$$\begin{array}{c|c} & n \\ \hline m & \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix} \end{array}$$

$c_i \in \mathbb{R}^m$

column-rank $\leq n$

column-rank $\leq m$

column-rank $\leq \min(m,n)$

~~full-rank matrix~~

full-column rank : col-rank $= n$

full-row rank : row-rank $= m$

```
L = []
for i in 1..n
    if c_i ∉ span(c_1 — c_{i-1})
        L.add(c_i)
```

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 7 \end{bmatrix}$$

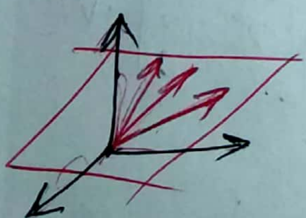
$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 3 & 5 & 1 \\ 3 & 4 & 7 & 2 \end{bmatrix} = \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 4 \end{bmatrix}} \underset{2\times 4}{\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}}$$

row-rank = col-rank

$$[\vec{a}_1\ \vec{a}_2]\begin{bmatrix} x \\ y \end{bmatrix} = x\vec{a}_1 + y\vec{a}_2$$

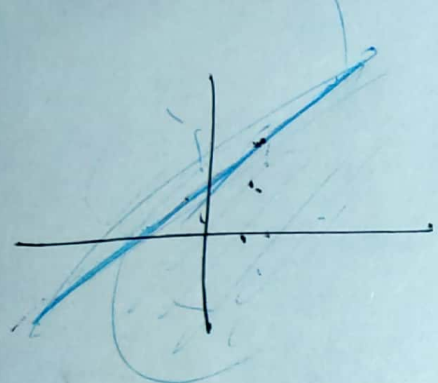
$$[x\ y]\begin{bmatrix} \vec{a}_1^T \\ \vec{a}_2^T \end{bmatrix} = x\vec{a}_1^T + y\vec{a}_2^T$$

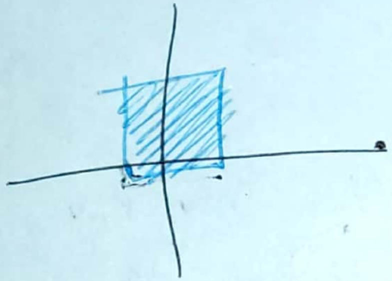
$$[\quad] = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 3 & 4 \\ 2 & 4 \\ 0 & 0 \end{bmatrix} [\quad]$$



$V_1 \quad V_2 \quad V_3$

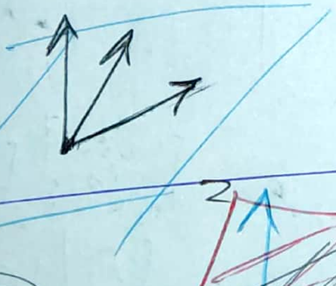
$\alpha_1 u_1 + \alpha_2 u_2 \quad \beta_1 u_1 + \beta_2 u_2 \quad \gamma_1 u_1 + \gamma_2 u_2$

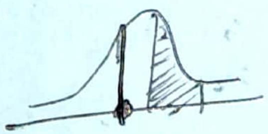




$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
A = np.random.randn(3,3)
```

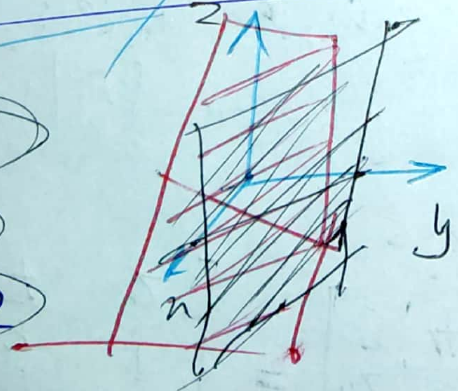




$x + y = 4$

$x - y = 3$

$x + y + z = 2$



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix} \Rightarrow x \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + y \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}$$

LA8 III

$$A\vec{x} = \vec{b}$$

$m \times n$ $\mathbb{R}^n$ $\mathbb{R}^m$

$$\begin{bmatrix} \vec{a}_1 & \vec{a}_2 & \cdots & \vec{a}_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \vec{b}$$

$\mathbb{R}^m$ $\mathbb{R}^m$ $\mathbb{R}^m$ $\mathbb{R}^m$

$$x_1\vec{a}_1 + x_2\vec{a}_2 + \cdots + x_n\vec{a}_n = \vec{b}$$

$\vec{b} \in \text{span}(\vec{a}_1 - \vec{a}_n)$

$\vec{b} \in C(A)$

$\vec{b} \notin C(A)$ $A\vec{x} = \vec{b}$ has no solution!

$\vec{b} \in C(A)$ $A\vec{x} = \vec{b}$ has at least one solution!

A { square ($A \in \mathbb{R}^{n\times n}$)
has full rank ($\text{Rank}(A) = n$)

$A \in$ ~~rank(A) < n~~

$A \in \mathbb{R}^{n\times n}$ { $\text{rank}(A) < n$ singular
$\text{rank}(A) = n$ non-singular

A non-singular