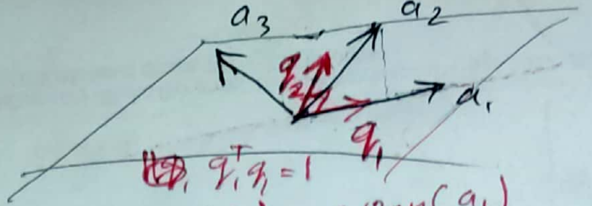


$q_1^T q_1 = 1$

$$u_1 = a_1 \qquad \text{span}(q_1) = \text{span}(a_1)$$
$$q_1 = \frac{u_1}{\|u_1\|}$$

$$u_2 = a_2 - q_1 q_1^T a_2 \qquad \begin{bmatrix} q_1^T \\ q_2^T \end{bmatrix} \begin{bmatrix} q_1 & q_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$q_2 = \frac{u_2}{\|u_2\|} \qquad \text{span}(q_1, q_2) = \text{span}(a_1, a_2)$$

$$u_3 = a_3 - q_1 q_1^T a_3 - q_2 q_2^T a_3 \qquad \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^T \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} = I$$
$$q_3 = \frac{u_3}{\|u_3\|} \qquad \text{span}(q_1, q_2, q_3) = \text{span}(a_1, a_2, a_3)$$

$\vdots$

$$u_i = a_i - \sum_{j=1}^{i-1} q_j q_j^T a_i$$
$$q_i = \frac{u_i}{\|u_i\|}$$

$$a_1, a_2, \dots, a_n \Longrightarrow \underbrace{q_1, q_2, \dots, q_n}_{\text{orthonormal}}$$
$$\text{span}(q_1, q_2, \dots, q_n) = \text{span}(a_1, a_2, \dots, a_n)$$

$$A \overset{?}{=} Q$$
$$\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \overset{?}{=} \begin{bmatrix} q_1 & q_2 & \cdots & q_n \end{bmatrix}$$

$$u_1 = a_1 \Longrightarrow a_1 = q_1 \|a_1\| \qquad \begin{bmatrix} a_1 \end{bmatrix} = \begin{bmatrix} q_1 \end{bmatrix} \begin{bmatrix} r_{11} \end{bmatrix}, \quad r_{11} = \|a_1\|$$
$$q_1 = \frac{u_1}{\|u_1\|}$$

$$u_2 = a_2 - q_1 q_1^T a_2 \Longrightarrow a_2 = u_2 + q_1 q_1^T a_2$$
$$q_2 = \frac{u_2}{\|u_2\|} \qquad a_2 = q_2 \underbrace{(\|u_2\|)} + q_1 \underbrace{(q_1^T a_2)}$$
$$\Rightarrow a_2 = r_{12} q_1 + r_{22} q_2 \qquad \begin{bmatrix} a_1 & a_2 \end{bmatrix} = \begin{bmatrix} q_1 & q_2 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} \\ 0 & r_{22} \end{bmatrix}$$

$$u_i = a_i - \sum_{j=1}^{i-1} q_j \underbrace{q_j^T a_i} \Longrightarrow a_i = \|u_i\| q_i + \sum_{j=1}^{i-1} (q_j^T a_i) q_j$$
$$q_i = \frac{u_i}{\|u_i\|} \qquad = r_{ii} q_i + \sum r_{ji} q_j$$

Scanned with CamScanner

LA 19 (II)

$$\underset{m\times n}{\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}} = \underset{m\times n}{\begin{bmatrix} q_1 & q_2 & \cdots & q_n \end{bmatrix}} \underset{n\times n}{\begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{1n} \\ 0 & r_{22} & r_{23} & r_{2n} \\ 0 & 0 & r_{33} & \vdots \\ 0 & 0 & 0 & r_{nn} \end{bmatrix}}$$

upper-triangular

$$[A] = Q\,R$$

R → upper triangular

$Q^TQ = I$ (Q has orthonormal columns)

$m = n$

$$\underset{n\times n}{A} = \underset{n\times n}{Q}\ \underset{n\times n}{R}$$

R → upper-triangular

Q → orthogonal

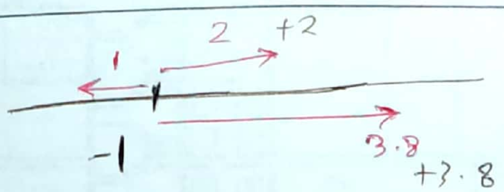
$$Q^TQ = QQ^T = I$$

Q-R decomposition

(QRQ, QL, LQ)

