Graph Connectivity in Sparse Subspace Clustering

Behrooz Nasihatkon

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Outline

Outline

Example: Rigid Body

Frame 1:

- $\mathbf{x}_i = [\ \ A_1\ \ ]_{2\times 4} \begin{bmatrix} \mathbf{X}_i \\ 1 \end{bmatrix}_{4\times 1}$

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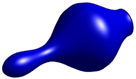

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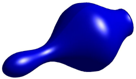

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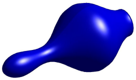

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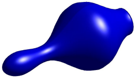

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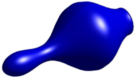

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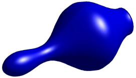

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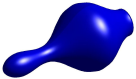

$$\begin{bmatrix} \mathbf{x}_{11} \cdots \mathbf{x}_{1n} \\ \mathbf{x}_{21} \cdots \mathbf{x}_{2n} \\ \vdots \quad\quad \vdots \\ \mathbf{x}_{F1} \cdots \mathbf{x}_{Fn} \end{bmatrix}_{2F \times n} = \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_F \end{bmatrix}_{2F \times 4} \begin{bmatrix} \mathbf{X}_1 \cdots \mathbf{X}_n \\ 1 \cdots 1 \end{bmatrix}_{4 \times n}$$

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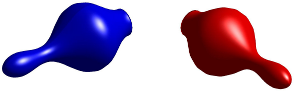
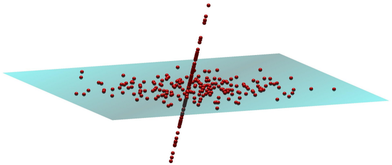
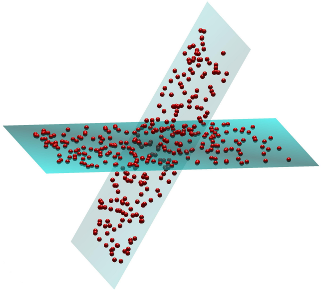


$$\begin{bmatrix} \mathbf{x}_{11} \cdots \mathbf{x}_{1n} \\ \mathbf{x}_{21} \cdots \mathbf{x}_{2n} \\ \vdots \quad \vdots \\ \mathbf{x}_{F1} \cdots \mathbf{x}_{Fn} \end{bmatrix}_{2F \times n} = ?$$

Subspace Clustering

- Data lying on a mixture of subspaces.





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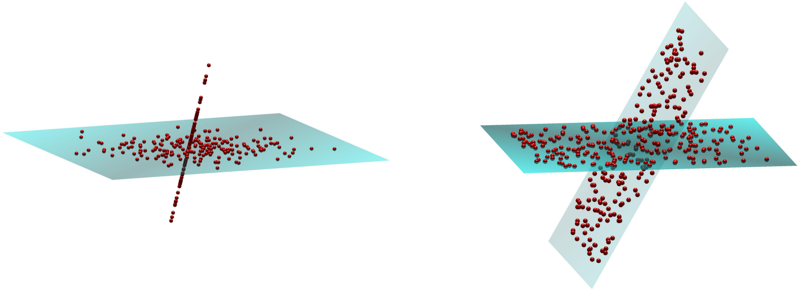

- Number of subspaces

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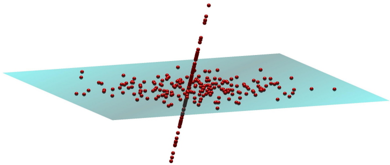

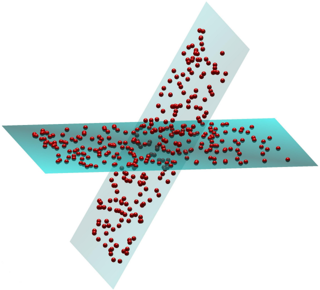

- No. of subspaces + their dimensions

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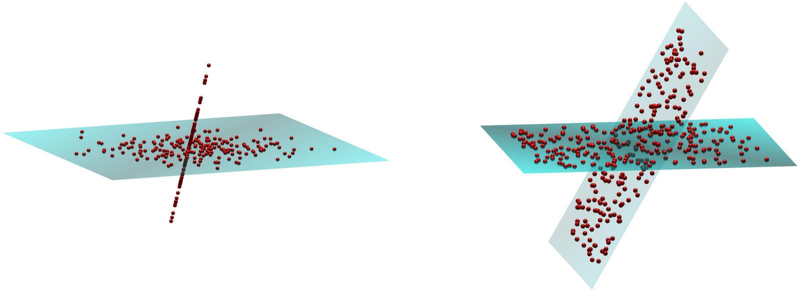

