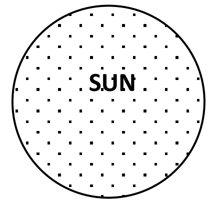
۱. خورشید را یک جسم سیاه با دمای سطح 10400 R در نظر بگیرید. نرخ انرژی تابشی مادون قرمز گسیل شده از سطح را در بازه طول موج $\lambda = 0.76 - 100\ \mu m$ بدست آورید.

Analysis T = 10400 R = 5778 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 11-2 to be

$$\lambda_1 T = (0.76\ \mu\text{m})(5778\ \text{K}) = 4391.3\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.547370$$

$$\lambda_2 T = (100\ \mu\text{m})(5778\ \text{K}) = 577{,}800\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 1.0$$



Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 1.0 - 0.547 = 0.453 \quad \text{(or 45.3\%)}$$

The total blackbody emissive power of the sun is determined from Stefan-Boltzman Law to be

$$E_b = \sigma T^4 = (0.1714 \times 10^{-8}\ \text{Btu/h.ft}^2.\text{R}^4)(10{,}400\ \text{R})^4 = 2.005 \times 10^7\ \text{Btu/h.ft}^2$$

Then,

$$E_{\text{infrared}} = (0.451)E_b = (0.453)(2.005 \times 10^7\ \text{Btu/h.ft}^2) = \mathbf{9.08 \times 10^6\ Btu/h.ft^2}$$

۲. طراحی یک لامپ به صورتی است که حداکثر انرژی تابشی در طول موج $\lambda = 0.47\ \mu m$ ساطع می شود. دمای این لامپ و نسبت انژی تابشی ساطع شده را در بازه طول موج مرئی $\lambda = 0.40 - 0.76\ \mu m$ بدست آورید.

Radiation emitted by a light source is maximum in the blue range. The temperature of this light source and the fraction of radiation it emits in the visible range are to be determined.
Assumptions The light source behaves as a black body.
Analysis The temperature of this light source is

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K} \longrightarrow T = \frac{2897.8\ \mu\text{m}\cdot\text{K}}{0.47\ \mu\text{m}} = \mathbf{6166\ K}$$

The visible range of the electromagnetic spectrum extends from $\lambda_1 = 0.40\ \mu\text{m}$ to $\lambda_2 = 0.76\ \mu\text{m}$. Noting that T =6166 K, the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$ are determined from Table 11-2 to be

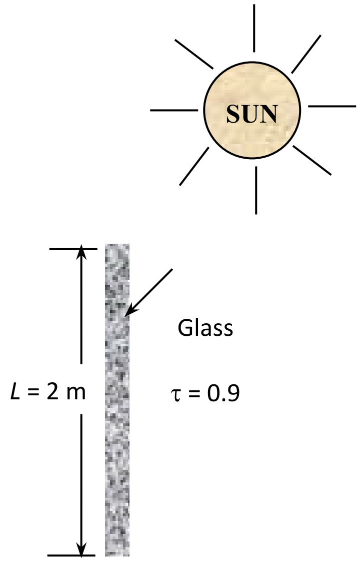
$$\lambda_1 T = (0.40\ \mu\text{m})(6166\ \text{K}) = 2466\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.15444$$

$$\lambda_2 T = (0.76\ \mu\text{m})(6166\ \text{K}) = 4686\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.59141$$

Then the fraction of radiation emitted between these two wavelengths becomes

$$f_{\lambda_2} - f_{\lambda_1} = 0.59141 - 0.15444 \cong \mathbf{0.437} \quad \text{(or 43.7\%)}$$

۳. طراحی یک پنجره 2 m*2 m به گونه ای است که 90% انرژی تابشی در بازه طول موج $\lambda = 0.3 - 3.0\ \mu m$ از شیشه عبور می کند و بازای سایر طول موج ها شیشه مانند یک جسم کاملا مات رفتار می کند. اگر خورشید یک جسم سیاه در دو دمای 5800 K و 1000 K در نظر گرفته شود، نرخ انرژی تابشی عبوری از پنجره را بدست آورید.



