a fuzzy non-linear utility function for traders because traders (like all humans) are thinking and making decisions in a fuzzy way. Our aim is to design an optimal double auction mechanism. To find optimum in this non-linear optimization problem, we use particle swarm optimization (PSO).

1. Introduction

Auctions are one of the oldest forms of market and some pinpoint their origin to Babylon back in 500 BC. The term auction comes from the Latin root 'auctio' which means 'increase'. In essence, auctions constitute a method of allocating goods based upon competition; consist of explicit set of rules among the interested parties. In double or auctions, multiple sellers and multiple buyers participate in order to trade a commodity [1] [2].

In a double auction (DA) market for multi unit exchanges, first sellers and buyers submit their asks and bids. A trade can be made if a buyers bid exceeds a sellers ask. A sellers ask may match several buyers bids and a buyers bid may satisfy several sellers asks. The trading rule of a market defines the organization, information exchange process, trading procedure and clearance rules of the market. It is notable that DA is mostly used in business to business (B2B) markets.

In this research we concentrate on how to design a trading mechanism for double auctions [7]. Therefore the main challenge is how we design a utility function which represents each participant's preference [8]. Almost all of the previous researches on double auctions suggested linear functions for the utility of the buyers and sellers. But in real world double auctions each buyer and seller can have different non-linear utility functions for trading goods.

Ho et al. in [9] proposed a double auction mechanism that each participant could express diverse utility functions on the goods. They called that mechanism the utility-based double auction (UDA) mechanism. They used two kinds of non-linear functions (early growth style and late growth style) to show the behavior of the buyers and sellers. The drawback of their work is that they limited the human behavior into two types of functions. And it is hard for human mind to change its utility preference in a fully mathematical model at each period in synchronous double auctions (SDA). Another problem with their work [9] is that they had an unrealistic assumption about the quantity of goods that the buyers want to buy. They assumed that a buyer's utility function is in early growth or late growth style, that both of them are not realistic. For example a buyer wants to buy about 150 unit of a specific good, which cannot be shown by early growth or late growth functions. But it can be easily showed with the fuzzy sets.

We use fuzzy sets to show the preference and the utility of buyers and sellers. In our approach a price or quantity is mapped to a value between zero and one. We will call our system fuzzy utility double auction (FUDA) mechanism through the rest of this article. To accommodate multiple complex utility functions in FUDA and find the optimal market solution of FUDA mechanism, we propose a particle swarm optimization

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(PSO) method as a tool for solving the non-linear optimization model discussed above.

The rest of the paper is organized as follows; in part 2 basic concepts of designing an auction will be introduced. In section 3 we will describe concept of utility function and explain why we use fuzzy sets. In section 4 we briefly discuss the PSO. Finally in section 5, simulation results are shown.

2. The mathematical model of double auctions

One ideal way of organizing an efficient double auction (DA) market is to let the buyers (sellers) submit bids (asks) about how any item they want to purchase (sell) and at what reservation price. Then market maker solves an optimization problem to determine how many units each agent should purchase (sell) and at what price to maximize the total profit of the market. The formulation of double auction (DA) for the optimal bidder selection model is as follows [7]:

$$\max\left(\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{a=1}^{k}(P_{i,j,a}-P_{i,a}^{\min})Q_{i,j,a}+\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{a=1}^{k}(P_{i,a}^{\max}-P_{i,j,a})Q_{i,j,a}\right)$$

With following constraints:

$$\sum_{i=1}^{m}Q_{i,a}+\sum_{j=1}^{n}Q_{j,u}=0$$

$$\forall i \in S, \forall j \in B, \forall a \in \{1,\dots,k\}$$

$$P_{i,j,a} \geq P_{i,a}^{\min} \quad \forall i \in S, \forall j \in B, \forall a \in \{1,\dots,k\}$$

$$P_{i,j,a} \leq P_{j,a}^{\max} \quad \forall i \in S, \forall j \in B, \forall a \in \{1,\dots,k\}$$

$$0 \leq Q_{i,a} \leq Q_{i,a}^{MAX} \quad \forall i \in (S \cup B), \forall a \in \{1,\dots,k\}$$

Where

m	total number of sellers
n	total number of buyers
k	total number of items
$P_{i,j,a}$	price of item a traded between seller i and buyer j
$Q_{i,j,a}$	quantity of item a traded between seller i and buyer j
$P_{i,a}^{min}$	minimum acceptable selling price
$P_{j,a}^{max}$	maximum acceptable buying price
Q^{MAX}	maximum acceptable quantity for an item
S	set of sellers
B	set of buyers

Constraints state that a seller sells no more than what he possesses and a buyer will not buy more than he needs.

3. Fuzzy utility based double auction

Utility is a customary terminology to say that if one world state is preferred to another. A utility function maps a state (or a sequence of states) onto a real number, which describes the associated degree of happiness [11]. It is obvious that each buyer and seller has its utility function of the goods they are trading. In general case a seller assign higher utilities to higher prices and a buyer assign higher utilities to lower prices. Or a seller wants to sell as much as it is possible.

