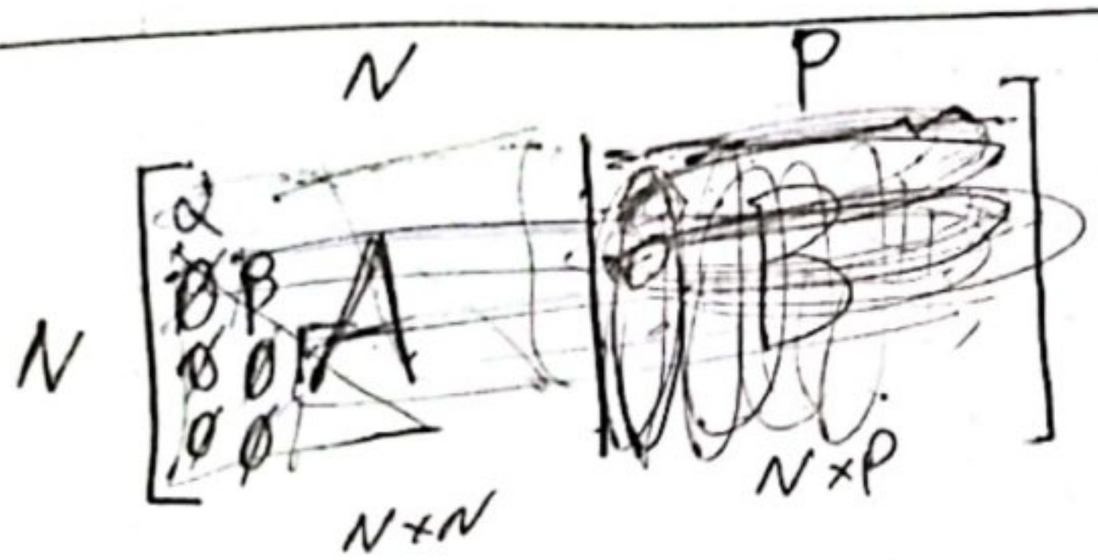
$Ax = b$ $\quad AX = B$

$[A \mid B] \rightarrow [\mathbb{I} \mid B'] \rightarrow [I \mid B'']$

$N \; [A \mid I]^{N \; N} \rightarrow [\mathbb{I} \mid G] \quad G = A^{-1}$



$(N-1+P)(N-1)$

$+ (N-2+P)(N-2)$

$+ (1+P)(1)$

$(N-1)^2 + (N-2)^2 + \cdots + 1^2 + P(1+2+\cdots+N-1)$

$\frac{(N-1)N(2N-3)}{6} +$

$O(N^3) + P\frac{N(N-1)}{2}$

$O(N^3) + O(PN^2) = O(N^3 + PN^2)$

$\geq$ Inverse $O(N^3)$

$Ax = b$ $\quad$ solve(A, b) $\quad$ solve(A, B)

inv(A) @ b $\quad$ inv(A) @ B

$A^{-1}B = A^{-1}[b_1 \; b_2 \cdots b_p] \quad N \times P$

$(AB)^{-1}AB = I$

$B^{-1}A^{-1}AB = B^{-1}B = I$

$(AB)^{-1}$ $A, B \in \mathbb{R}^{n\times n}$ LA II (II)

$(AB)^{-1} = B^{-1}A^{-1}$

$\boxed{\text{rank}(A) \geq \text{rank}(AB)}$

$x \in C($

$\text{rank}(A)$ $\text{rank}(AB)$

$\dim(C(A))$ $\dim(C(AB))$

T S

$A = [\vec{a}_1\ \vec{a}_2 \cdots \vec{a}_n]$

$z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$

Az

$[\vec{a}_1\ \vec{a}_2 \cdots \vec{a}_n] \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix}$

$z_1\vec{a}_1 + z_2\vec{a}_2 + \cdots + z_n\vec{a}_n$

$x \in C(AB) \Rightarrow \exists z \quad x = (AB)z$

$x = A(\underbrace{Bz}_{y}) \Rightarrow x = Ay \Rightarrow x \in C(A)$

$\dim(C(AB)) \leq \dim(C(A))$

$\text{rank}(AB) \leq \text{rank}(A)$

$\text{rank}(ABCD) \leq \min(\text{rank}(A), \text{rank}(B), \text{rank}(D), \text{rank}(C))$

$(A^T)^{-1}$ $A \in \mathbb{R}^{n\times n}$

$AA^{-1} = I$

$(AA^{-1})^T = I^T$

$(A^{-1})^T A^T = I$

$(A^{-1})^T = (A^T)^{-1} = A^{-T}$

$\begin{bmatrix} a & b \\ & c \end{bmatrix}$

LA II (III)

