Extra Material

Variants of SIMPLE: SIMPLER & SIMPLEC

SIMPLER Algorithm: Motivation

- The SIMPLE algorithm has been used widely but suffers from a drawback
- Dropping the $\sum_{nb} a_{nb}u_{nb}^{'}$, $\sum_{nb} a_{nb}v_{nb}^{'}$ terms produces too-large pressure corrections, needing under-relaxation when correcting pressure
- Under-relaxation factors are problem-dependent
- Velocity corrections are good though
  - » Are configured to guarantee velocity fields are continuity-satisfying every iteration

Motivation (Cont'd)

- Idea is to use pressure-correction to correct velocities but not pressure
- Create a separate equation for pressure computation

Another Take…

- Let’s say we start with a good guess of velocity but a bad guess of pressure
- First step in SIMPLE algorithm is to solve for u*,v* using bad guess p*
- Even if velocity guess is good, bad p* destroys good velocity guess right away
- Rest of the iterative procedure is basically trying to recover the good velocity field
- Need to compute a good pressure field from good velocity guess instead of having to guess it.

SIMPLER

- Semi-Implicit Method for Pressure-Linked Equations (Revised)
- Creates a separate equation for pressure (rather than using p' equation)
- Uses p' equation only to correct velocity

Pressure Equation

- Discrete momentum equations:

$$a_e u_e = \sum_{\text{nb}} a_{\text{nb}} u_{\text{nb}} + \Delta y (p_P - p_E) + b_e$$

$$a_n v_n = \sum_{\text{nb}} a_{\text{nb}} v_{\text{nb}} + \Delta x \left(p_P - p_N\right) + b_n$$

- Find velocities

$$u_e = \frac{\sum_{\text{nb}} a_{\text{nb}} u_{\text{nb}} + b_e}{a_e} + d_e (p_P - p_E)$$

$$v_n = \frac{\sum_{\text{nb}} a_{\text{nb}} v_{\text{nb}} + b_n}{a_n} + d_n \left(p_P - p_N\right)$$

Pressure Equation (Cont'd)

- Define hat velocities

$$\hat{u}_e = \frac{\sum_{\mathrm{nb}} a_{\mathrm{nb}} u_{\mathrm{nb}} + b_e}{a_e^u}$$

$$\hat{v}_n = \frac{\sum_{\mathrm{nb}} a_{\mathrm{nb}} v_{\mathrm{nb}} + b^n}{a_e}$$

- Momentum equations:

$$u_e = \hat{u}_e + d_e \left(p_P - p_E \right)$$

$$v_n = \hat{v}_n + d_n \left(p_P - p_N \right)$$

Pressure Equation (Cont'd)

- Multiplying hat velocities by density and area:

$$\begin{aligned} \hat{F}_e &= \rho_e \hat{u}_e \Delta y \\ \hat{F}_n &= \rho_n \hat{v}_n \Delta x \end{aligned}$$

- Similarly, from the momentum equations:

$$\begin{aligned} F_e &= \hat{F}_e + \rho_e d_e \Delta y \left(p_P - p_E \right) \\ F_n &= \hat{F}_n + \rho_n d_n \Delta x \left(p_P - p_N \right) \end{aligned}$$

Continuity Equation

- Discrete continuity equation for main control volume:

$$(\rho u)_e \Delta y - (\rho u)_w \Delta y + (\rho v)_n \Delta x - (\rho v)_s \Delta x = 0$$

- Substitute for flow rates in terms of pressure and hat velocities to obtain equation for pressure

Discrete Pressure Equation

$$a_P p_P = \sum_{\mathrm{nb}} a_{\mathrm{nb}} p_{\mathrm{nb}} + b$$

$$\begin{aligned}
a_E &= \rho_e d_e \Delta y \\
a_W &= \rho_w d_w \Delta y \\
a_N &= \rho_n d_n \Delta x \\
a_S &= \rho_s d_s \Delta x \\
a_P &= \sum_{\mathrm{nb}} a_{\mathrm{nb}} \\
b &= \hat{F}_w - \hat{F}_e + \hat{F}_s - \hat{F}_n
\end{aligned}$$

- Scarborough criterion satisfied in the equality
- Notice p coefficients are the same as p' coefficients
- b is in terms of $\hat{F}$, not F* -- not mass imbalance
- No approximations – if velocity is exact, pressure is exact

Overall SIMPLER Algorithm

1. Guess velocity field
2. Compute momentum coefficients and store.

   Compute $\hat{u}, \hat{v}$
3. Compute pressure coefficients and store.