From an auctioneer point of view, the goal is to design the auction mechanism in such a way that buyers (sellers) who value the goods the most (least) receive the goods (money) in the end so that the total utility function is maximized. In this research we will bring the power of fuzzy sets and systems to double auction problem, and we will open new horizons in it.

Because fuzzy logic is near to human thinking, we can imagine the utility of buyers and sellers as fuzzy (linguistic) values. Suppose that there is seller who 100% is satisfied when she sells a good at $200 and 0% satisfied when she sells that specific good at $100, then linear assumption is like figure 1. This is quite not true in real world trading.

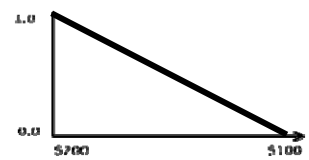

Figure 1. Linear utility assumption when a seller sells a good

In the case of figure 1 the utility of selling at $150 is 0.5 or 50%. But in most of the cases the behavior is non-linear, for example selling at $150 has utility about 0.2, like figure 2.

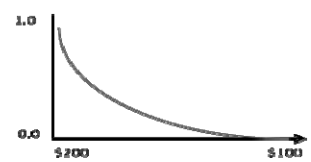

Figure 2. Non-linear utility assumption when a seller sells a good

There are some basic fuzzy membership functions that we can use them for modeling behavior of buyers and sellers. One famous function that can be used is sigmoidal function Yager (or Sugeno) complements:

$$N(x)=\left(1-x^{w}\right)^{1/w}$$

They can easily show the late growing and early

growing forms, for both buyers and sellers (figure 3 and 4).

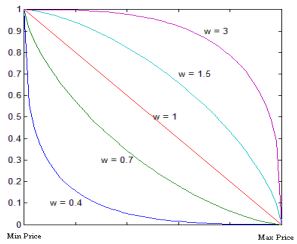

Figure 3. Yager membership function using for the buyer's utility

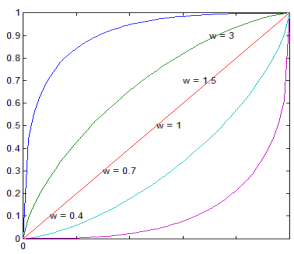

Figure 4. An extension of Yager member function (1-N(x)) for a seller's selling price utility

$$guassian(x;c,\sigma) = e^{-\frac{1}{2}(\frac{x-c}{\sigma})^2}$$

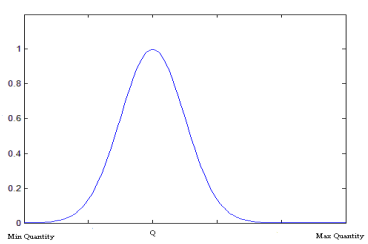

Figure 5. Guassian membership function; when a buyer prefers to buy about Q quantity

Because buying price is in a late growth increasing style, selling price is like the opposite of buying price and selling quantity for a seller is an early growth function. But buying quantity for a buyer is different, buying quantity has a minimum and a maximum threshold and a preferred value. For example buyer i at least wants 50 units of good k and maximum 100 units, but he prefers 70 units. This is a fuzzy expression too, and we can show this kind of behavior with Gaussian or generalized bell membership functions (figure 5 and 6).

$$bell(x;a,b,c) = \frac{1}{1+|\frac{x-c}{a}|^{2b}}$$

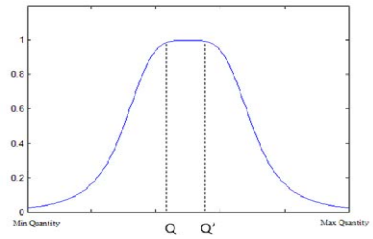

Figure 6. Generalized bell membership function to show the buyer's utility when any quantity between Q and Q' is indifferent to him

In double auction a buyer or a seller can updates its utility functions. Regardless of the membership functions parameters, there are some other operations on linguistic values to modify the utility functions (membership functions). The three famous operators [13] are concentration, dilation and contrast intensification, with the following definitions:

$$Concentration(A) = A^2$$

$$Dilation(A) = A^{0.5}$$

$$Intensification(A) = \begin{cases} 2A^2 & , \quad 0 \le \mu_A(x) \le 0.5 \\ \neg 2(\neg A)^2 , & 0.5 \le \mu_A(x) \le 1 \end{cases}$$

Assume a trader wants to update its utility function in synchronous double auction (SDA), so he can insist on his policy by using concentration operator, or he can change his policy by choosing dilation operator. Using contrast intensification several times we can reduce the fuzziness and change the fuzzy utility function to a crisp one [13]. Using fuzzy utility functions we can rewrite the mathematical model as following equation:

$$\max \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{a=1}^{k}(\mu(P_{i,j,a}) - P_{i,a}^{min})\mu(Q_{i,j,a}) + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{a=1}^{k}(P_{i,a}^{max} - \mu(P_{i,j,a}))\mu(Q_{i,j,a})$$

We call this mechanism fuzzy utility based double auction (FUDA).

4. Particle swarm optimization

Particle Swarm Optimization (PSO) is a parallel population-based computation technique developed by Kennedy and Eberhart [14]. In these groups the movement of the whole swarm is based on his own knowledge and on a leader, the one with the best

performance. Each individual of the swarm has a position in the solution hyperspace and a velocity, which is changed, at each step, to update individual position.