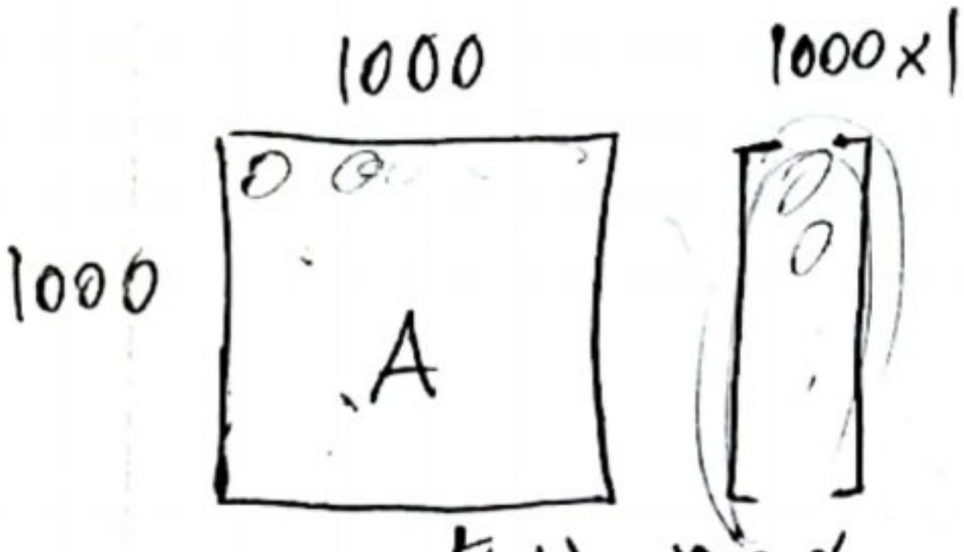
$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & cz \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}^{-1} = \begin{bmatrix} 1/a & 0 & 0 \\ 0 & 1/b & 0 \\ 0 & 0 & 1/c \end{bmatrix}$$

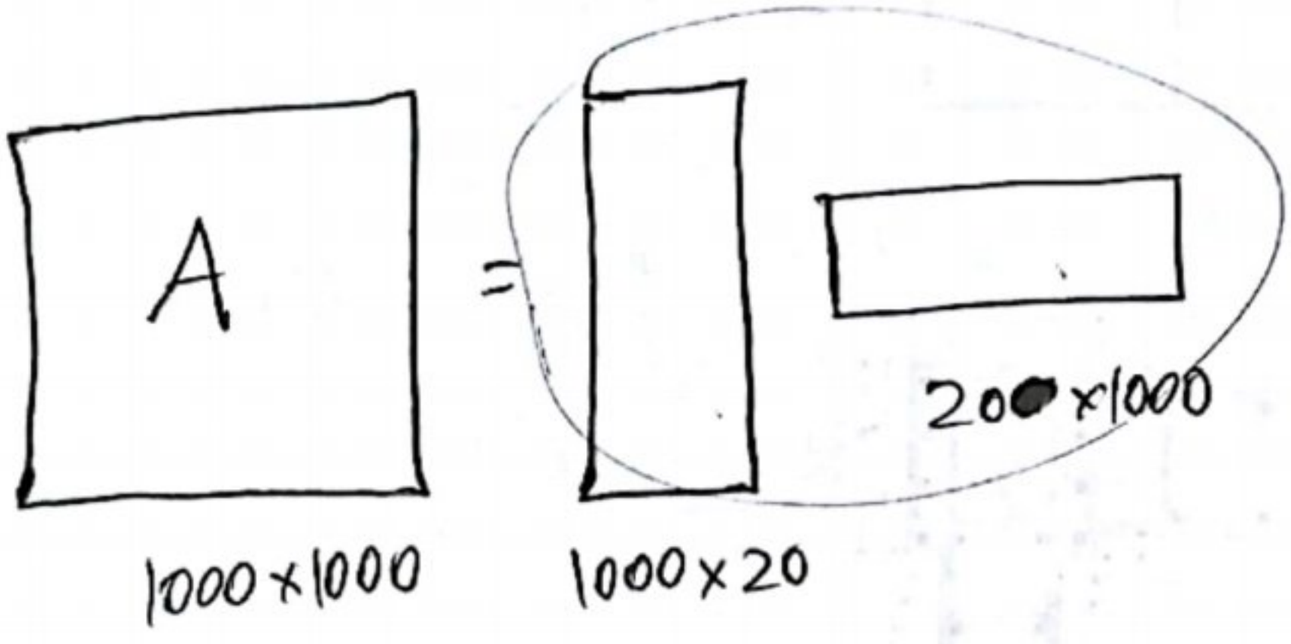
$$A \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix}$$

