$\mathrm{span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n) = \{a_1\vec{u}_1 + a_2\vec{u}_2 + \dots + a_n\vec{u}_n \mid a_1, a_2, \dots a_n \in \mathbb{R}\}$

$\vec{u}_i \in \mathbb{R}^m$

$$= \left\{ \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \cdots & \vec{u}_n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \;\middle|\; a_1, \dots, a_n \in \mathbb{R} \right\}$$

$$= \{ \underset{m\times n}{U} \underset{n\times 1}{\vec{a}} \mid \vec{a} \in \mathbb{R}^n \} = \{ U\vec{a} \mid \vec{a} \in \mathbb{R}^n \}$$

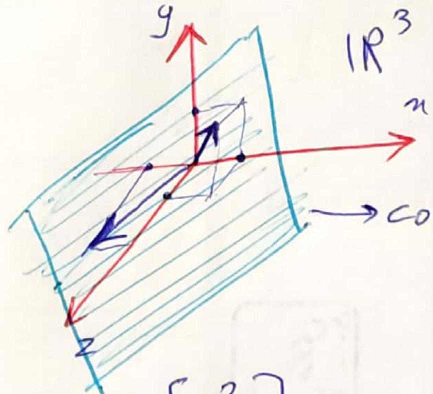
$A \in \mathbb{R}^{m\times n}$ column-space of A فضای ستونی

$A = [\vec{a}_1 \; \vec{a}_2 \; \cdots \; \vec{a}_n]$ $\quad C(A) = \mathrm{span}(\vec{a}_1, \vec{a}_2, \dots, \vec{a}_n)$

range A = column-space $A = \{ A\vec{b} \mid \vec{b} \in \mathbb{R}^m \}$

$$A = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$$

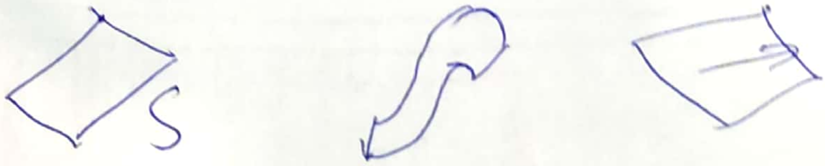
$u_1 \quad u_2$



$u_1 = \begin{bmatrix} 1 \\ 4 \\ 6 \end{bmatrix} \quad u_2 = \begin{bmatrix} 7 \\ 8 \\ -1 \end{bmatrix} \quad \cdots \quad u_n = \begin{bmatrix} -2 \\ 1.5 \\ 4 \end{bmatrix}$

$$[u_1 \; u_2 \; \cdots \; u_n] = \begin{bmatrix} 1 & 7 & \cdots & -2 \\ 4 & 8 & \cdots & 1.5 \\ 6 & -1 & \cdots & 4 \end{bmatrix}$$

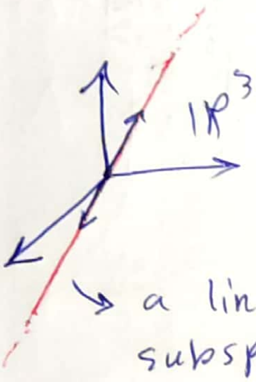
$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 7 & 8 & 9 \\ 3 & 4 & 10 & 11 & 12 \end{bmatrix}$$



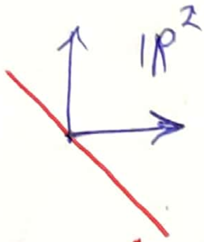
linear subspace S of a vector space V

$S \subseteq V$

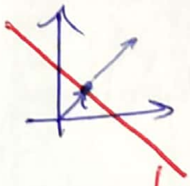
$$\begin{cases} \vec{u} \in S \Rightarrow \alpha \vec{u} \in S \quad \forall \alpha \in \mathbb{R} \\ \vec{u}, \vec{v} \in S \Rightarrow \vec{u} + \vec{v} \in S \end{cases}$$



