

$$\begin{cases} \text{Eq1} & x+y=7 \\ \text{Eq2} & 2x-y=5 \end{cases}$$

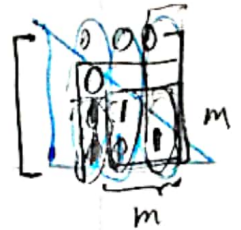
$$\text{Eq2} \xrightarrow{-2\text{Eq1}} \begin{cases} x+y=7 \\ 0x-3y=-9 \end{cases} \Rightarrow \boxed{y=3} \Rightarrow \boxed{x+3=7} \Rightarrow \boxed{x=4}$$

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$

$$r_2 \xrightarrow{-2r_1} \begin{bmatrix} 1 & 1 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$$

بالاصفح

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Backward Substitution

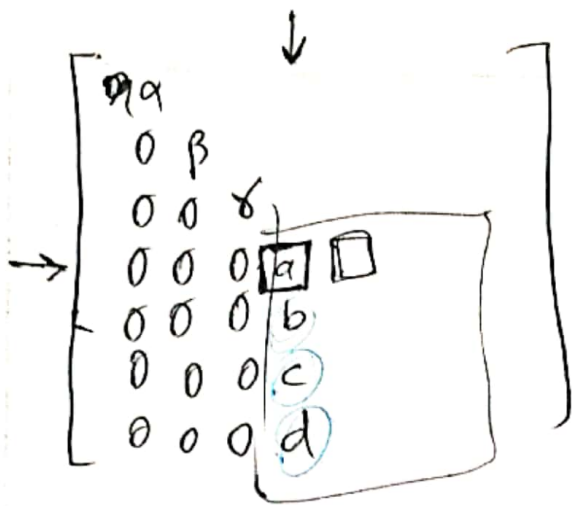
$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \\ 11 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & | & 6 \\ 3 & 1 & -1 & | & -2 \\ 2 & -3 & 1 & | & 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & | & 6 \\ 0 & -5 & -10 & | & -20 \\ 2 & -3 & 1 & | & 11 \end{bmatrix} \xrightarrow{r_3 \xrightarrow{-2r_1}} \begin{bmatrix} 1 & 2 & 3 & | & 6 \\ 0 & -5 & -10 & | & -20 \\ 0 & -7 & -5 & | & -1 \end{bmatrix}$$

$$r_3 \xrightarrow{-\frac{7}{5}r_2} \begin{bmatrix} 1 & 2 & 3 & | & 6 \\ 0 & -5 & -10 & | & -20 \\ 0 & 0 & 9 & | & 27 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -20 \\ 27 \end{bmatrix}$$

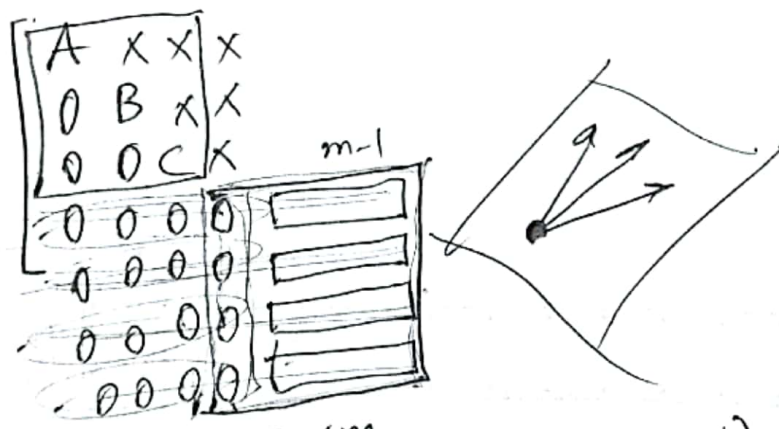
$$r_2 \xrightarrow{1=-5} \begin{bmatrix} 1 & 2 & 3 & | & 6 \\ 0 & 1 & 2 & | & 4 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{r_1 \xrightarrow{-3r_3}} \begin{bmatrix} 1 & 2 & 0 & | & -3 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \xrightarrow{r_2 \xrightarrow{-2r_3}} \begin{bmatrix} 1 & 2 & 0 & | & -3 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix}$$

$$r_1 \xrightarrow{-2r_2} \begin{bmatrix} 1 & 0 & 0 & | & 1 \\ 0 & 1 & 0 & | & -2 \\ 0 & 0 & 1 & | & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 6 \\ 3 & 1 & -1 & -2 \\ 2 & -3 & 1 & 11 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 6 \\ 0 & -5 & -10 & -20 \\ 2 & -3 & 1 & 11 \end{bmatrix}$$

$$E_{12} \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & -1 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = E_{12} \begin{bmatrix} 6 \\ -2 \\ 11 \end{bmatrix}$$



np. linalg. solve (A, \underline{B})

$$\begin{matrix} A & X & = & B \\ n \times n & n \times p & & n \times p \end{matrix}$$

$$\begin{matrix} [A | B] \\ \downarrow G-J \\ [I | C] \end{matrix}$$

$$\begin{matrix} n \times p & n \times p \\ IX & = C \\ X & = C \end{matrix}$$

$$A \underline{X} = I$$

$n \times n \quad n \times n \quad n \times n$

$$\begin{matrix} [A \ I] \\ \downarrow G-J \\ [I \ A^{-1}] \end{matrix}$$

$$Ax = b \begin{cases} x = \text{np.linalg.solve}(A, b) \\ x = \text{np.linalg.inv}(A) @ b \end{cases}$$

↓ ↓
A b

$$\begin{array}{c} n \times 1 \\ [A \quad b] \\ \downarrow \text{G-J} \\ [I \quad x] \end{array}$$

$$\begin{array}{c} n \times n \\ [A \quad I] \\ \downarrow \text{G-J} \\ [I \quad A^{-1}] \end{array}$$

$x = A^{-1}b$

$AX = B$

$$\begin{array}{c} [A \quad B] \\ [I \quad X] \end{array}$$

$$\begin{array}{c} [A \quad I] \\ [A \quad I] \end{array}$$

$X = A^{-1}B$