- No. of subspaces + dimensions + A basis for each subspace

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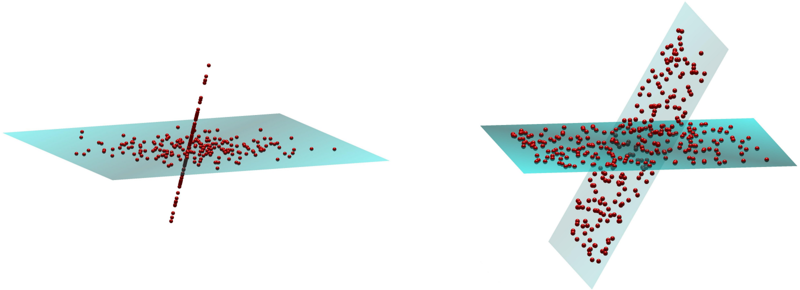

- No. of subspaces + dimensions + bases + data segmentation

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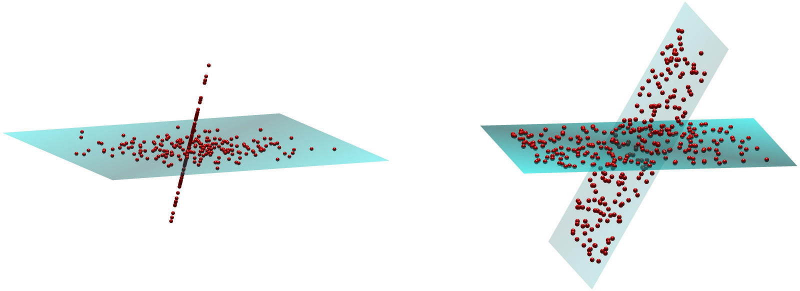

- No. of subspaces + dimensions + bases + segmentation

Applications

NICTA

- Motion Segmentation *[Rene Vidal et al. 2008]*
- Video Shot Segmentation *[Le Lu and R. Vidal 2006]*
- Illumination Invariant Clustering *[J. Ho et al. 2003]*
- Image Segmentation *[Alen Yang et al. 2008]*
- Image Representation and Compression *[Wei Hong et al. 2005]*
- Linear Hybrid Systems Identification *[Rene Vidal et al. 2003]*

Solutions

- Random Sample Consensus (RANSAC) *[Martin Fischler and R. Bolles 1981]*
- Mixture of Probabilistic PCA *[Michael Tipping and C. Bishop 1999]*
- Generalized PCA (GPCA) *[Rene Vidal et al. 2005]*
- Locally Linear Manifold Clustering (LLMC) *[Alvina Goh and R. Vidal 2007]*
- Agglomerative Lossy Compression (ALC) *[Yi Ma et al. 2007]*
- Sparse Subspace Clustering (SSC) *[Ehsan Elhamifar and R. Vidal 2009]*
- Low-Rank Subspace Clustering (LLR) *[Guangcan Li et al. 2010]*

Outline

NICTA

1. Subspace Clustering
2. Sparse Subspace Clustering
   - Sparse Representation
   - Main Theorem
   - Noise and Outliers
   - Open Problems
3. Graph Connectivity
4. Conclusion

Sparse Representation

- Represent $\mathbf{y}$ as a linear combination of the smallest possible subset of the vectors $\{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$.

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$$\min \|\mathbf{a}\|_0 \quad \text{s.t.} \quad \mathbf{y} = \mathbf{X}\mathbf{a}$$

where $\mathbf{X} = [\mathbf{x}_1 \mathbf{x}_2 \cdots \mathbf{x}_n]$.

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- NP-hard

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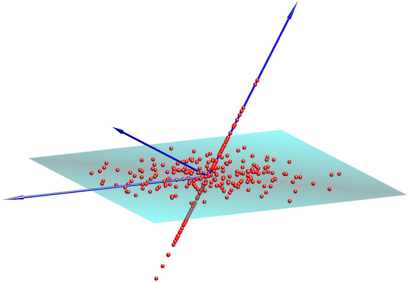
- L^1/L^0 equivalence