   Solve pressure equation and obtain pressure
4. Solve momentum equations using stored momentum coefficients and just-computed pressure. Find u* and v*
5. Find b term in pressure-correction equation using u* and v*

SIMPLER Algorithm (Cont’d)

6. Solve p’ equation to find pressure correction field
7. Correct velocities:

$$
\begin{aligned}
u &= u^* + u' \\
v &= v^* + v'
\end{aligned}
$$

Do not correct pressure!

8. Compute other scalar fields if necessary
9. Check for convergence. If not converged, go to 2. Else stop.

Discussion

- SIMPLER is found to be faster than SIMPLE (about 30-50% fewer iterations)
- Does not need good pressure guess – finds it from velocity guess
- About 50% larger computational effort typically
  - » Two pressure-like equations
  - » Expensive because of lack of Dirichlet conditions
- Need extra storage for pressure and momentum coefficients
  - » Can recompute instead

Discussion (Cont'd)

- P' equation only used for velocity correction – pressure correction does not correct pressure
  - » No need to under-relax pressure correction
- May under-relax pressure equation directly, but usually not necessary
- May need to under-relax momentum equations to account for non-linearities, sequential solution procedure (as with SIMPLE)

SIMPLEC Algorithm

- SIMPLE-Corrected (SIMPLEC) algorithm seeks to mitigate the effects of dropping velocity neighbor correction terms:

$$a_e u'_e = \sum_{\text{nb}} a_{\text{nb}} u'_{\text{nb}} + \Delta y \left(p'_P - p'_E \right)$$

SIMPLEC Velocity Correction Equation

- SIMPLEC retains neighbor velocity correction terms, but makes an approximation:

$$\sum_{\text{nb}} a_{\text{nb}} u'_{\text{nb}} \approx u'_e \sum_{\text{nb}} a_{\text{nb}}$$

$$\sum_{\text{nb}} a_{\text{nb}} v'_{\text{nb}} \approx v'_e \sum_{\text{nb}} a_{\text{nb}}$$

- Thus

$$\left(a_e - \sum_{\text{nb}} a_{\text{nb}}\right) u'_e = \Delta y \left(p'_P - p'_E\right)$$

$$\left(a_n - \sum_{\text{nb}} a_{\text{nb}}\right) v'_n = \Delta x \left(p'_P - p'_N\right)$$

Velocity Correction (Cont'd)

- Therefore, re-define *d* coefficients:

$$d_e = \frac{\Delta y}{\left(a_e - \sum_{\text{nb}} a_{\text{nb}}\right)}$$

$$d_n = \frac{\Delta x}{\left(a_n - \sum_{\text{nb}} a_{\text{nb}}\right)}$$

Need under-relaxation in the momentum equation to avoid division by zero for S_p =0, steady flow

Discrete Pressure Correction Equation

$$a_P p'_P = \sum_{nb} a_{nb} p'_{nb} + b$$

$$\begin{aligned}
a_E &= \rho_e d_e \Delta y \\
a_W &= \rho_w d_w \Delta y \\
a_N &= \rho_n d_n \Delta x \\
a_S &= \rho_s d_s \Delta x \\
a_P &= \sum_{\text{nb}} a_{\text{nb}} \\
b &= F_w^* - F_e^* + F_s^* - F_n^*
\end{aligned}$$

- Form of equation is the same as for SIMPLE
- b term is still the amount by which the starred velocities do not satisfy continuity
- Only *d* coefficients are different from SIMPLE

