$$\begin{bmatrix}2&3\\4&5\\6&7\end{bmatrix}+\begin{bmatrix}7&8\\1.5&2\\-1&4\end{bmatrix} \qquad \mathbb{R}^{3\times 2}$$

$$\begin{bmatrix}x_1 & y_1\\ x_2 & y_2\\ \vdots & \\ x_{96} & y_{96}\end{bmatrix} \qquad \cancel{2}\times 96\times 2$$

$$192\left\{\begin{bmatrix}x_1\\ y_1\\ \vdots\\ x_{96}\\ y_{96}\end{bmatrix}\right.$$

$f: X \longrightarrow Y$

دامنه — domain

هم دامنه — codomain

$f(x)=2x+1$ $\quad f:\mathbb{R}\longrightarrow\mathbb{R}$

$f(x)=2x+1$

$x \longmapsto 2x+1$

maps to

$Range(f)=\mathbb{R}$

$$\text{range}(f)=\{f(x)\mid x\in X\}\subseteq Y$$

$f:\mathbb{R}\longrightarrow\mathbb{R} \qquad f(x)=x^2$

$x\longmapsto x^2$

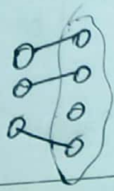
$$\text{range}(f)=\mathbb{R}^+=\{x\in\mathbb{R}\mid x\geqslant 0\}$$

