Course outlines:

Part I
A brief introduction to CFD and review of fluid mechanics

Part II
Standard Numerical Analysis of partial differential equations

Part III
Advanced topics in CFD

- Introduction, what is CFD, examples
- Elementary numerical analysis, integration of ordinary differential equations
- Elementary numerical analysis, accuracy, stability, partial differential equations
- Review of fluid mechanics: the governing equations
- Finite Difference solution of the Navier-Stokes Equations in vorticity/streamfunction form
- First order Partial Differential Equations (PDF’s) Characteristics
- Classification of Second Order PDF’s
- Algorithms for Hyperbolic equations, Shock capturing, The Euler equations
- Algorithms for Parabolic equations
- Algorithms for Elliptic equations
- Putting it together, solving the Navier-Stokes Equations in Primitive Variables, the MAC Method
- Grid Generation
Course Outlines Details:

- What is CFD
- A few examples
- Computational tools
- Short history

Elementary Numerical Analysis: Finite Difference Approximations

- Time integration of an ordinary differential equation
  - Integration methods
  - Matlab code
  - Error analysis
- Solving partial differential equations
  - Finite difference approximations
  - The linear advection-diffusion equation
  - Matlab code
  - Accuracy and error quantification
  - Stability
  - Consistency
  - Multidimensional problems
  - Boundary value problems (Steady state)
- A Finite Difference Code for Navier-Stokes Equations in Vorticity/Streamfunction Form
  - The Driven Cavity Problem
  - The Navier-Stokes Equations in Vorticity/Streamfunction form
  - Boundary Conditions
  - The Grid
  - Finite Difference Approximation of the Vorticity/Streamfunction equations
  - Finite Difference Approximation of the Boundary Conditions
  - Iterative Solution of the Elliptic Equation
  - The Code
  - Results
  - Convergence Under Grid Refinement

Elementary Numerical Analysis: Finite Volume Approximations

- Integral versus differential form of a model equation
- Finite volume approximations
Flux function
Advection-diffusion equation in finite volume form

Equations Governing Fluid Motion
Derivation of the equations governing fluid flow in integral form
  - Conservation of Mass
  - Conservation of Momentum
  - Conservation of Energy
Differential form
Summary
Incompressible flows
Inviscid compressible flows
Vorticity Form

Brief Review of Fluid Mechanics
Role of advection, pressure, and viscous diffusion
Nondimensional numbers—the Reynolds number
Advection and diffusion—Boundary layers

Theory of Partial Differential Equations
Basic Properties of PDE
Quasi-linear First Order Equations
  - Characteristics
  - Linear and Nonlinear Advection Equations
Quasi-linear Second Order Equations
  - Classification: hyperbolic, parabolic, elliptic
  - Domain of Dependence/Influence
Conservative form
Navier-Stokes equations

Numerical Methods for Hyperbolic Equations
FTCS and upwind
Stability in terms of fluxes
Generalized upwind
Second order schemes for smooth flow
Modified Equation
Course Title: Computational Fluid Dynamics

1. Conservation
2. Flows with shocks, Solutions to Burgers eqn. Entropy condition
3. Conservation and shock speed
4. Advecting a shock with several schemes
5. Godunov's Theorem
6. Godunov's Method
7. Higher Order Upwind schemes
8. Artificial Viscosity
9. Flux Vector Splitting

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Numerical Methods for the Euler Equations

- The Euler Equations
  - Characteristics
  - One dimensional shock tube–exact solution
- Numerical solutions
  - Upwind/flux splitting
  - Lax-Wendroff/artificial viscosity
- Two dimensional problems

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Numerical Methods for Parabolic Equations

- Solution Methods for Parabolic Equations
- One-Dimensional Problems
  - Explicit, implicit, Crank-Nicolson
  - Accuracy, stability
  - Various schemes
- Multi-Dimensional Problems
  - Alternating Direction Implicit (ADI)
  - Approximate Factorization of Crank-Nicolson
- Splitting
- Stability in terms of fluxed

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Numerical Methods for Elliptic Equations

- Examples of elliptic equations
- Direct Methods for 1D problems
- Elementary Iterative Methods
- Iteration as Time Integration
- Example
- Boundary Conditions
- Convergence of Iterative Methods
  - 1D Example
  - Formal Discussion
- Multigrid methods
- Fast Direct Method
- Krylov Methods

The Advection-Diffusion Equation
- Advection Diffusion equation
- The cell Reynolds number
- Higher order in space
  - QUICK
  - Compact schemes
- Conservation of energy
- Higher order in time
  - the $\omega - \psi$ formulation

A Code for the Navier-Stokes Equations in Velocity/Pressure Form
- Equations
- Discrete Form
- Solution Strategy
- Boundary Conditions
- Code and Results

Elementary Grid Generation
- Stretched grids for rectangular geometries
- Bilinear Interpolation
- Elliptic grid generation
- Unstructured hexahedron grids and block-structured grids
- Imbedded boundaries
- Adaptive Mesh Refinement

Other CFD Methods
- Spectral Methods
- Finite Element Methods
- Vortex Methods
- A Few words about:
  - Lattice Boltzmann Methods
  - Molecular dynamics

Course Resources:
The following text books are recommended for this course: