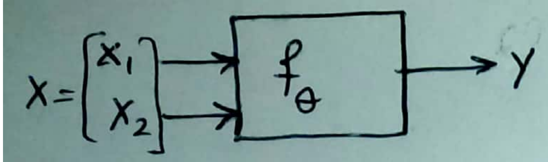

$$y = f_\theta(x) = f(\theta, x) = a x_1^2 + b x_2^2 + c x_1 x_2 + d x_1 + e x_2 + f$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \theta = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} \in \mathbb{R}^6$$

$x \in \mathbb{R}^2$

$$f : \underset{\theta}{\mathbb{R}^6} \times \underset{x}{\mathbb{R}^2} \longrightarrow \mathbb{R}$$

$$f(\theta, x) = \underbrace{\begin{bmatrix} a & b & c & d & e & f \end{bmatrix}}_{\theta^T} \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$= \theta^T \phi(x) \qquad \phi(x) = \phi\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 \\ x_2^2 \\ x_1 x_2 \\ x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$f(\theta, x) = a x_1^2 + b x_2^2 + c x_1 x_2 + d x_1 + e x_2 + f$$

$$= \underbrace{\begin{bmatrix} x_1 & x_2 \end{bmatrix}}_{x^T} \underbrace{\begin{bmatrix} a & c/2 \\ c/2 & b \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} d & e \end{bmatrix}}_{v^T} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + f$$

$$= x^T A x + v^T x + f \qquad (A \text{ symmetric})$$

$$\theta = (\underset{\text{sym}}{A}, \underset{\in \mathbb{R}^2}{v}, \underset{\in \mathbb{R}}{f})$$

$f(x) = Ax + b \qquad \theta = (A, b) \qquad (A_1, b_1) + (A_2, b_2) = (A_1 + A_2, b_1 + b_2)$

$\alpha(A, b) = (\alpha A, \alpha b)$