Sparse Subspace Clustering

- If we had the basis $[\mathbf{b}_1 \mathbf{b}_2 \dots \mathbf{b}_m]$



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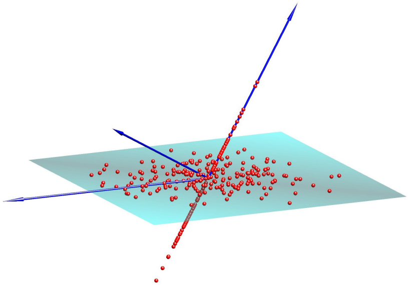
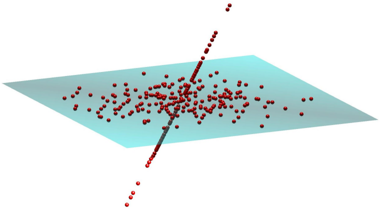


- Represent each $\mathbf{x}_i$ as a sparse combination of $\{\mathbf{b}_1 \mathbf{b}_2 \ldots \mathbf{b}_m\}$

Sparse Subspace Clustering

- We do not have the basis



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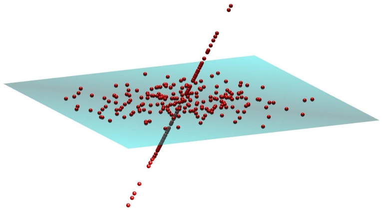

- Represent each $\mathbf{x}_i$ as a sparse combination of $X - \{\mathbf{x}_i\}$

Main Theorem

- For each $\mathbf{x}_i$ solve:

$$\mathbf{a}^i = \arg\min \|\mathbf{a}\|_1 \quad \text{s.t.} \quad \mathbf{x}_i = \mathbf{X}\,\mathbf{a}, \quad a_i = 0$$

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- Construct a graph whose nodes are $\mathbf{x}_i$ and each node $\mathbf{x}_i$ is connected to the node $\mathbf{x}_j$ if the j-th element of $\mathbf{a}^i$ is nonzero.

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Theorem

If the subspaces are independent, the graphs of different subspaces are disconnected.

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Theorem

If the subspaces are independent, the graphs of different subspaces are disconnected.

- Find the connected components
- In practice spectral clustering using $[\mathbf{a}^1\mathbf{a}^2 \ldots \mathbf{a}^n]$

Example

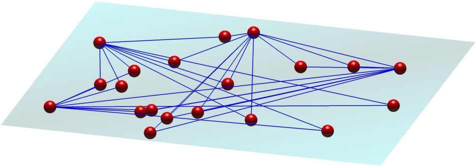

Figure: An example of an SSC graph

Noise and Outliers

- Noise

$$
\begin{aligned}
&\min \|\mathbf{a}\|_1 + \lambda \|\mathbf{e}\|_2 \\
&\ s.t. \\
&\quad \mathbf{x}_i = \mathbf{X}_{-i}\mathbf{a} + \mathbf{e} \\
&\quad a_i = 0
\end{aligned}
$$

Noise and Outliers

- Noise

$$\min \|\mathbf{a}\|_1 + \lambda \|\mathbf{e}\|_2$$
$$s.t.$$
$$\mathbf{x}_i = \mathbf{X}_{-i}\mathbf{a} + \mathbf{e}$$
$$a_i = 0$$

- Outliers

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$$s.t.$$
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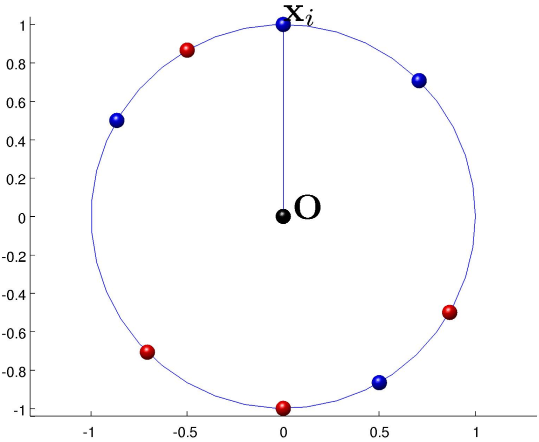
Open Problems

- Noise and Outliers
- Extension to manifolds
- Graph Connectivity in each subspace

Outline

1. Subspace Clustering
2. Sparse Subspace Clustering
3. Graph Connectivity
   - Example
   - Preparation
   - 2D Case
   - 3D Case
   - N-D Case
4. Conclusion

