Fundamentals of Computer Vision (Undergraduate course) Spring 1398 - Final Exam	Instructor: B. Nasihatkon	دانشگاه صنعتی خواجه نصیرالدین طوسی K. N. TOOSI UNIVERSITY OF TECHNOLOGY
Name:	ID:	Khordad 1398 - June 2019

Question 1 - Image Transforms/RANSAC (20 points)

Consider the transformation $\phi : R^2 \rightarrow R^2$ defined as

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \phi(x, y) = \begin{pmatrix} a\,x + b\,y + c\,x^2 + d\,y^2 + e\,xy + f \\ g\,x + h\,y + i\,x^2 + j\,y^2 + k\,xy + l \end{pmatrix}$$

A) How many correspondences between pairs of points (x,y) and (x', y') in two images are needed to fully determine the transformation ϕ? Why? (6 points)

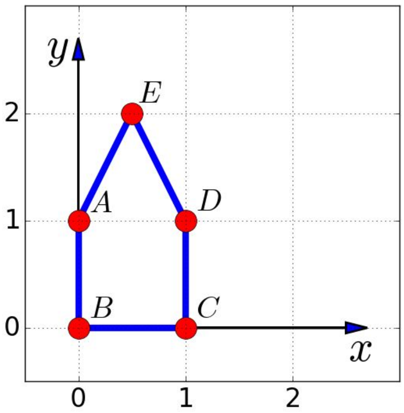
B) Assume that we have a large number of point correspondences out of which at least 60 per cent are inliers. Derive a lower bound on the number of RANSAC iterations so that there is a 0.999999 chance of success (14 points).

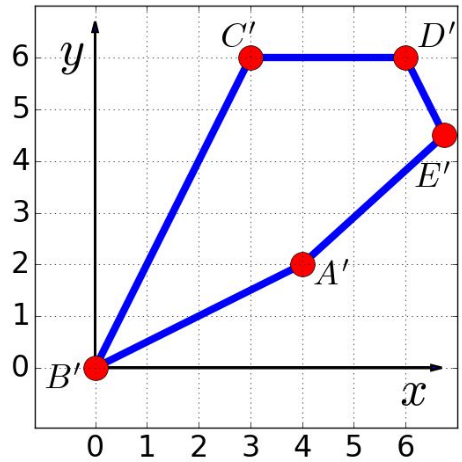
Here is the relation between the probability of getting at least one sample with all inliers (p), the minimum number of point matches to compute the transformation (s), and the proportion of the outliers (e):

$$(1-p) = (1-(1-e)^s)^N$$

Question 2 - Image Transforms/Homography (30 points)

In the images below we have $A = (0,1)$, $B = (0,0)$, $C = (1,0)$, $D = (1,1)$, $E = (0.5,2)$ and $A' = (4,2)$, $B' = (0,0)$, $C' = (3,6)$, $D' = (6,6)$. The images are related by a *homography*.





A) Compute the Homography matrix taking the point in the left image to the right image. Write down the full derivations. (Hint: start with the correspondence $B \rightarrow B'$) (24 points)

B) Compute the coordinates of E'. (6 points)

Question 3 - Scale Space / SIFT Detection (20 points)

The matrices in the left column are the output of applying Gaussian filters with different bandwidths for a single octave in the SIFT detection algorithm. On the right, we have the Difference of Gaussian images.

Fill in the blank areas in the Gaussian filtered images so that there are only 2 sift keypoints located at
(x=2, y=1, scale=2).
and
(x=1, y=2, scale=3),
as marked by "X" in the difference of Gaussian images. (This is **before** removing the edges and low contrast points, and sub-pixel tuning). Also, Fill in the Difference of Gaussian values. Explain why we have key points in the above-mentioned locations, and why we do not have keypoints in other locations.

Notice that the key points are found at the locations of extrema in the scale-space.

The keypoints cannot exist in the first and last scale.

Gaussian Filtered Images

25	22	20	17
25	28	19	17
20	19	19	17
15	15	15	15

25	20	20	16
25			16
19			16
13	13	14	14

24	18	20	14
25			15
18			14
12	12	13	13

22	15	20	12
24			14
16			13
11	11	11	12

20	10	20	8
20			8
16			10
12	10	9	10

Difference of Gaussian Images

scale = 1

scale = 2

		X	

scale = 3

	X		

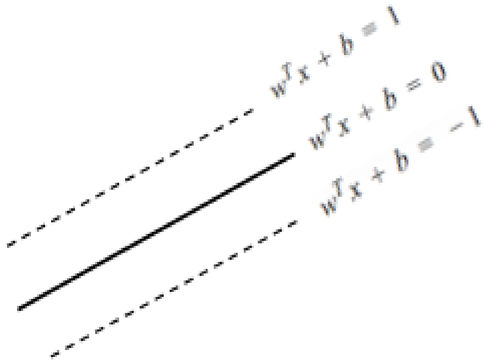
scale = 4

Question 4 - SVM (30 points)

Consider a 2-class linear SVM to be trained with the following training data. The negative samples (y_i = -1) include

$$N = \{(\sin(k\pi/3) + 1,\ \cos(k\pi/3) - 1) \mid k \in \{0, 1, 2, 4, 5\}\}$$

and the positive (y_i = +1) samples are



$$P = \{(\cos(k\pi/4) - 1,\ \sin(k\pi/4) + 1) \mid k \in \{0, 1, 2, 4, 5, 6, 7\}\}$$

Determine the support vectors and the optimal **w** and **b**. Hint: find all potential sets of support vectors, for each of them, compute **w** and **b,** and choose the one with the largest margin).