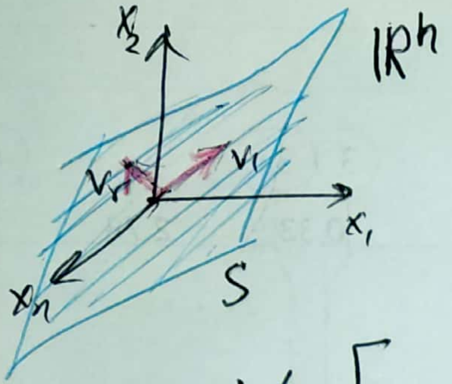
LAB ①



How to represent a subspace?

$\dim(S) = r \leq n$

choose a basis $v_1, v_2, \dots, v_r \in S$

represent $V = \begin{bmatrix} v_1 & v_2 & \cdots & v_r \end{bmatrix}$ $\quad V = \begin{bmatrix} v_1 & v_2 & \cdots & v_r \end{bmatrix} \in \mathbb{R}^{n\times r}$

represent S with $V = \begin{bmatrix} v_1 & v_2 & \cdots & v_r \end{bmatrix} \in \mathbb{R}^{n\times r}$

$S = C(V)$

V a representation of $S \Rightarrow$ So is VH

for any invertible matrix $H \in \mathbb{R}^{r\times r}$

$$\underbrace{\begin{bmatrix} V \end{bmatrix}}_{m\times r} \underbrace{\begin{bmatrix} H \end{bmatrix}}_{r\times r} = \underbrace{\begin{bmatrix} \ \end{bmatrix}}_{m\times r} \qquad C(V) = C(VH)$$

$$\begin{bmatrix} \boxed{1} & 2 & 3 & 4 & 5 \\ 0 & 0 & \boxed{2} & 3 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \\ u \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ z \\ t \\ u \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 2 & 4 & 5 \\ 0 & 3 & -2 \end{bmatrix} \begin{bmatrix} y \\ t \\ u \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} y + \begin{bmatrix} 3 \\ 2 \end{bmatrix} z + \begin{bmatrix} 4 \\ 3 \end{bmatrix} t + \begin{bmatrix} 5 \\ -2 \end{bmatrix} u$$

$$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}x\\z\end{bmatrix}+\begin{bmatrix}2 & -1/2 & 8\\0 & 3/2 & -1\end{bmatrix}\begin{bmatrix}y\\t\\u\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}x\\z\end{bmatrix}=\begin{bmatrix}-2 & 1/2 & -8\\0 & -3/2 & +1\end{bmatrix}\begin{bmatrix}y\\t\\u\end{bmatrix}$$

$$\begin{bmatrix}y\\t\\u\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{matrix}0\\1\\0\end{matrix}$$

$$\begin{bmatrix}x\\y\\z\\t\\u\end{bmatrix}=\begin{bmatrix}-2 & 1/2 & -8\\1 & 0 & 0\\0 & -3/2 & 1\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$A\in\mathbb{R}^{m\times n}$ $\quad b\in\mathbb{R}^m$

$Ax=b$ $\quad A=\begin{bmatrix}a_1 & a_2 & \cdots & a_n\end{bmatrix}$

$$Ax=\begin{bmatrix}a_1 & a_2 & - & a_n\end{bmatrix}\begin{bmatrix}x_1\\x_2\\|\\x_n\end{bmatrix}=x_1\vec{a}_1+x_2\vec{a}_2+\cdots+x_n\vec{a}_n=b$$

$b\notin C(A)\Rightarrow Ax=b$ has no solution

$b\in C(A)$

$A\vec{x}=\vec{b}$ $\quad b\in C(A)$

Let $\vec{x}_p$ be one solution to $A\vec{x}=b\Rightarrow A\vec{x}_p=\vec{b}$

~~$\Rightarrow A\vec{x}_p=b$~~

Take $\vec{x}_n\in N(A)\Rightarrow A\vec{x}_n=0$

Is $x'=x_p+x_n$ a solution? $Ax'=A(x_p+x_n)=\underbrace{Ax_p}_{b}+\underbrace{Ax_n}_{0}=b$

Let x_p be a solution to $Ax=b$

$$\Rightarrow Ax_p = b$$

Let x be ~~another~~ also a solution $=$

$$\Rightarrow Ax = b$$

$$\Rightarrow Ax - Ax_p = 0 \Rightarrow A(x-x_p) = 0 \Rightarrow x - x_p \in N(A)$$

Take $x_n \doteq x - x_p$. $x = x_p + x_n$

particular solution $\rightarrow \in N(A)$

$A \in \mathbb{R}^{m \times n}$ $b \in C(A)$, x_p a solution to $Ax=b$

All solutions to $Ax=b$

$$= \{ x_p + x_n \mid x_n \in N(A) \}$$

$A \in \mathbb{R}^{m\times n}$ $\operatorname{rank}(A) = r$

Let columns of $N \in \mathbb{R}^{n \times (n-r)}$ form a basis for $N(A)$

$$x_n \in N(A) \Rightarrow x_n = Nz \quad z \in \mathbb{R}^{n-r}$$

$$\text{Solutions} = \{ x_p + Nz \mid z \in \mathbb{R}^{n-r} \}$$