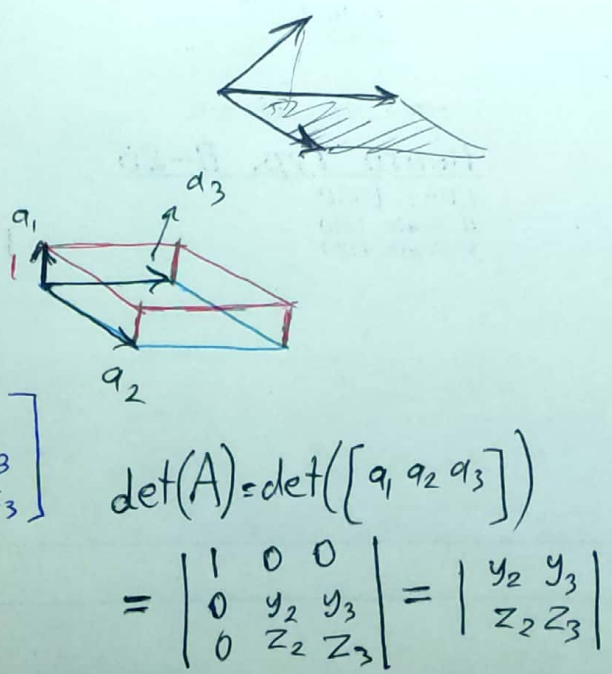
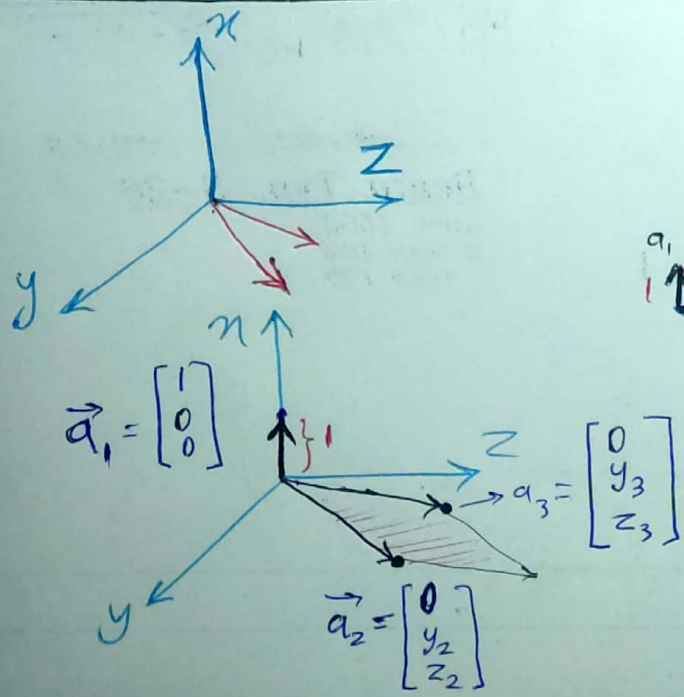


$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & b & c \\ 0 & e & f \\ 0 & h & i \end{vmatrix} + \begin{vmatrix} 0 & b & c \\ d & e & f \\ 0 & h & i \end{vmatrix} + \begin{vmatrix} 0 & b & c \\ 0 & e & f \\ g & h & i \end{vmatrix} \\
 = \begin{vmatrix} a & b & c \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} a & 0 & c \\ 0 & e & f \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} a & 0 & c \\ 0 & 0 & 0 \\ 0 & h & i \end{vmatrix} + \dots + \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g & h & i \end{vmatrix}$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = \begin{vmatrix} a & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & i \end{vmatrix} + \begin{vmatrix} a & 0 & 0 \\ 0 & 0 & f \\ 0 & h & 0 \end{vmatrix} + \begin{vmatrix} 0 & b & 0 \\ d & 0 & 0 \\ 0 & 0 & i \end{vmatrix} \\
 \quad \quad \quad (aei) \quad - (ahf) \quad - (dbi) \\
 \begin{vmatrix} 0 & 0 & c \\ d & 0 & 0 \\ 0 & h & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 & c \\ 0 & e & 0 \\ g & 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & b & 0 \\ 0 & 0 & f \\ g & 0 & 0 \end{vmatrix} \\
 dch - ceg + bfg$$



