Mathematics for AI

Lecture 1

Introduction and Logistics

Mathematics for Artificial Intelligence

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- Graduate Course
- 3 credits, 32 sessions
- Saturday, Wednesday, 15:30-17:30
- Instructor: Behrooz Nasihatkon
- Email: nasihatkon@kntu.ac.ir
- Office: Room 402

Exam Dates

- Midterm Exam: Thursday, 17 Aban, 9:00-12:00
- Final Exam: look at the schedule

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Ask Questions

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Special Needs

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Eating in class

How to get help

How to give feedback?

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Anonymous form: https://goo.gl/zPxBAS

Join the Telegram channel

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https://t.me/math4AI4031

My Telegram Channel

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t.me/behrooznasihatkon

دانشجویان ارشد ورودی ۱۴۰۳

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https://t.me/cemaster1403

Why this course?

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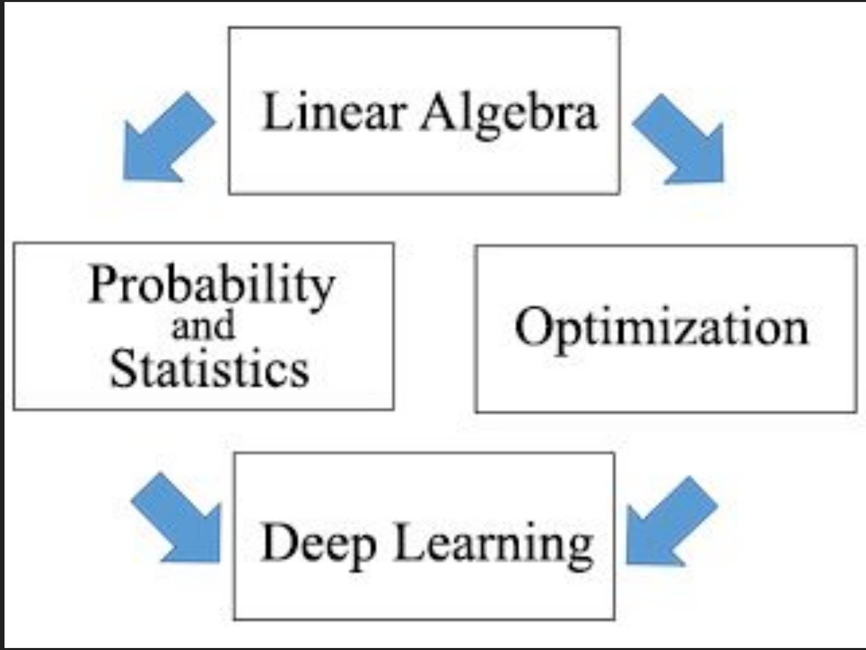
-

Why this course?

Why this course?



https://ocw.mit.edu/courses/18-065-matrix-methods-in-data-analysis-signal-processing-and-machine-learning-spring-2018/