Analysis The surface area of the glass window is

$$A_s = 4\ \text{m}^2$$

(*a*) For a blackbody source at 5800 K, the total blackbody radiation emission is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{kW/m}^2.\text{K})^4(5800\ \text{K})^4(4\ \text{m}^2) = 2.567\times10^5\ \text{kW}$$

The fraction of radiation in the range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\ \mu\text{m})(5800\ \text{K}) = 1740\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.03345$$

$$\lambda_2 T = (3.0\ \mu\text{m})(5800\ \text{K}) = 17{,}400\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.97875$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.97875 - 0.03345 = 0.9453$$

Noting that 90% of the total radiation is transmitted through the window,

$$E_{\text{transmit}} = 0.90\Delta f E_b(T)$$

$$= (0.90)(0.9453)(2.567\times10^5\ \text{kW}) = \mathbf{2.184\times10^5\ kW}$$

(*b*) For a blackbody source at 1000 K, the total blackbody emissive power is

$$E_b(T) = \sigma T^4 A_s = (5.67\times10^{-8}\ \text{W/m}^2.\text{K}^4)(1000\ \text{K})^4(4\ \text{m}^2) = 226.8\ \text{kW}$$

The fraction of radiation in the visible range of 0.3 to 3.0 μm is

$$\lambda_1 T = (0.30\ \mu\text{m})(1000\ \text{K}) = 300\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0000$$

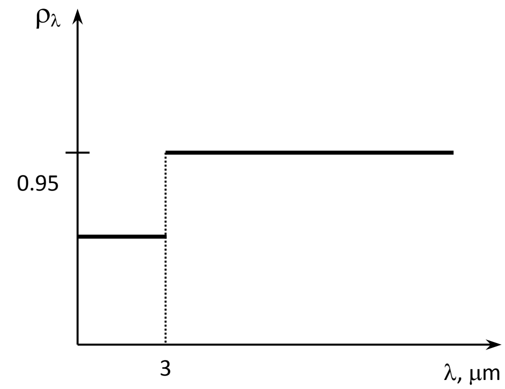
$$\lambda_2 T = (3.0\ \mu\text{m})(1000\ \text{K}) = 3000\ \mu\text{mK} \longrightarrow f_{\lambda_2} = 0.273232$$

$$\Delta f = f_{\lambda_2} - f_{\lambda_1} = 0.273232 - 0$$

and

$$E_{\text{transmit}} = 0.90\Delta f E_b(T) = (0.90)(0.273232)(226.8\ \text{kW}) = \mathbf{55.8\,kW}$$

۴. ضریب انعکاس یک صفحه آلومینیومی پوشیده شده با سولفات مس بازای طول موج های کمتر از $3\ \mu m$ برابر 0.35 و بازای طول موج های بیشتر از $3\ \mu m$ برابر 0.95 است. در دو دمای 5800 K و 300 K ضریب انعکاس متوسط سطح (reflectivity)، ضریب گسیل (emissivity) و جذب (absorptivity) را بدست آورید.



Analysis The average reflectivity of this surface for solar radiation (T = 5800 K) is determined to be

$$\lambda T = (3\ \mu\text{m})(5800\ \text{K}) = 17400\ \mu\text{mK} \rightarrow f_\lambda = 0.978746$$

$$\rho(T) = \rho_1 f_{0-\lambda_1}(T) + \rho_2 f_{\lambda_1-\infty}(T)$$
$$= \rho_1 f_{\lambda_1} + \rho_2 (1 - f_{\lambda_1})$$
$$= (0.35)(0.978746) + (0.95)(1 - 0.978746)$$
$$= \mathbf{0.362}$$

Noting that this is an opaque surface, $\tau = 0$

At T = 5800 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.362 = \mathbf{0.638}$

