

# Homework 1

The following proves that the inverse of a bijective linear map is also linear. Your task is to write it under LaTeX.

Let  $f: U \rightarrow V$  be a ~~linear~~ bijective linear map, and  $f^{-1}: V \rightarrow U$  be its inverse. Consider two points  $y_1, y_2 \in V$ . We want to show that

$$f^{-1}(\alpha y_1 + \beta y_2) = \alpha f^{-1}(y_1) + \beta f^{-1}(y_2), \quad (1)$$

for any pair of real numbers  $\alpha, \beta$ .

Let  $x_1 = f^{-1}(y_1)$  and  $x_2 = f^{-1}(y_2)$ . We have

$$\begin{aligned} f^{-1}(\alpha y_1 + \beta y_2) &= f^{-1}(\alpha f(x_1) + \beta f(x_2)) \\ &= f^{-1}(f(\alpha x_1 + \beta x_2)) \\ &= \alpha x_1 + \beta x_2 \\ &= \alpha f^{-1}(y_1) + \beta f^{-1}(y_2), \end{aligned} \quad (2)$$

where the ~~second~~ second line of (2) ~~comes~~ follows from the fact that  $f$  is linear. This prove that  $f^{-1}$  is also linear.  $\square$

You have to follow the following rules:

- Your document must contain a title, a date, and your name as the author. Also, write your affiliation as "K. N. Toosi University of Technology".
- Write all scalars with regular italic letters ( $\$a\$, \$b\$, \$\alpha\$, \$\beta\$$ ).
- Write all vectors with bold letters ( $\mathbf{\$x\$}$ ).

- Write the spaces  $U$ , and  $V$  using calligraphic letters ( $\mathcal{U}$ ).
- You can define macros to make your life easier (for instance `\newcommand{\tU}{\mathcal{U}}`)
- Use the `equation`, `align`, or similar environments for equations (1) and (2). Notice that equation (2) spans multiple lines. The equal signs must be well aligned (look at the `align` environment).
- Write the above in a **proof** environment. Also, state what you are trying to prove in a **theorem** or **proposition** environment.
  - look here for more details  
[https://www.overleaf.com/learn/latex/Theorems\\_and\\_proofs](https://www.overleaf.com/learn/latex/Theorems_and_proofs)
- Submit two files name **homework1.pdf** file and a **homework1.tex** file.