A Simple Example



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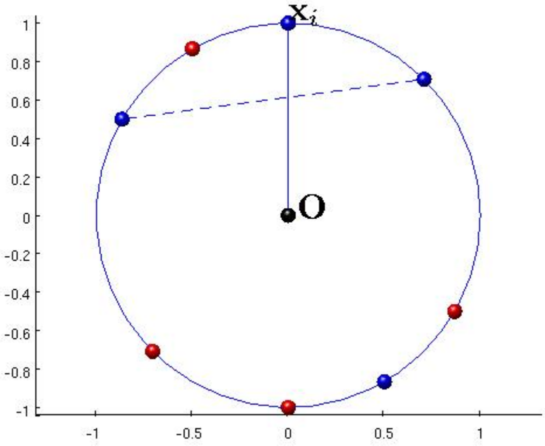

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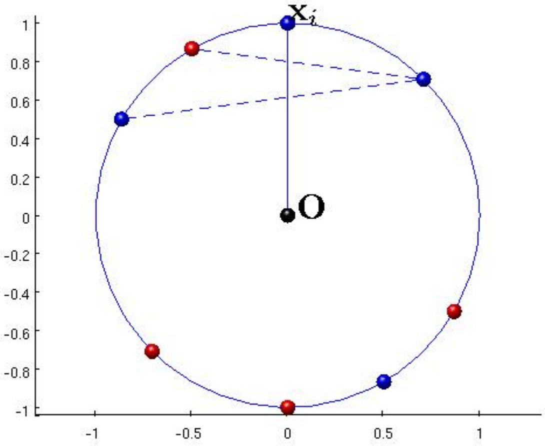

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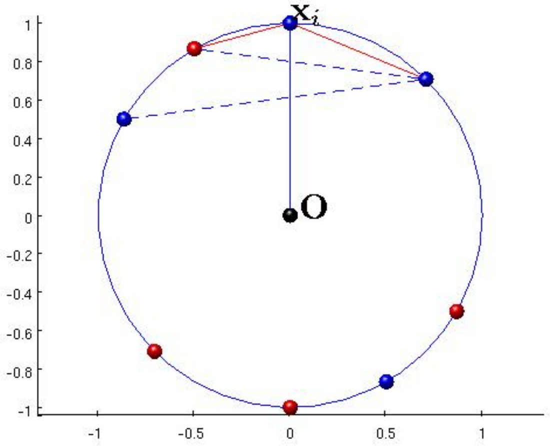

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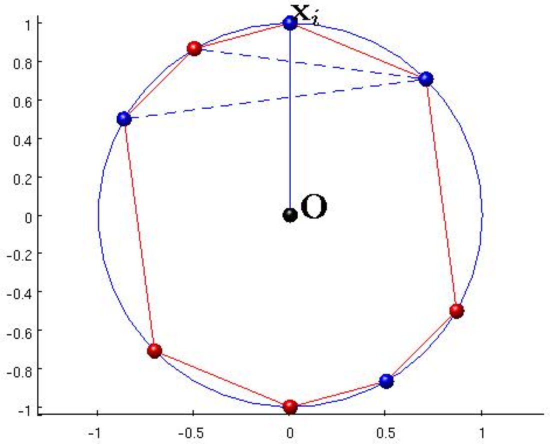
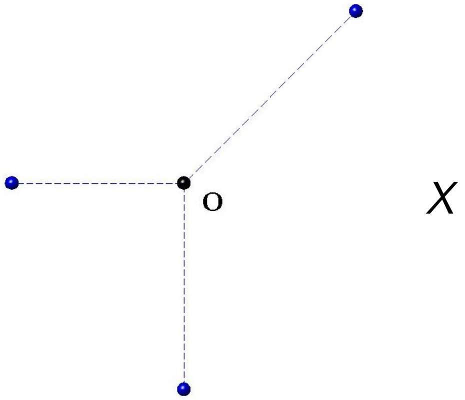


Preparation

Adding Negative Points

- $\mathbf{x}_i = \sum_{j \neq i} a_j \mathbf{x}_j$



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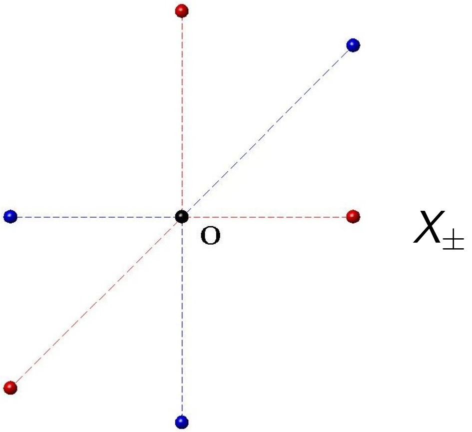
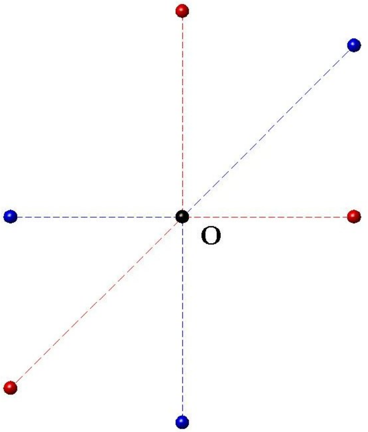