Repeating calculations for radiation coming from surfaces at T = 300 K,

$$\lambda T = (3\ \mu\text{m})(300\ \text{K}) = 900\ \mu\text{mK} \longrightarrow f_{\lambda_1} = 0.0001685$$

$$\rho(T) = (0.35)(0.0001685) + (0.95)(1 - 0.0001685) = \mathbf{0.95}$$

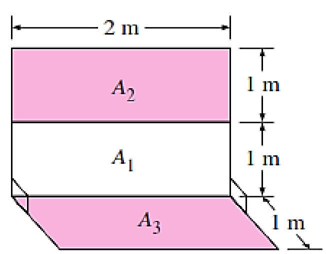
At T = 300 K: $\alpha + \rho = 1 \longrightarrow \alpha = 1 - \rho = 1 - 0.95 = \mathbf{0.05}$

and $\varepsilon = \alpha = \mathbf{0.05}$

The temperature of the aluminum plate is close to room temperature, and thus emissivity of the plate will be equal to its absorptivity at room temperature. That is,

$$\varepsilon = \varepsilon_{room} = 0.05$$
$$\alpha = \alpha_s = 0.638$$

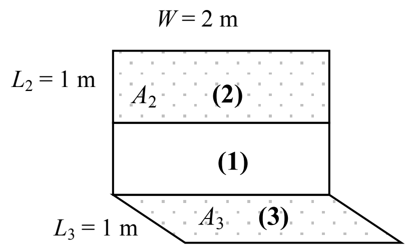
۵. زوایای دید؟



Assumptions The surfaces are diffuse emitters and reflectors.

Analysis From Figure:

$$\left.\begin{array}{l}\dfrac{L_3}{W}=\dfrac{1}{2}=0.5\\ \dfrac{L1}{W}=\dfrac{1}{2}=0.5\end{array}\right\}F_{31}=0.24$$



and

$$\left.\begin{array}{l}\dfrac{L_3}{W}=\dfrac{1}{2}=0.5\\ \dfrac{L_1+L_2}{W}=\dfrac{2}{2}=1\end{array}\right\}F_{3\rightarrow(1+2)}=0.29$$

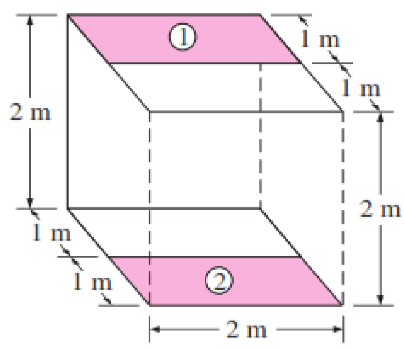
We note that $A_1 = A_3$. Then the reciprocity and superposition rules give

$$\mathrm{A}_1F_{13}=A_3F_{31}\longrightarrow F_{13}=F_{31}=\mathbf{0.24}$$

$$F_{3\rightarrow(1+2)}=F_{31}+F_{32}\longrightarrow\quad 0.29=0.24+F_{32}\longrightarrow F_{32}=0.05$$

Finally, $A_2=A_3\longrightarrow F_{23}=F_{32}=\mathbf{0.05}$

۶. زوایای دید؟



From Fig.12-5,

$$\left.\begin{array}{l}\dfrac{L_2}{D}=\dfrac{2}{2}\\ \dfrac{L_1}{D}=\dfrac{2}{2}\end{array}\right\}F_{(2+4)\rightarrow(1+3)}=0.20\quad\text{and}\quad\left.\begin{array}{l}\dfrac{L_2}{D}=\dfrac{2}{2}\\ \dfrac{L_1}{D}=\dfrac{1}{2}\end{array}\right\}F_{14}=0.12$$

superposition rule: $F_{(2+4)\rightarrow(1+3)}=F_{(2+4)\rightarrow1}+F_{(2+4)\rightarrow3}$