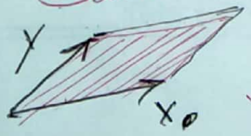
1D $x \in \mathbb{R}$ طول

signed lenght طول علامت دار



$SL(x) = x$

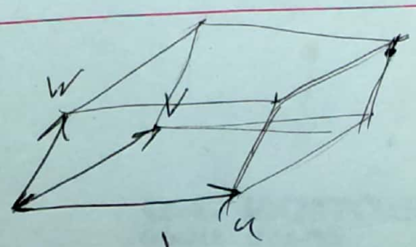
2D Area مساحت



SA: signed Area

$SA(X,Y) = -SA(Y,X)$

3D Volume حجم



SV: signed volume

$SV(u,v,w) = -SV(u,v,-w)$

$SV(u,v,w) = -SV(u,w,v)$

N-D
N-Dimensional

ابرحجم

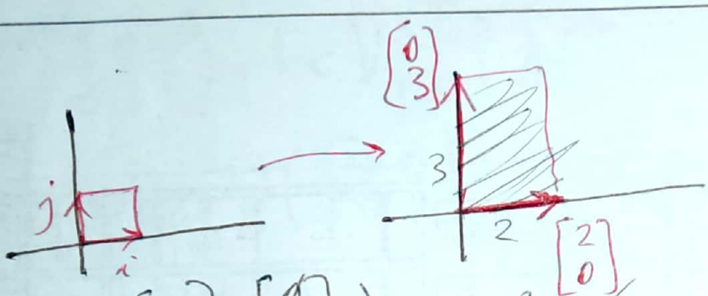
Hyper-Volume ~~Cloud~~

$V(a_1, a_2, \dots, a_N)$ $\quad a_i \in \mathbb{R}^N$ $\quad [a_1, a_2, \dots, a_N] \in \mathbb{R}^{N\times N}$

$$A = [\vec{a}_1\ \vec{a}_2 \dots \vec{a}_n] \in \mathbb{R}^{n\times n} = [\quad]$$

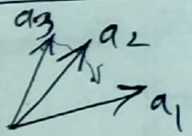
$$V(a_1, a_2, \dots, a_n) = ?$$

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$



$$V(A) = V(a_1, a_2) = V\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \end{bmatrix}\right) = 2\times 3 = 6$$

$$V\left(\begin{bmatrix} d_1 & & & 0 \\ & d_2 & & \\ & & \ddots & \\ 0 & & & d_n \end{bmatrix}\right) = d_1 \cdot d_2 \cdots d_n = \prod_{i=1}^{n} d_i$$

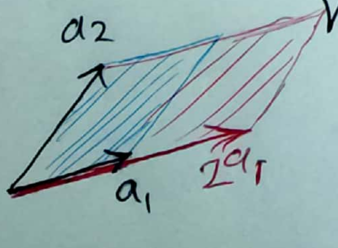


$A = [a_1\ a_2\ a_3] \in \mathbb{R}^{3\times 3}$ $\quad a_i \in \mathbb{R}^3$

a_1, a_2, a_3 dependent $\Longrightarrow V(A) = V(a_1, a_2, a_3) = 0$

A singular

2D case



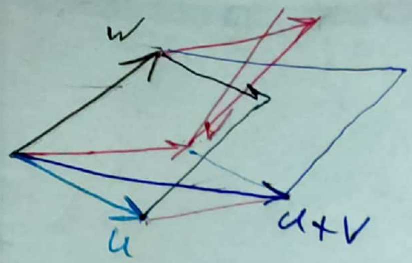
$$V(a_1, a_2)$$

$$V(2\vec{a}_1, \vec{a}_2) = 2V(a_1, a_2)$$

$$V(-a_1, a_2) = -V(a_1, a_2)$$

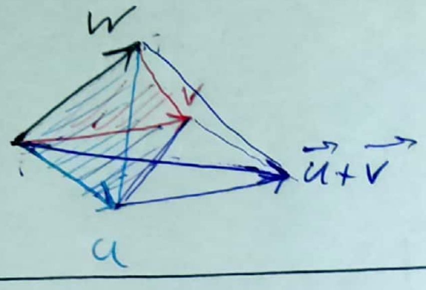
$$V(\beta a_1, a_2) = \beta V(a_1, a_2)$$

2D



$$V(u+v, w) = V(u, w) + V(v, w)$$

$$V(\alpha u + \beta v, w) = \alpha V(u, w) + \beta V(v, w)$$

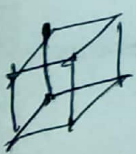


$V(a_1, a_2, \dots, a_n)$ is multilinear. $a_i \in \mathbb{R}^n$

linear in each argument

$$V(\underline{\alpha a_1 + \beta a_1'}, a_2, \dots, a_n) = \alpha V(a_1, a_2, \dots, a_n) + \beta V(a_1', a_2, \dots, a_n)$$

$$V(a_1, a_1, a_3, a_4, \dots, a_n) = 0$$



$$V(I) = 1 = V\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\\0\end{bmatrix}, \begin{bmatrix}\ \end{bmatrix} \dots, \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right) = 1$$

$V(x, y, z)$ $\quad x, y, z \in \mathbb{R}^3$

$V(y, x, z) = ?$

$$0 = V(x+y, \underline{x+y}, z) = V(x, x+y, z) + V(y, x+y, z)$$

$$= \underbrace{V(x, x, z)}_{0} + V(x, y, z) + \underbrace{V(y, y, z)}_{0} + V(y, x, z) = 0$$

$$\Rightarrow V(x, y, z) = -V(y, x, z)$$

$$V\left(\begin{bmatrix} a & c \\ b & d \end{bmatrix}\right) = V\left(\begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} c \\ d \end{bmatrix}\right)$$

$$V\left(a\begin{bmatrix} 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 0 \\ 1 \end{bmatrix}, c\begin{bmatrix} 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$aV\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, c\begin{bmatrix} 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) + bV\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}, c\begin{bmatrix} 1 \\ 0 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$

$$ac\,\underbrace{V\left(\cancel{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}}\right)}_{0} + adV\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) + bcV\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + bd\,\underbrace{\cancel{V\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}}}_{0}$$

$$ad\,V\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) + bc\,V\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)$$

$$ad - bc\,V\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = ad - bc$$

$$V\left(\begin{bmatrix} a & c \\ b & d \end{bmatrix}\right) = ad - bc$$

determinant