

## Question 2- Linear filtering

Your task is to find the 3 by 3 filter below (i.e. the values  $A, B, C, D, E, F, G, H, I$ ).

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix}$$

These are what we know about the filter:

- A) Correlating the input image below with the filter results in the image on the right.

**Input image**

1	2	0	0
-2	-1	0	1
2	0	0	0
-1	0	0	0

**Correlated**

x	x	x	x
x	-5	6	x
x	1	2	x
x	x	x	x

- B) The filter is separable. Notice that a filter is separable if and only if it can be written as the product of a column vector by a row vector. For a 3x3 filter, it means that there exist real values  $x, y, z, u, v, w$  such that

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} u & v & w \end{bmatrix}$$

Hint: The choice of  $x, y, z, u, v, w$  is not unique:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} u & v & w \end{bmatrix} = \begin{bmatrix} \alpha x \\ \alpha y \\ \alpha z \end{bmatrix} \begin{bmatrix} u/\alpha & v/\alpha & w/\alpha \end{bmatrix}$$

So, unless  $A=B=C=0$ , you may assume  $x = 1$ .

- C)  $A = 1$ .

Write down the full derivations. Do not forget to report the values of  $A, B, C, D, E, F, G, H, I$ .  
Good Luck!