Evaluation

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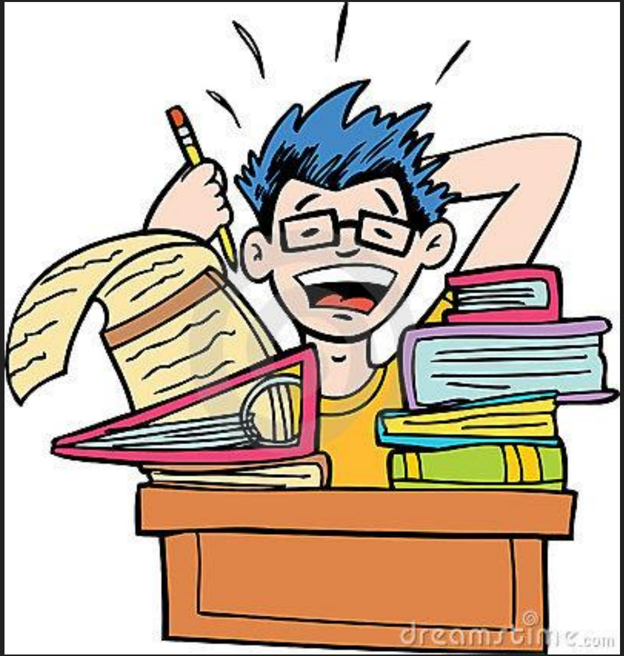
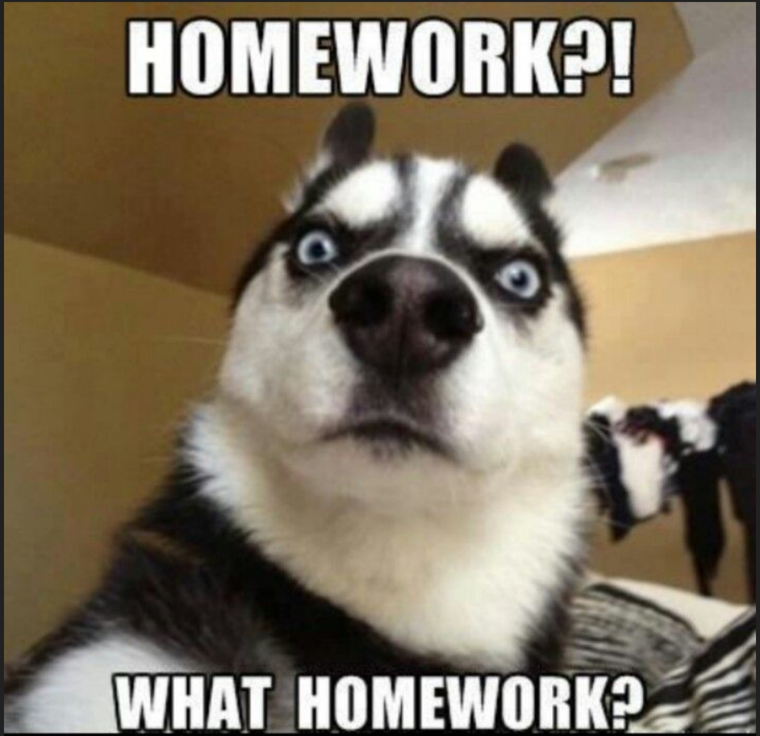
- Lab Sessions (~2.5 points)
- Homework (~2.5 points)
- Projects (~3 points)