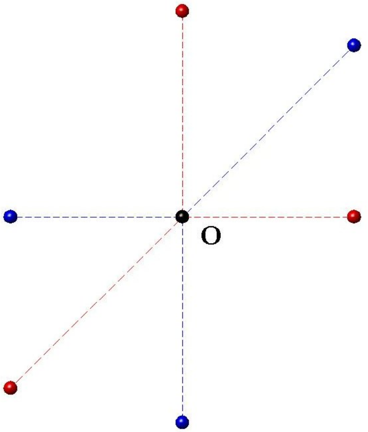
- $\mathbf{x}_i = \sum_{j \neq i} a_j \mathbf{x}_j$
- $a_j \mathbf{x}_j = -a_j(-\mathbf{x}_j)$



$X_{\pm}$

Preparation
Adding Negative Points

- $\mathbf{x}_i = \sum_{j \neq i} a_j \mathbf{x}_j$
- $a_j \mathbf{x}_j = -a_j(-\mathbf{x}_j)$
- $a_j \geq 0$



$X_{\pm}$

Preparation

NICTA

$$\mathbf{a}^i = \arg\min \|\mathbf{a}\|_1$$

s.t.

$$\mathbf{x}_i = \mathbf{X}\,\mathbf{a},$$

$$a_i = 0$$

$$\mathbf{a}^i = \arg\min \mathbf{1}^T\mathbf{a}$$

s.t.

$$\mathbf{x}_i = \mathbf{X}_{\pm}\,\mathbf{a},$$

$$\mathbf{a} \succeq \mathbf{0},$$

$$a_i = 0$$

NICTA

- L^1 minimization and geometry of polytopes *[David Donoho 2005]*

Geometry of Polytopes

- L^1 minimization and geometry of polytopes *[David Donoho 2005]*

$$\mathbf{x}_i = \mathbf{X}_{-i}\mathbf{b} = \mathbf{X}_{-i}\frac{\mathbf{b}}{\|\mathbf{b}\|_1} \cdot \|\mathbf{b}\|_1$$

NICTA

- L^1 minimization and geometry of polytopes *[David Donoho 2005]*

$$\mathbf{x}_i = \mathbf{X}_{-i}\mathbf{b} = \mathbf{X}_{-i}\frac{\mathbf{b}}{\|\mathbf{b}\|_1} \cdot \|\mathbf{b}\|_1$$

$$\mathbf{x}_i = \mathbf{X}_{-i}\,\mathbf{p}\,\alpha,$$

where

$$\|\mathbf{p}\| = 1, \mathbf{p} \succeq \mathbf{0}$$

α : to be minimized

- L^1 minimization and geometry of polytopes *[David Donoho 2005]*

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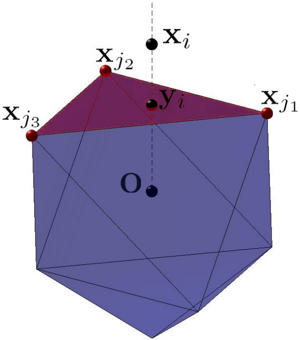
$\mathbf{X}_{-i}\,\mathbf{p}$: convex hull of X_{-i}

Geometry of Polytopes

$$\text{maximize } \beta \qquad s.t. \qquad \beta \mathbf{x}_i \in \mathbf{hull}(X_{-i})$$



Assumptions

- Indivisible subspaces?