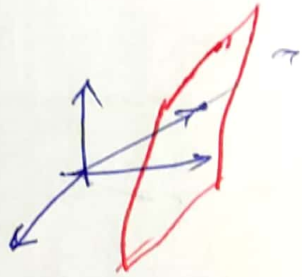
→ a linear subspace of $\mathbb{R}^3$

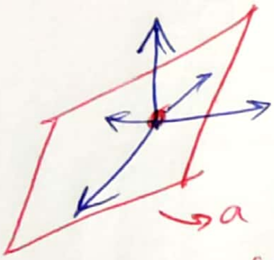


→ a linear subspace $\mathbb{R}^2$



→ not a linear subspace





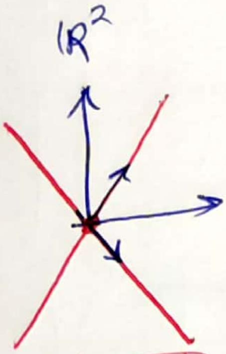
→ a linear subspace of $\mathbb{R}^3$

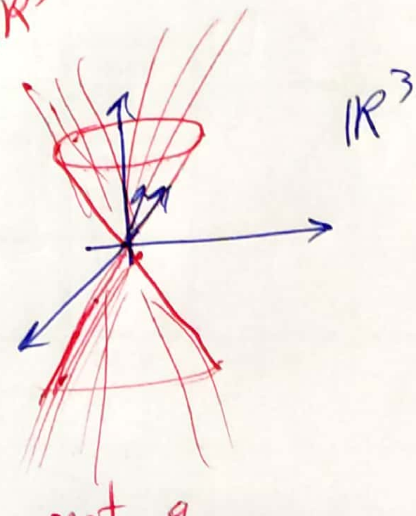
$\vec{u} \in S \Rightarrow \alpha \vec{u} \in S$

$0 \vec{u} \in S$

$\vec{0} \in S$

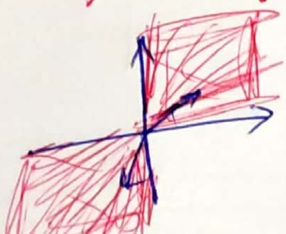
مبدأ مختصات ←





not a linear subspace

$\left.\begin{matrix} \mathbb{R}^3 \\ \vec{0} \end{matrix}\right\}$ linear subspace of $\mathbb{R}^3$



$$X \in \mathbb{R}^m \rightarrow \boxed{f(\cdot)} \rightarrow Y = f(X) \in \mathbb{R}^n$$

$$\underbrace{X_4 \quad X_3 \quad X_2 \quad X_1}_{\text{batch}} \rightarrow \boxed{f(\cdot)} \rightarrow Y_4 \quad Y_3 \quad Y_2 \quad Y_1$$

$X_4 \in \mathbb{R}^m$, $X_2 \in \mathbb{R}^m$, $X_1 \in \mathbb{R}^m$

$f(X_4) \quad f(X_2) \quad f(X_1)$

$$X_1, X_2, X_3, \dots, X_N \in \mathbb{R}^m$$

$$[X_1, X_2, X_3, \dots, X_N] = X \in \mathbb{R}^{m \times N}$$

$$\begin{bmatrix} \vec{X}_1^T \\ \vec{X}_2^T \\ \vdots \\ \vec{X}_N^T \end{bmatrix}$$

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix} \qquad a^T = (1 \quad 2 \quad 3]$$

$$A_{ij} = (A^T)_{ji}$$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix} \quad \begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 6 \\ 7 & 8 & 10 \\ & \vdots & \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 4 & 6 \\ 7 & 8 & 9 \\ 10 & 0 & 2 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & -1 & 7 & 10 \\ 2 & 4 & 8 & 0 \\ 3 & 6 & 9 & 2 \end{bmatrix}$$

$$A \in \mathbb{R}^{m \times n}$$

$$u \in C(A) \Rightarrow \exists b \in \mathbb{R}^n \quad \underset{m\times n}{A}\,\underset{n\times 1}{b} = u$$

$$C(A) = R(A^T)$$

R → row-space

$$C(A^T) = R(A)$$