- Midterm Exam (~5 points)
- Final Exam (~7 points)



Homework/Projects/Labs



What is considered cheating?

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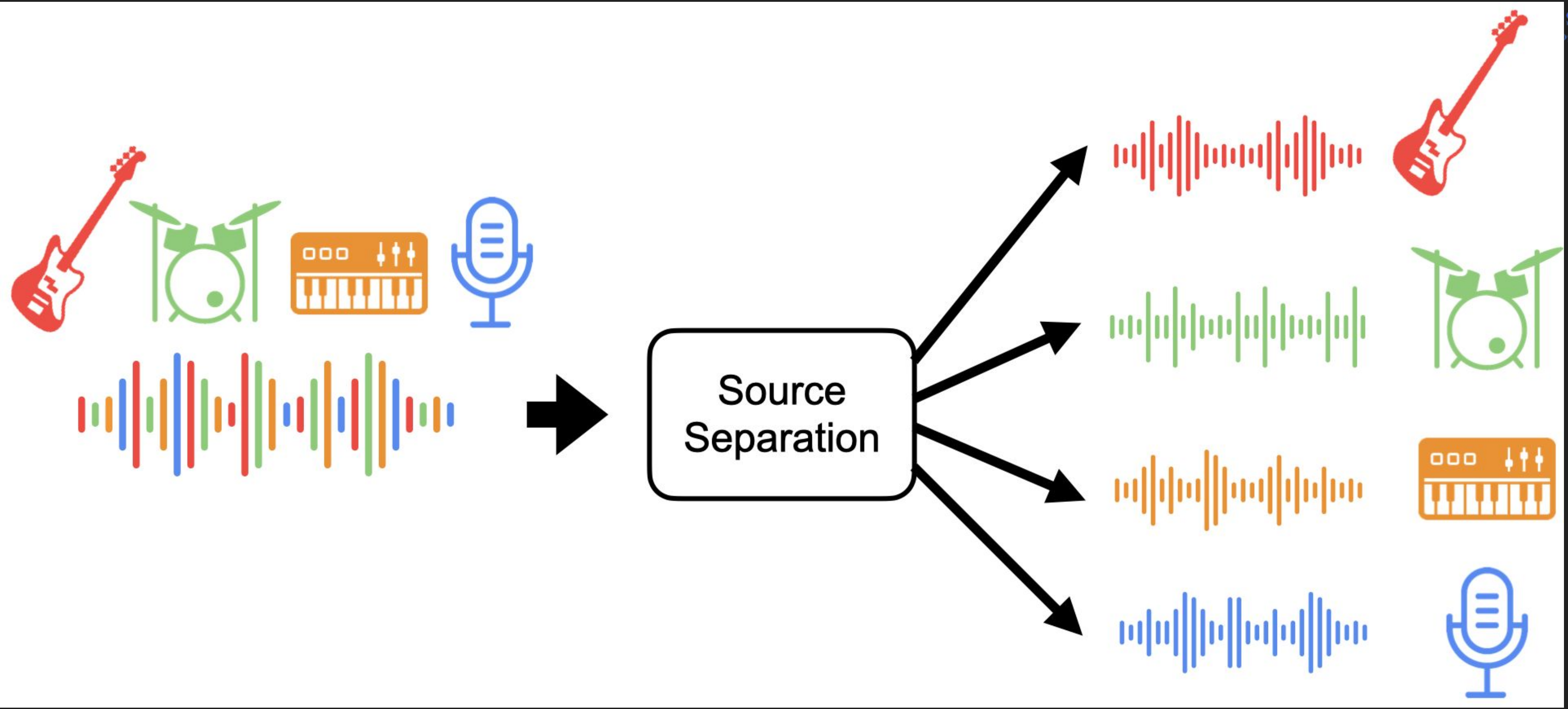
Programming Languages

