بسمه تعالی

درس دینامیک سیالات محاسباتی ۱، دوره کارشناسی ارشد

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Computational Fluid Dynamics-1(CFDI)

Course outlines:

Part I

A brief introduction to CFD and review of fluid mechanics

Part II

Standard Numerical Analysis of partial differential equations

Part III

Advanced topics in CFD

- **Introduction, what is CFD, examples**
- **Elementary numerical analysis, integration of ordinary differential equations**
- **Elementary numerical analysis, accuracy, stability, partial differential equations**
- **Review of fluid mechanics: the governing equations**
- **Finite Difference solution of the Navier-Stokes Equations in vorticity/streamfunction form**
- **First order Partial Differential Equations (PDF's) Characteristics**
- **Classification of Second Order PDF's**
- **Algorithms for Hyperbolic equations, Shock capturing, The Euler equations**
- **Algorithms for Parabolic equations**
- **Algorithms for Elliptic equations**
- **Putting it together, solving the Navier-Stokes Equations in Primitive Variables, the MAC Method**
- **Grid Generation**

Course Outlines Details:

- What is CFD
- A few examples
- Computational tools
- Short history

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Elementary Numerical Analysis: Finite Difference Approximations

- Time integration of an ordinary differential equation
  - ✓ Integration methods
  - ✓ Matlab code
  - ✓ Error analysis
- Solving partial differential equations
  - ✓ Finite difference approximations
  - ✓ The linear advection-diffusion equation
  - ✓ Matlab code
  - ✓ Accuracy and error quantification
  - ✓ Stability
  - ✓ Consistency
  - ✓ Multidimensional problems
  - ✓ Boundary value problems (Steady state)
- A Finite Difference Code for Navier-Stokes Equations in Vorticity/Streamfunction Form
  - ✓ The Driven Cavity Problem
  - ✓ The Navier-Stokes Equations in Vorticity/Streamfunction form
  - ✓ Boundary Conditions
  - ✓ The Grid
  - ✓ Finite Difference Approximation of the Vorticity/Streamfunction equations
  - ✓ Finite Difference Approximation of the Boundary Conditions
  - ✓ Iterative Solution of the Elliptic Equation
  - ✓ The Code
  - ✓ Results
  - ✓ Convergence Under Grid Refinement

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Elementary Numerical Analysis: Finite Volume Approximations

- Integral versus differential form of a model equation
- Finite volume approximations

- Flux function
- Advection-diffusion equation in finite volume form

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Equations Governing Fluid Motion

- Derivation of the equations governing fluid flow in integral form
  - ✓ Conservation of Mass
  - ✓ Conservation of Momentum
  - ✓ Conservation of Energy
- Differential form
- Summary
- Incompressible flows
- Inviscid compressible flows
- Vorticity Form

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Brief Review of Fluid Mechanics

- Role of advection, pressure, and viscous diffusion
- Nondimensional numbers-the Reynolds number
- Advection and diffusion-Boundary layers

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Theory of Partial Differential Equations

- Basic Properties of PDE
- Quasi-linear First Order Equations
  - ✓ Characteristics
  - ✓ Linear and Nonlinear Advection Equations
- Quasi-linear Second Order Equations
  - ✓ Classification: hyperbolic, parabolic, elliptic
  - ✓ Domain of Dependence/Influence
- Conservative form
- Navier-Stokes equations

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Numerical Methods for Hyperbolic Equations

- FTCS and upwind
- Stability in terms of fluxes
- Generalized upwind
- Second order schemes for smooth flow
- Modified Equation

- Conservation
- Flows with shocks, Solutions to Burgers eqn. Entropy condition
- Conservation and shock speed
- Advecting a shock with several schemes
- Godunov's Theorem
- Godunov's Method
- Higher Order Upwind schemes
- Artificial Viscosity
- Flux Vector Splitting

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Numerical Methods for the Euler Equations

- The Euler Equations
  - ✓ Characteristics
  - ✓ One dimensional shock tube-exact solution
- Numerical solutions
  - ✓ Upwind/flux splitting
  - ✓ Lax-Wendroff/artificial viscosity
- Two dimensional problems

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Numerical Methods for Parabolic Equations

- Solution Methods for Parabolic Equations
- One-Dimensional Problems
  - ✓ Explicit, implicit, Crank-Nicolson
  - ✓ Accuracy, stability
  - ✓ Various schemes
- Multi-Dimensional Problems
  - ✓ Alternating Direction Implicit (ADI)
  - ✓ Approximate Factorization of Crank-Nicolson
- Splitting
- Stability in terms of fluxed

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Numerical Methods for Elliptic Equations

- Examples of elliptic equations
- Direct Methods for 1D problems
- Elementary Iterative Methods
- Iteration as Time Integration

- Example
- Boundary Conditions
- Convergence of Iterative Methods
  - ✓ 1D Example
  - ✓ Formal Discussion
- Multigrid methods
- Fast Direct Method
- Krylov Methods

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The Advection-Diffusion Equation

- Advection Diffusion equation
- The cell Reynolds number
- Higher order in space
  - ✓ QUICK
  - ✓ Compact schemes
- Conservation of energy
- Higher order in time
  - ✓ the $\omega - \psi$ formulation

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A Code for the Navier-Stokes Equations in Velocity/Pressure Form

- Equations
- Discrete Form
- Solution Strategy
- Boundary Conditions
- Code and Results

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Elementary Grid Generation

- Stretched grids for rectangular geometries
- Bilinear Interpolation
- Elliptic grid generation
- Unstructured hexahedron grids and block-structured grids
- Imbedded boundaries
- Adaptive Mesh Refinement

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Other CFD Methods

- Spectral Methods

- **Finite Element Methods**
- **Vortex Methods**
- **A Few words about:**
- **Lattice Boltzmann Methods**
- **Molecular dynamics**

Course Resources:

The following text books are recommended for this course:

- **K. A. Hoffmann and S. T. Chiang, Computational Fluid Dynamics (Vol. 1), Engineering Education System, 2000.**
- **H. K. Versteeg and W. Malalasekera, An Introduction to Computational Fluid Dynamics: The Finite Volume Method, Prentice Hall, 1995, 2007.**
- **S. V. Patankar, Numerical Heat Transfer and Fluid Flow, Hemisphere Publishing Corporation, 1980.**
- **J. H. Ferziger and M. Peric, Computational Methods for Fluid Dynamics, Springer (3rd ed) 2002.**