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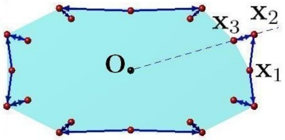
- Degenerate Cases



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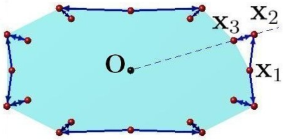

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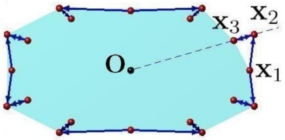

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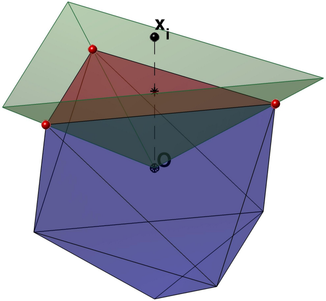
Assumption

The facet of each **polytope**(X_{-i}) *on which* $\mathbf{y}_i$ *lies has exactly d points on it.*

Neighbourhood Cones



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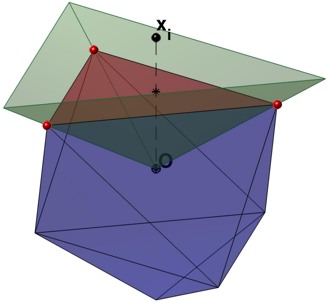
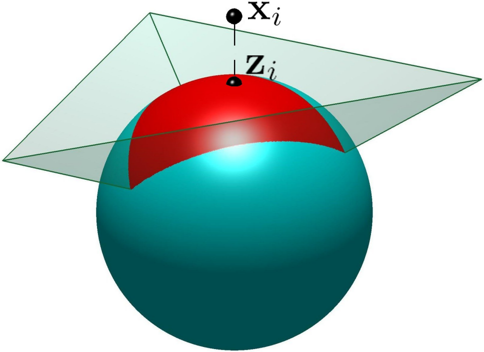


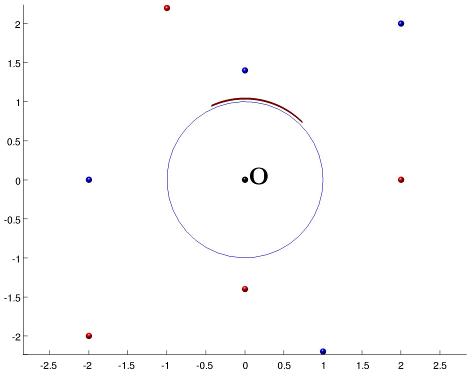
Theorem

Two points are neighbours iff their neighbourhood cones strictly intersect.