- Matlab
- C++
- Python
- R
- Julia
- ...

Example: Source Separation



https://source-separation.github.io/tutorial/landing.html

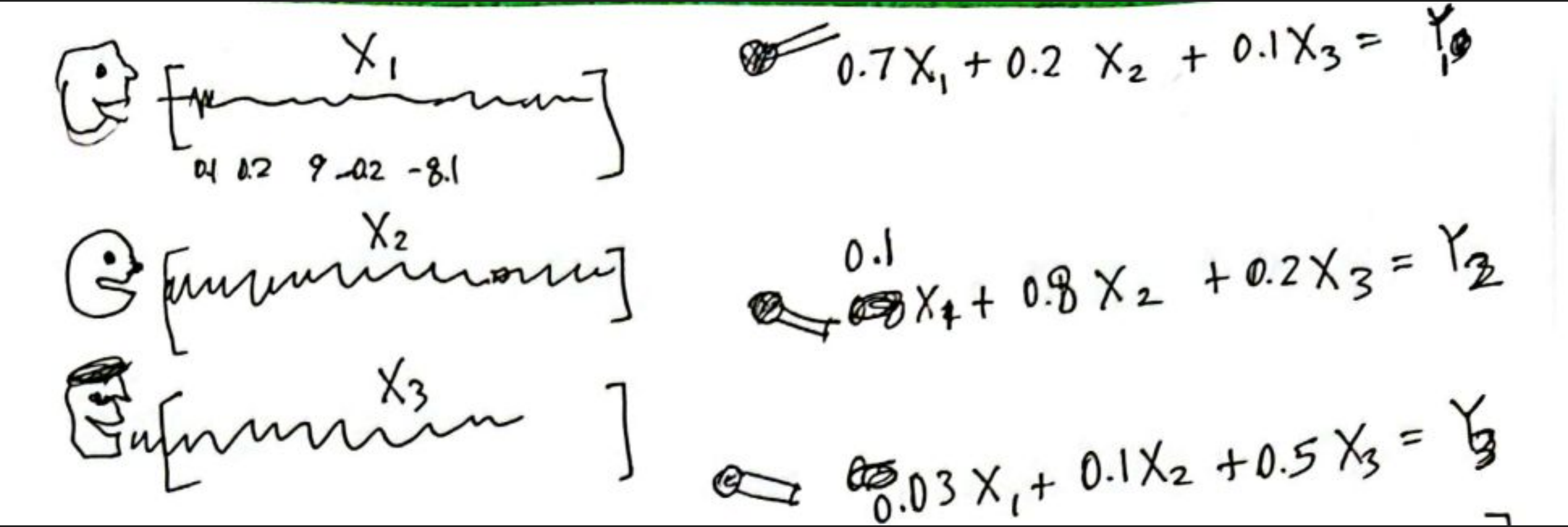
Source Separation

https://youtu.be/n7y2rLAnd5I

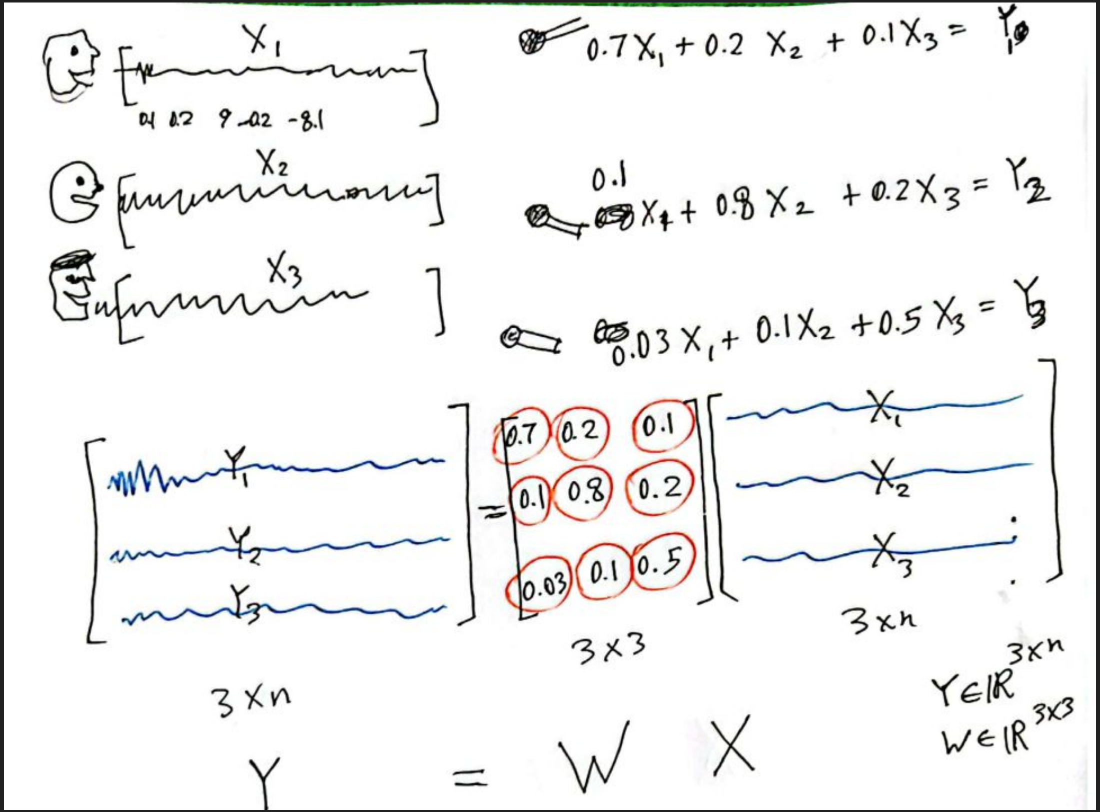
Source Separation

https://www.youtube.com/watch?v=tkkm6zVUDXo

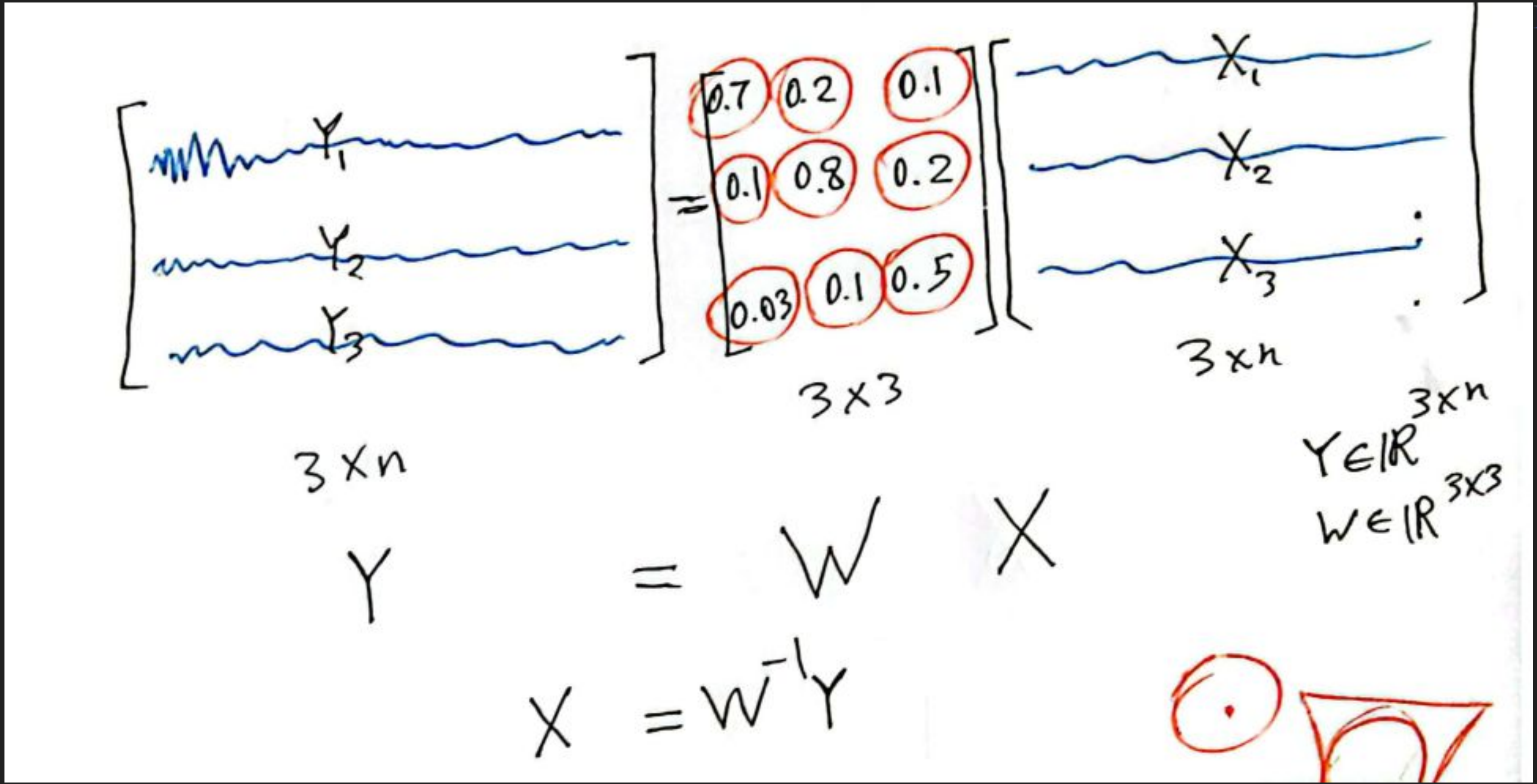
Example: Source Separation



Example: Source Separation



Example: Source Separation



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Example: Source Separation

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$$\underset{3\times n}{Y} = \underset{3\times 3}{W} \; \underset{3\times n}{X}$$

decompose Y into $W \in \mathbb{R}^{3\times 3}$ by $X \in \mathbb{R}^{3\times n}$

$$Y = I\,Y$$

$$Y = WX = \underbrace{WA}_{W'}\,\underbrace{A^{-1}X}_{X'} = W'X'$$

Example: Source Separation

- Linear Algebra:

  Y = A X   => X = A^{-1} Y
- Probability and Statistics:

  Mutual Information

$$I(X_1;X_2;X_3) = \int_{x_1}\int_{x_2}\int_{x_3} p(x_1,x_2,x_3)\log\left(\frac{p(x_1,x_2,x_3)}{p(x_1)p(x_2)p(x_3)}\right)dx_1dx_2dx_3$$