یک به یک

one-to-one

injective

$x, x'\in X$

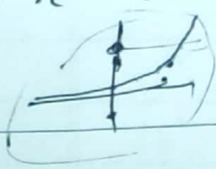


$$f(x)=f(x') \Longrightarrow x=x'$$

$f(x)=\frac{\cancel{2x}\,x}{2}+1$

$f(x)=\cancel{\log}\ e^x$

$f:\mathbb{R}\longrightarrow\mathbb{R}$

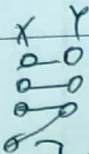


پوشا

onto

surjective

$f: X\longrightarrow Y$



$\forall y\in Y \quad \exists x \quad f(x)=y$

$f(x)=\frac{x}{2}+1$

$f(x)=x^3$

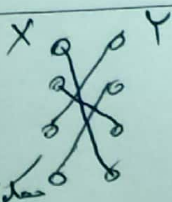
$\mathbb{R}\rightarrow\mathbb{R}$

یک به یک و پوشا

one-to-one & onto

bijective

intertible معکوس پذیر

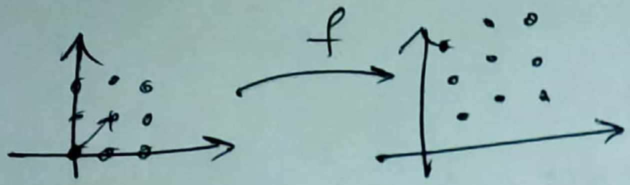


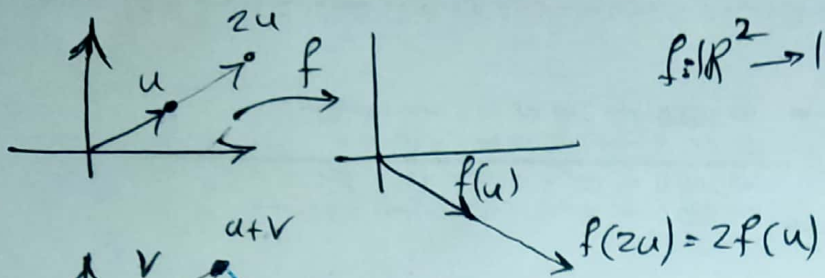
$\exists f^{-1}: Y\rightarrow X$

inverse of f

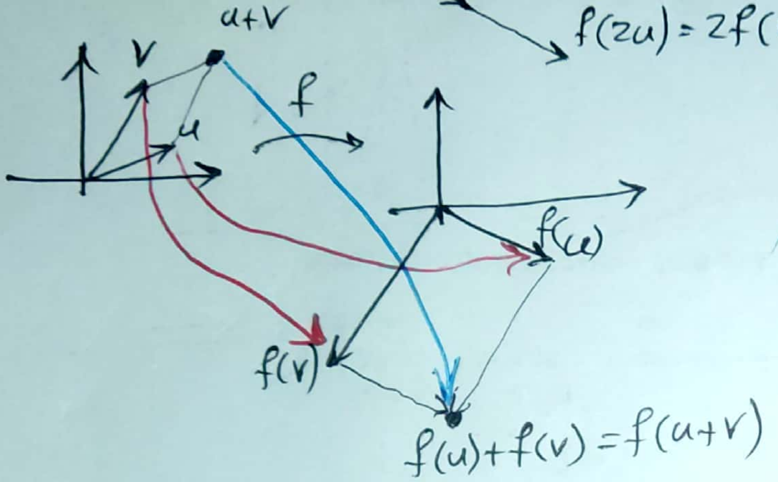
$f^{-1}(f(x))=x$

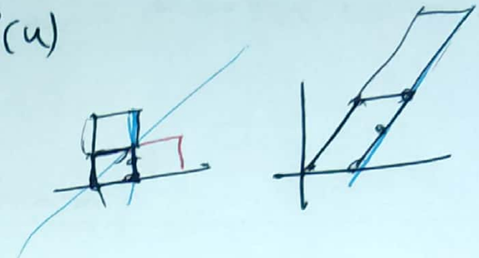
$f(f^{-1}(x))=x$





$f: \mathbb{R}^2 \to \mathbb{R}^2$





$$\boxed{f(\alpha u+\beta v) = \alpha f(u) + \beta f(v)}$$

$\mathbb{R} \to \mathbb{R}$

$f(x) = 4x$

خطی

$$f(\alpha x+\beta y) = 4(\alpha x+\beta y)$$
$$= \alpha 4x + \beta 4y$$
$$= \alpha f(x) + \beta f(y)$$

$f(x) = 4x+1$

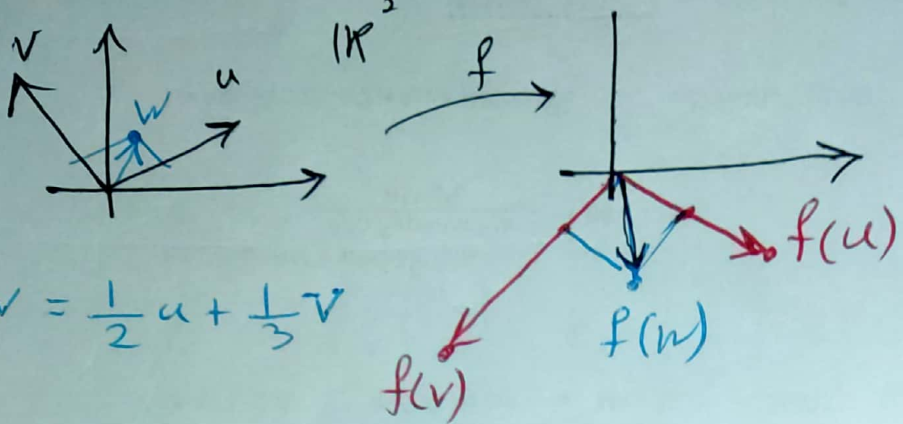
خطی نیست

$$f(2x) = 4(2x)+1 = 8x+1$$
$$\neq$$
$$2f(x) = 8x+2$$

Assume that we do not know the formulae for f

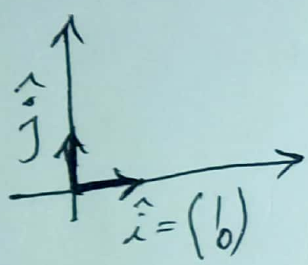
we know $\begin{cases} f: \text{linear} \\ \text{we know } f(v_1), f(v_2), \dots, f(v_n) \text{ for the basis } v_1, v_2, \dots, v_n \end{cases}$