$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & & & \\ \vdots & & & \\ 0 & & & \end{bmatrix} \cdot B$$

$$\det(A) = \det(B)$$

$$M_{11} = |B| \quad \text{1-minor}$$

$$A = \begin{bmatrix} 0 & 0 & \overset{j}{1} & 0 & 0 & 0 \\ \vdots & & & & & \\ B_1 & 0 & & & & B_2 \\ \vdots & & & & & \\ 0 & & & & & \end{bmatrix}$$

$$B = [B_1, B_2] \quad |B| = M_{ij}$$

$$\det(A) = (-1)^{j+1} \det(B)$$

$$A = \begin{bmatrix} B_1 & 0 & B_2 \\ \vdots & & \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & & & & & \\ B_3 & 0 & & & & B_4 \end{bmatrix}$$

$$B = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}$$

$$\det(A) = (-1)^{j+1} (-1)^{i+1} \det(B) \\ = (-1)^{i+j} \det(B)$$

$$M_{ij} = \det(B)$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} 1 & b & c \\ 0 & e & f \\ 0 & h & i \end{vmatrix} + d \begin{vmatrix} 0 & b & c \\ 1 & e & f \\ 0 & h & i \end{vmatrix} + g \begin{vmatrix} 0 & b & c \\ 0 & e & f \\ 1 & h & i \end{vmatrix}$$

$$= a \begin{vmatrix} 1 & 0 & 0 \\ 0 & e & f \\ 0 & h & i \end{vmatrix} + d \begin{vmatrix} 0 & b & c \\ 1 & 0 & 0 \\ 0 & h & i \end{vmatrix} + g \begin{vmatrix} 0 & b & c \\ 0 & e & f \\ 1 & 0 & 0 \end{vmatrix}$$

$$= a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - d \begin{vmatrix} b & c \\ h & i \end{vmatrix} + g \begin{vmatrix} b & c \\ e & f \end{vmatrix}$$

$$M_{11} - d M_{21} + g M_{31}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix} = a_{11} M_{11} - a_{21} M_{21} + a_{31} M_{31} - \dots + (-1)^{n+1} a_{n1} M_{n1}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

$$|A| = \sum_{i=1}^n a_{ij} \underbrace{M_{ij} (-1)^{i+j}}_{C_{ij}} = \sum_{i=1}^n a_{ij} C_{ij}$$

1-Minor

$$\text{Cofactor } C_{ij} = (-1)^{i+j} M_{ij}$$

$$C = \begin{bmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & & C_{2n} \\ \vdots & \vdots & & \vdots \\ C_{n1} & C_{n2} & \dots & C_{nn} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$\langle C[:,1], A[:,1] \rangle = \det(A)$$

$$\langle C[:,j], A[:,j] \rangle = \det(A)$$

$$\langle C[:,1], A[:,2] \rangle = ? \det$$

$$= \det \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

$$\begin{bmatrix} a_{12} & a_{12} & \dots & a_{1n} \\ a_{22} & a_{22} & & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n2} & a_{n2} & \dots & a_{nn} \end{bmatrix} = 0$$

$$C_{:k}^T a_{:j} = \langle C_{:k}, a_{:j} \rangle = \begin{cases} |A| & k=j \\ 0 & k \neq j \end{cases} = (C^T A)_{kj}$$

$$C^T A = \begin{bmatrix} |A| & 0 & 0 & 0 \\ 0 & |A| & 0 & 0 \\ 0 & 0 & |A| & 0 \\ 0 & 0 & 0 & |A| \end{bmatrix} = |A| I$$

$$\Rightarrow \left(\frac{1}{|A|} C^T \right) A = I \Rightarrow A^{-1} = \frac{1}{|A|} C^T$$

$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

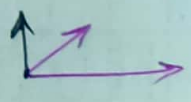
\downarrow \downarrow
 \vec{x} \vec{y}

$$\begin{bmatrix} c_{13} \\ c_{23} \\ c_{33} \end{bmatrix}$$

\downarrow
 \vec{c}_3

$\vec{c}_3 \perp x$
 $\vec{c}_3 \perp y$

$$c_3 = \begin{bmatrix} |x_2 y_2| \\ |x_3 y_3| \\ -|x_1 y_1| \\ |x_3 y_3| \\ |x_1 y_1| \\ |x_2 y_2| \end{bmatrix} = \begin{bmatrix} x_2 y_3 - y_2 x_3 \\ -x_1 y_3 + y_1 x_3 \\ x_1 y_2 - y_1 x_2 \end{bmatrix} = \vec{x} \times \vec{y}$$



$$A = \begin{bmatrix} a_1 & a_2 & \dots & a_{n-1} & a_n \end{bmatrix}$$

$\vec{a}_1, \vec{a}_2, \dots, \vec{a}_{n-1} \in \mathbb{R}^n$

$$\begin{bmatrix} a_1 & a_2 & \dots & a_{n-1} \end{bmatrix} \in \mathbb{R}^{n \times (n-1)}$$

~~$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$~~

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$$A = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix} \quad C = \begin{bmatrix} c_1 & c_2 & \dots & c_n \end{bmatrix}$$

$$A_2 = \begin{bmatrix} a_1 & a_3 & \dots & a_n \end{bmatrix} \quad b$$

$b^T C_2 = \det(A_2)$

$$Ax = b \Rightarrow x = A^{-1}b = \frac{1}{|A|} C^T b = \frac{1}{|A|} \begin{bmatrix} c_1^T \\ c_2^T \\ \vdots \\ c_n^T \end{bmatrix} b$$

$$x_i = \frac{1}{|A|} c_i^T b = \frac{|A_i|}{|A|}$$