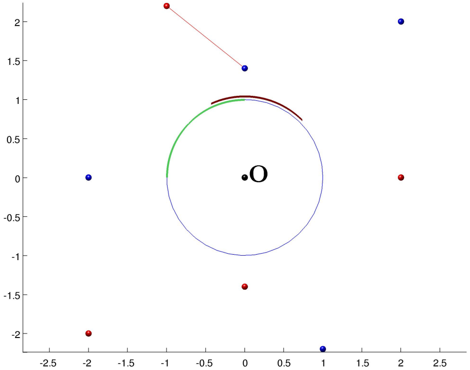
Projection on Unit Hypersphere



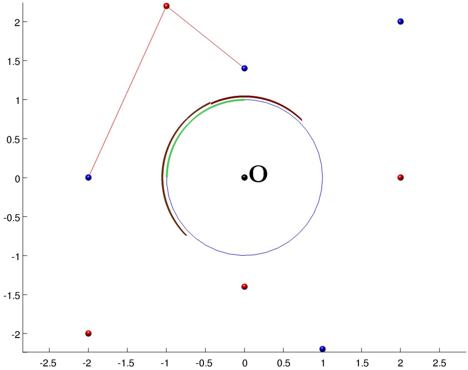
2D Case



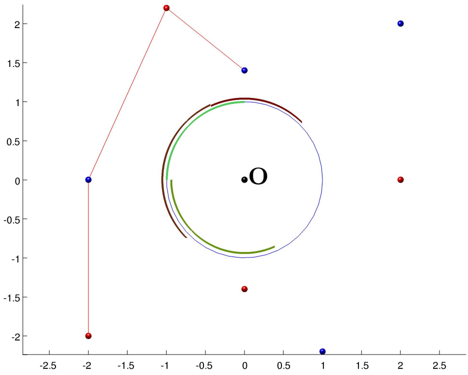
2D Case



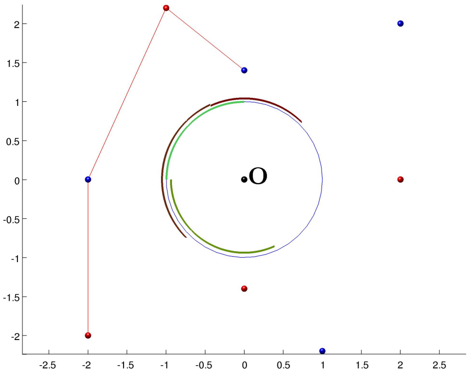
2D Case

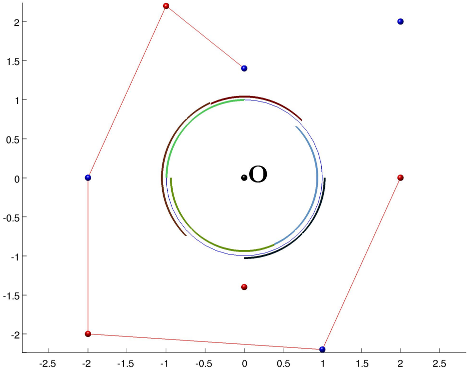


2D Case

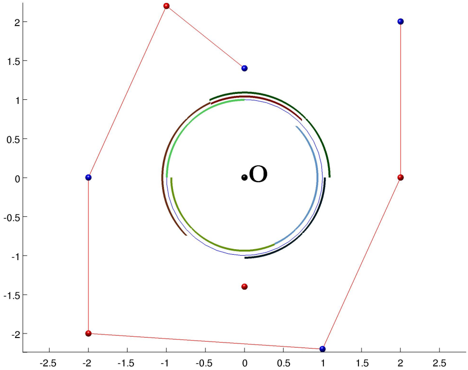


2D Case

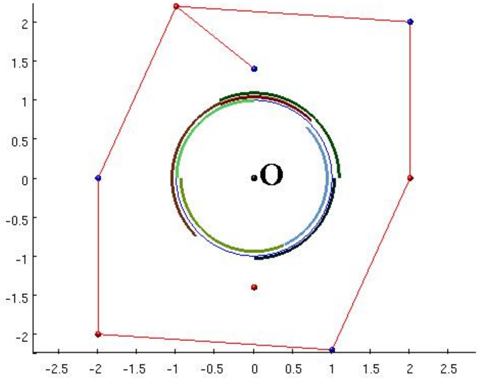



O
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5

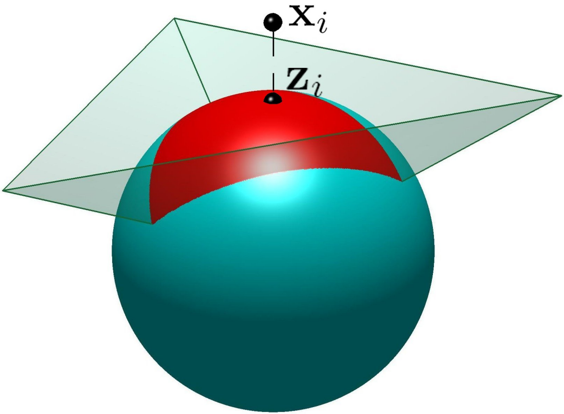
2D Case

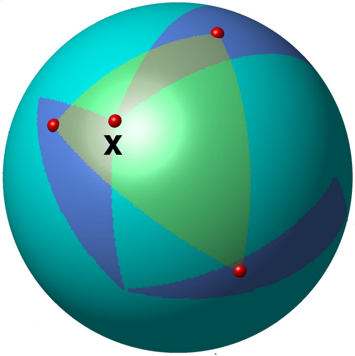


2D Case

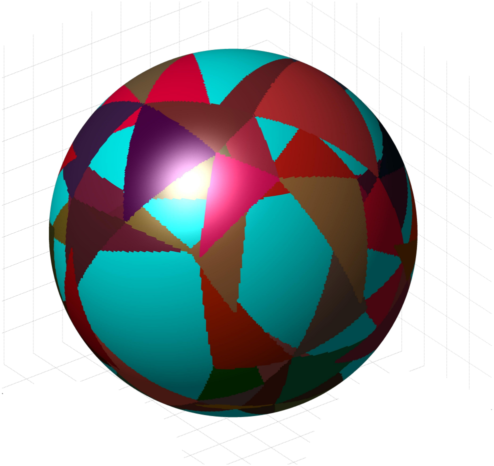


3D Case

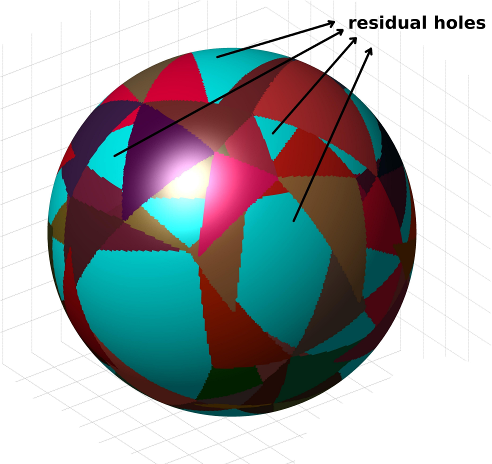



x

3D Case



3D Case



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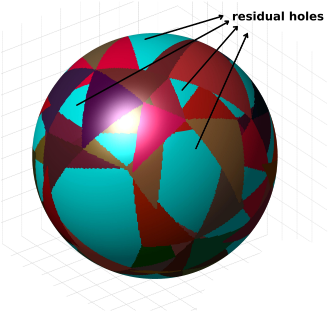


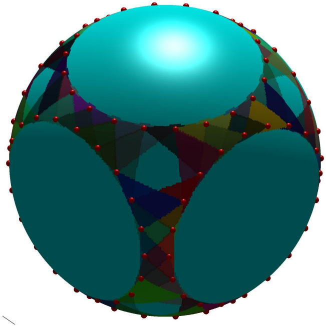
- **residual holes** for one connected component
  - Topologically open disks.
  - Area: $<$ half-sphere. ($\Leftarrow$ Gauss-Bonnet Theorem)

What about $d \geq 4$?

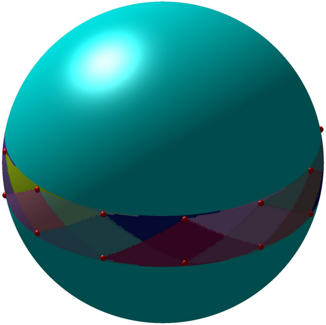
- No residual holes
- Search for counterexamples

Observations from 3D

Towards a Counterexample



Towards a Counterexample



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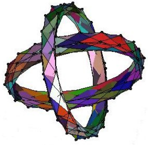
- Data around two great circles:
  - $[\cos\theta, \sin\theta, \quad 0, \quad 0]^T$
  - $[\quad 0, \quad 0, \cos\theta, \sin\theta]^T$