SIMPLEC Algorithm

- The overall algorithm is identical to that for SIMPLE
- Only the *d* coefficients used to drive the p’ equation are different.
- With SIMPLEC, because the $\sum_{nb} a_{nb} u'_{nb}$ and $\sum_{nb} a_{nb} v'_{nb}$ are not dropped, there is no need to under-relax the p’ correction. Thus:

$$p = p^* + p'$$

SIMPLEC Performance

- SIMPLEC is found to converge faster that SIMPLE
  - » Gains of 20-30% can be found for many problems
  - » Does not solve extra equations like SIMPLER
  - » Cost per iteration approximately the same as SIMPLE
- However, like SIMPLE, the SIMPLEC algorithm will also destroy a good velocity field unless there is also a good pressure guess.

Relationship Between SIMPLE an SIMPLEC

- SIMPLE and SIMPLEC can be shown to be closely related to each other.
- Recall that pressure correction for SIMPLE is

$$p = p^* + \alpha_p p'$$

- For SIMPLEC, it is: $p = p^* + p'$

- Can think of SIMPLEC as solving for a variable

$$\hat{p}' = \alpha_P p'$$

SIMPLE and SIMPLEC (Cont'd)

- The SIMPLEC pressure correction equation can be written as:

$$a_P \hat{p}'_P = \sum_{\mathrm{nb}} a_{\mathrm{nb}} \hat{p}'_{\mathrm{nb}} + b$$

with coefficients of the form:

$$a_{\mathrm{nb}} = \frac{\rho \Delta y^2}{a_e - \sum_{\mathrm{nb}} a_{\mathrm{nb}}}$$

$$d_e = \frac{\Delta y}{a_e - \sum_{nb} a_{nb}}$$

a_{nb} for pressure correction equation is of the form

$\rho_e d_e \Delta y$

SIMPLE and SIMPLEC (Cont'd)

- Consider the case when S_p=0. Momentum coefficient is given by:

$$a_e = \frac{\sum_{\text{nb}} a_{\text{nb}}}{\alpha_u}$$

- Thus, neighbor coefficients in SIMPLEC pressure correction equation are of the form

$$a_{\text{nb}} = \frac{\rho \Delta y^2}{\frac{1-\alpha_u}{\alpha_u} \sum_{\text{nb}} a_{\text{nb}}}$$

Momentum neighbor coefficients

SIMPLE and SIMPLEC (cont'd)

- Now look at p' equation for SIMPLE:

$$a_P p_P' = \sum_{nb} a_{nb} p_{nb}' + b$$

$$\left(\frac{a_P}{\alpha_P}\right)\left(\alpha_P p_P'\right) = \sum_{nb}\left(\frac{a_{nb}}{\alpha_P}\right)\left(\alpha_P p_{nb}'\right) + b$$

$$\left(\frac{a_P}{\alpha_P}\right)\hat{p}' = \sum_{nb}\left(\frac{a_{nb}}{\alpha_P}\right)\hat{p}_{nb}' + b$$

SIMPLE and SIMPLEC (Cont'd)

- Therefore

$$a_{nb} = \frac{\rho d_e \Delta y}{a_e} = \frac{\rho \Delta y^2}{\left(\sum_{nb} a_{nb}\right)_e / \alpha_u}$$

or

$$\frac{a_{nb}}{\alpha_p} = \frac{\rho \Delta y^2}{\frac{\alpha_p}{\alpha_u}\left(\sum_{nb} a_{nb}\right)_e}$$

- Compare SIMPLE and SIMPLEC coefficients

SIMPLE and SIMPLEC (Cont'd)

- If we chose

$$\alpha_p = 1 - \alpha_u$$

in the SIMPLE algorithm, we would effectively get the same corrected pressure each iteration for both SIMPLE and SIMPLEC

- So SIMPLE and SIMPLEC are not very different
  - » SIMPLE run with $\alpha_p = 1 - \alpha_u$ is the same as SIMPLEC (when $S_p = 0$)