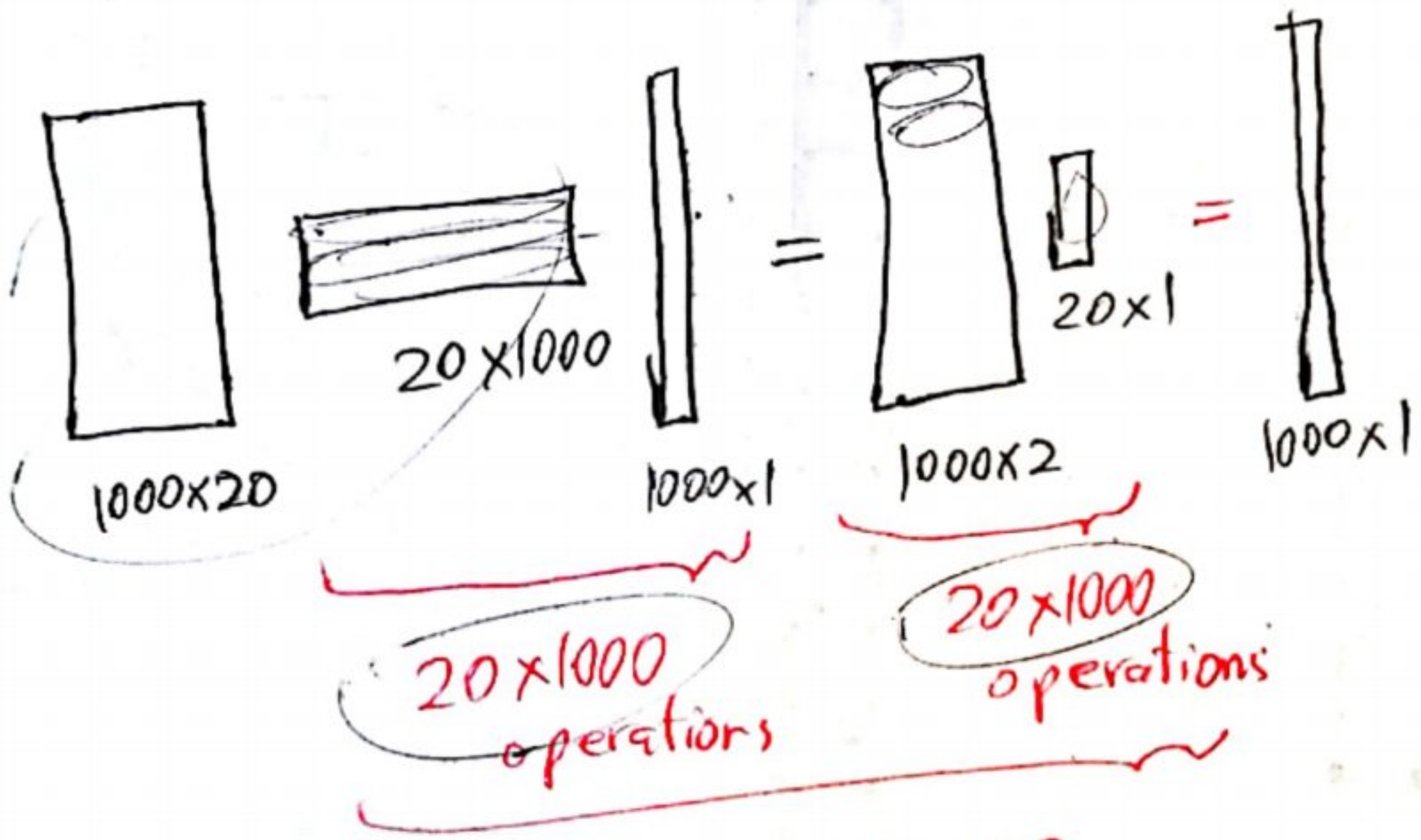
$$= \begin{bmatrix} \alpha a_1 & \beta a_2 & \gamma a_3 \end{bmatrix}$$

$$\begin{bmatrix} a & 0 & 0 \\ d & b & 0 \\ f & e & c \end{bmatrix}^{-1} \begin{bmatrix} x & 0 & 0 \\ u & v & w \\ p & q & r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



1000x1000 multiplications





1000000

40000

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad r_2 -= 3r_1 \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

L U

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix}$$

$$A = L \quad U$$

$r_2 := \alpha_{21} r_1$

$r_3 := \alpha_{31} r_1$

$r_3 := \alpha_{41} r$

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & \ell \\ m & n & o & p \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21} & 1 & 0 & 0 \\ \alpha_{31} & 0 & 1 & 0 \\ \alpha_{41} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c & d \\ 0 & f' & g' & h' \\ 0 & j' & k' & \ell' \\ 0 & n' & o' & p' \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21} & 1 & 0 & 0 \\ \alpha_{31} & 0 & 1 & 0 \\ \alpha_{41} & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \alpha_{32} & 1 & 0 \\ 0 & \alpha_{42} & 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c & d \\ 0 & f' & g' & h' \\ 0 & 0 & k'' & \ell'' \\ 0 & 0 & o'' & p'' \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21} & 1 & 0 & 0 \\ \alpha_{31} & \alpha_{32} & 1 & 0 \\ \alpha_{41} & \alpha_{42} & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \alpha_{43} & 1 \end{bmatrix} \begin{bmatrix} a & b & c & d \\ 0 & f' & g' & h' \\ 0 & 0 & k'' & \ell'' \\ 0 & 0 & 0 & p''' \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21} & 1 & 0 & 0 \\ \alpha_{31} & \alpha_{32} & 1 & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix} \begin{bmatrix} a & b & c & d \\ 0 & \times & \times & \times \\ 0 & 0 & \times & \times \\ 0 & 0 & 0 & \times \end{bmatrix}$$

$$A = P \quad L \quad U$$

$$PA = LU$$

$$A = P^{\dagger} L U$$

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} \begin{bmatrix} x & y \\ 0 & z \end{bmatrix}$$