symmetry rule: $F_{(2+4)\rightarrow1}=F_{(2+4)\rightarrow3}$

Substituting symmetry rule gives

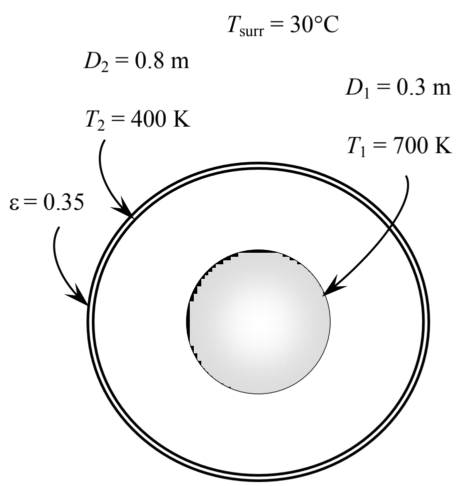
$$F_{(2+4)\to 1} = F_{(2+4)\to 3} = \frac{F_{(2+4)\to(1+3)}}{2} = \frac{0.20}{2} = 0.10$$

reciprocity rule: $A_1 F_{1\to(2+4)} = A_{(2+4)} F_{(2+4)\to 1} \longrightarrow (2)F_{1\to(2+4)} = (4)(0.10) \longrightarrow F_{1\to(2+4)} = 0.20$

superposition rule: $F_{1\to(2+4)} = F_{12} + F_{14} \longrightarrow 0.20 = F_{12} + 0.12 \longrightarrow F_{12} = 0.20 - 0.12 = \mathbf{0.08}$

۷. دو کره هم مرکز به قطر های $D_1 = 0.3\ m$ & $D_2 = 0.8\ m$ در دماهای ثابت $T_1 = 700\ K$ & $T_2 = 400\ K$ نگه داشته شده اند. ضرایب گسیل این دو کره $\varepsilon_1 = 0.5$ & $\varepsilon_2 = 0.7$ می باشند. نرخ انتقال حرارت خالص تابشی میان دو کره را بدست آورید. همچنین اگر دمای محیط اطراف $30\ ℃$ باشد ضریب انتقال حرارت جابجایی را در سطح خارجی کره ها بدست آورید. (ضریب گسیل سطح خارجی کره برابر 0.35 در نظر گرفته شود.)

Two concentric spheres are maintained at uniform temperatures. The net rate of radiation heat transfer between the two spheres and the convection heat transfer coefficient at the outer surface are to be determined.



Assumptions **1** Steady operating conditions exist **2** The surfaces are opaque, diffuse, and gray.

Analysis The net rate of radiation heat transfer between the two spheres is

$$\dot{Q}_{12} = \frac{A_1\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1^2}{r_2^2}\right)}$$

$$= \frac{[\pi(0.3\text{ m})^2](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(700\text{ K})^4 - (400\text{ K})^4]}{\frac{1}{0.5} + \frac{1-0.7}{0.7}\left(\frac{0.15\text{ m}}{0.4\text{ m}}\right)^2}$$

$$= \mathbf{1669\ W}$$

Radiation heat transfer rate from the outer sphere to the surrounding surfaces are

$$\dot{Q}_{rad} = \varepsilon F A_2 \sigma (T_2{}^4 - T_{surr}{}^4)$$
$$= (0.35)(1)[\pi(0.8\text{ m})^2](5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)[(400\text{ K})^4 - (30+273\text{ K})^4] = 685\text{ W}$$

The convection heat transfer rate at the outer surface of the cylinder is determined from requirement that heat transferred from the inner sphere to the outer sphere must be equal to the heat transfer from the outer surface of the outer sphere to the environment by convection and radiation. That is,

$$\dot{Q}_{conv} = \dot{Q}_{12} - \dot{Q}_{rad} = 1669 - 685 = 984\text{ W}$$

Then the convection heat transfer coefficient becomes

$$\dot{Q}_{conv.} = hA_2(T_2 - T_\infty)$$
$$984\text{ W} = h\left[\pi(0.8\text{ m})^2\right](400\text{ K} - 303\text{ K}) \longrightarrow h = \mathbf{5.04\ W/m^2\cdot{}^\circ C}$$