f: linear ($\mathbb{R}^2 \to \mathbb{R}^2$) / u, v basis for $\mathbb{R}^2$



$$w = \frac{1}{2}u + \frac{1}{3}v$$

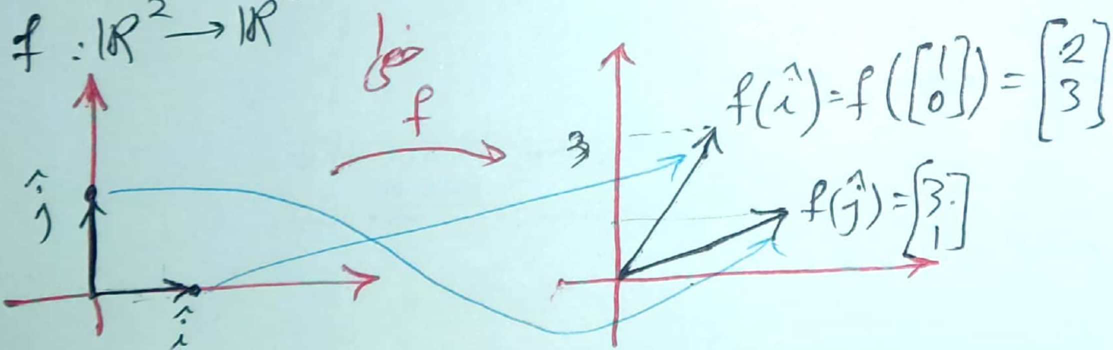
$$f(w) = f\left(\frac{1}{2}u + \frac{1}{3}v\right) = \frac{1}{2}f(u) + \frac{1}{3}f(v)$$



$\mathbb{R}^2$ $\quad \hat{i} = \begin{bmatrix}1\\0\end{bmatrix} \quad \hat{j} = \begin{bmatrix}0\\1\end{bmatrix}$ standard unit basis vector

$$\begin{pmatrix}3\\-2\end{pmatrix} = 3\begin{bmatrix}1\\0\end{bmatrix} + (-2)\begin{bmatrix}0\\1\end{bmatrix} = 3\hat{i} - 2\hat{j}$$

$f : \mathbb{R}^2 \to \mathbb{R}^2$



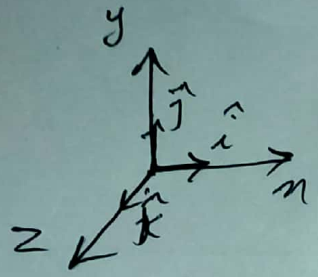
$$f\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = f\left(x\begin{bmatrix}1\\0\end{bmatrix} + y\begin{bmatrix}0\\1\end{bmatrix}\right) = f(x\hat{i} + y\hat{j})$$

$$= x f(\hat{i}) + y f(\hat{j})$$

$$= x\begin{bmatrix}2\\3\end{bmatrix} + y\begin{bmatrix}3\\1\end{bmatrix} = \begin{bmatrix}2x + 3y\\3x + y\end{bmatrix}$$

$$f\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}2x+3y\\3x+y\end{bmatrix} \qquad f\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}2 & 3\\3 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$f(\vec{u}) = A\vec{u}$$



$$e_1 \quad \hat{i} = \begin{bmatrix}1\\0\\0\end{bmatrix} \qquad e_2 \quad \hat{j} = \begin{bmatrix}0\\1\\0\end{bmatrix} \qquad e_3 \quad \hat{k} = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

standard unit vectors for $\mathbb{R}^3$

$\mathbb{R}^m$

$$e_1 = \begin{bmatrix}1\\0\\0\\ \vdots\\0\end{bmatrix} \qquad e_2 = \begin{bmatrix}0\\1\\0\\0\\ \vdots\\0\end{bmatrix} \qquad e_3 = \left.\begin{bmatrix}0\\0\\1\\0\\ \vdots\\0\end{bmatrix}\right\} m \qquad e_m = \begin{bmatrix}0\\0\\ \vdots\\0\\1\end{bmatrix}$$

$f: \mathbb{R}^m \longrightarrow \mathbb{R}^n$ linear $\qquad e_1, e_2, \dots, e_m$ s.b.v. for $\mathbb{R}^m$

$\vec{u} \in \mathbb{R}^m \qquad \exists A \in \mathbb{R}^{n \times m} : f(\vec{u}) = A\vec{u}$

$$A = \begin{bmatrix} f(e_1) & f(e_2) & \cdots & f(e_m) \end{bmatrix}$$

$n \times m$

$f: \mathbb{R}^m \longrightarrow \mathbb{R}^n \qquad f(\vec{x}) = A\vec{x} \qquad A \Rightarrow n \times m$

$$f(\alpha\vec{x} + \beta\vec{y}) = A(\alpha\vec{x} + \beta\vec{y}) = \alpha A\vec{x} + \beta A\vec{y}$$
$$= \alpha f(\vec{x}) + \beta f(\vec{y})$$