Each particle knows its position and the value of the fitness function for that position. Besides each particle keeps track of its coordinates in the problem space, which are associated with the best fitness value it has achieved so far. Every particle knows also the best position among all of the particles and its fitness value. Update of the particles' position is result of a comparison among three alternatives: following its current pattern of exploration; going back towards its best previous position; going towards the overall best position. The updating process is accomplished according to the following equations:

$$X_{pq}(k+1) = X_{pq}(k) + V_{pq}(k)$$

$$V_{pq}(k) = F.[w.V_{pq}(k-1) + V_{pq,local}(k-1) + V_{pq,global}(k-1)]$$

$$V_{pq,local}(k) = c_1.r.(X_{pq,local}(k) - X_{ij}(k))$$

$$V_{pq,global}(k) = c_2.r.(X_{pq,global}(k) - X_{pq}(k))$$

At each step, equation calculates a new velocity for each particle in swarm based on its velocity at previous step, the best position it has been achieved (local) and best position (global) that population has achieved yet. Then, using the resultant velocity value, position of each particle is updated by the first equation. About the coefficients in second equation, F is a constrictor factor to insure convergence, use of an inertia weight w provides to improve performance in a number of applications, while the random constants r and the coefficients c1 and c2 represent the weighting of the stochastic acceleration terms that pull each particle towards local best and global best locations. PSO is known as continuous space optimizer, but our optimization problem is a discrete one (the prices and quantities are discrete).

Consequently we need to extend the PSO to solve this discrete space problem, so we used the simplest way. We assumed that the problem is continuous and after reaching the final answer we convert the answer to discrete one by rounding the continuous value to its nearest discrete value. This assumption makes sense: the DA mechanism is mostly being used in B2B markets. B2B market has some specifications that lead us to assume that our problem (DA) is a semi-continuous problem. Let us look at the B2B market. International Data Corporation (IDC) [15] estimates worldwide B2B transaction volume at $1.4 trillion in 2003 and $2.4 trillion in 2004 and it will be around $10 trillion in 2009. The quantities are quite large enough that we can assume the quantity is a semi-continuous variable. The other variable in DA is the price. Currently there are some other units like micro-payments. For example $0.0001. So we can think of price as a semi-continuous variable too.

5. Simulation and results

To validate our method, the proposed algorithm was implemented using Matlab (v.7) and tested on an Intel Core 2 Dou 2 GHz computer. We made five double auction scenarios with random numbers of sellers, buyers and goods. We can see the information of scenarios in table 1.

The column that titled as "average of minimum price for sellers" in table 1 shows the average of minimum price that all of the sellers prefer for all of the goods in the related scenario. We have similar definitions for the five rightmost columns in table 1.

There are some parameters in PSO that they have to be set optimally. To acquire the proper values of these parameters according to problem characteristics, we run the algorithm several times with many different parameters with random initial solutions. After several experiments we set the parameters as in table 2.

Table 2. PSO parameters

Parameter	Value
C_1	1
C_2	1
F	0.685

Table 1. Simulation information

Scenarios	Number of sellers	Number of buyers	Number of goods	Average of minimum price for sellers	Average of maximum price for sellers	Average of minimum price for buyers	Average of maximum price for buyers	Average of quantity for buyers
I	5	5	4	4000	10000	2000	8000	80
II	10	10	5	10000	50000	35000	30000	30
III	25	25	7	200	1000	150	200	40
IV	50	50	20	3000	4000	1800	3500	60
V	120	120	30	100	600	50	400	70

To show the efficiency of the proposed method for all of seven scenarios we also solved them with linear fuzzy utility function. We can see result of the simulation in table 3. The results show the superiority of our method to the conventional linear utility. It is noticeable that for scenarios I and II we implemented synchronous double auction (SDA), this means that we changed the utility functions several times manually. But for other scenarios we used Yager function (figure 4) with w=[1.5, 3] for sellers selling prices and 1-

Yager function (figure 5) with w=[0.3, 0.7] to showing the buyers utility functions and sellers' quantity preferences. Gaussian utility function is used for buyers' quantity preferences. The results in table 3 express that our method works properly and better than simple linear-utility based models.

Table 3. Total achieved utility comparison

Scenarios	Utility in FUDA	Utility in simple DA
I	5756.74	3267.55
II	25876.54	13455.76
III	18544.97	8547.19
IV	143795.52	86904.83
V	194380.43	114943.94

6. Conclusion and future works

Auction is one of the most well-known trading mechanisms. With rapid growth of e-commerce, various kinds of e-auctions have been developed in the Internet. Consequently many researchers have been attracted in this field to develop efficient auction mechanism. In this paper we focused on a fuzzy utility based double auction, and our main contribution was bringing the power of fuzzy sets to this problem in order to help traders to gain more profit. As result of our work, now traders can offer a bid with more flexibility which is closer to the way they trade in real world. One of the future works might be on how to compute the fuzzy similarity between traders and sellers and then assigning the buyers to sellers which are more similar to them.

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