- $X_{\pm} = X_{C_1} \cup X_{C_2}$
  - $X_{C_1} : [\cos\frac{k\pi}{m}, \quad \sin\frac{k\pi}{m}, \quad \pm\delta, \quad \pm\delta]^T$
  - $X_{C_2} : [\quad \pm\delta, \quad \pm\delta, \quad \cos\frac{k\pi}{m}, \quad \sin\frac{k\pi}{m}]^T$

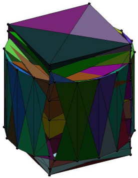
- Disconnected for:
  - $\delta \in (\Delta, \frac{\sqrt{2}}{2} - \Delta)$
  - $\Delta = \mathrm{f}(m)$

(a) $\delta = \Delta + \epsilon$



(b) $\delta = \Delta + \epsilon$

(c) $\delta = \frac{\sqrt{2}}{2} - \Delta - \epsilon$

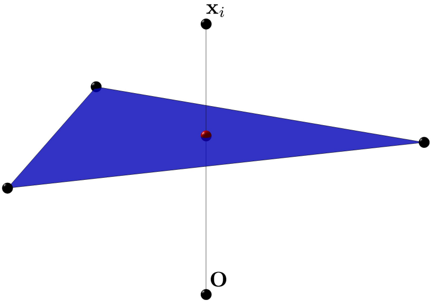


(d) $\delta = \frac{\sqrt{2}}{2} - \Delta + \epsilon$

Figure: Orthographic projection to 3D

Is Connectivity Generic?

- In the counterexample $\mathbf{ray}(\mathbf{x}_i)$ hits the interior of the facet.



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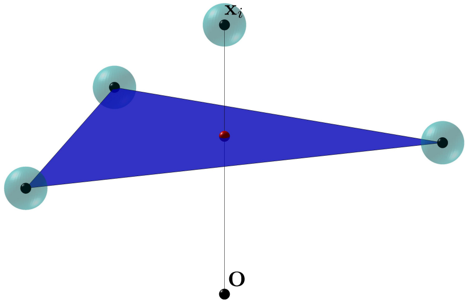

Conclusion

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- Advantages of Sparse Subspace Clustering

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Conclusion

- The importance of Subspace Clustering
- Advantages of Sparse Subspace Clustering
- For $d = 2$ and $d = 3$ it will not fail,
- Caution must be taken for $d \geq 4$,
  - A post processing stage, etc.

NICTA

Thanks

Questions?

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