- Optimization

$$\max_{A,X}\ I(X_1;X_2;X_3) \quad \text{subject to} \quad Y = AX$$

Machine Learning

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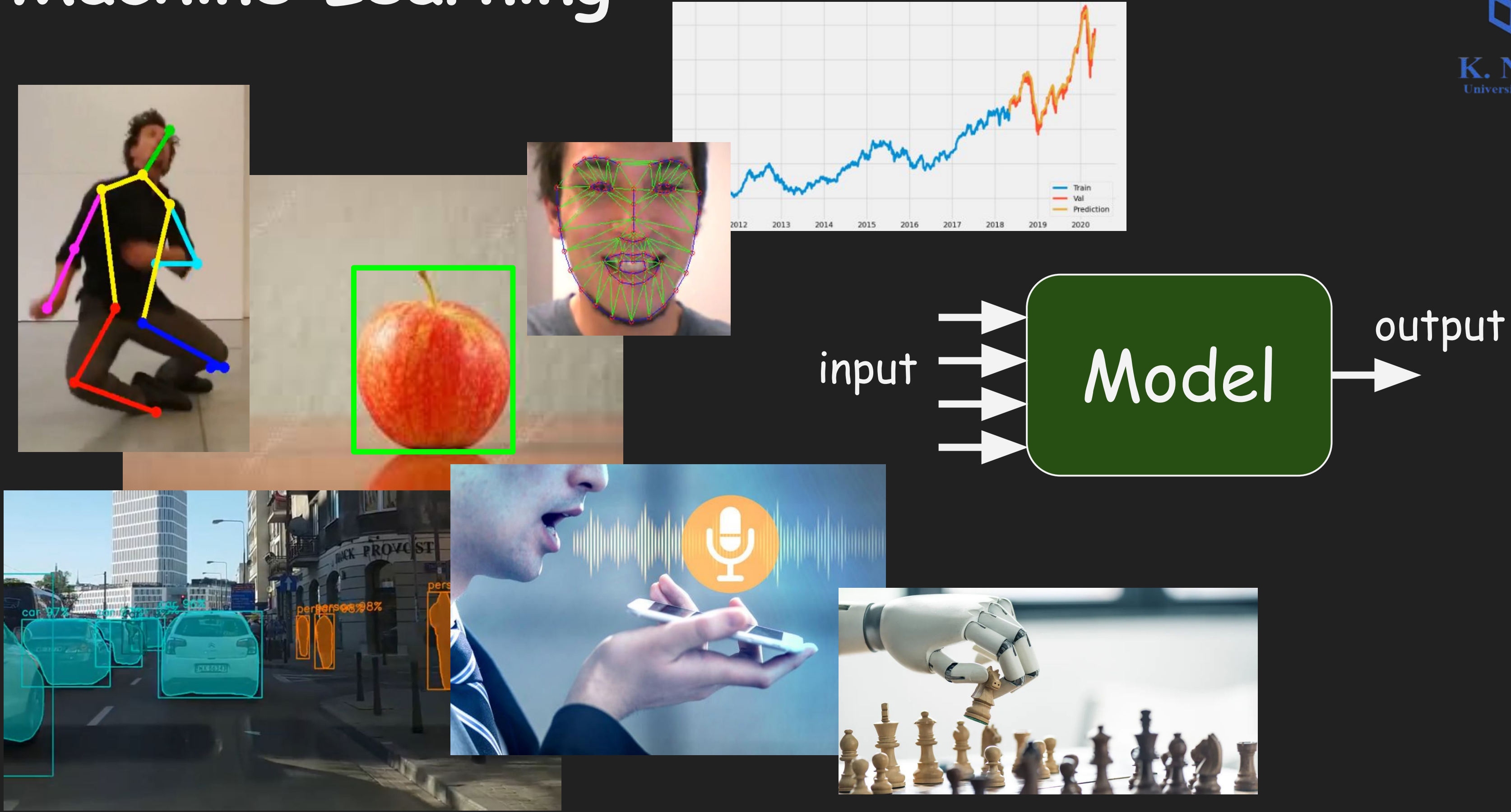
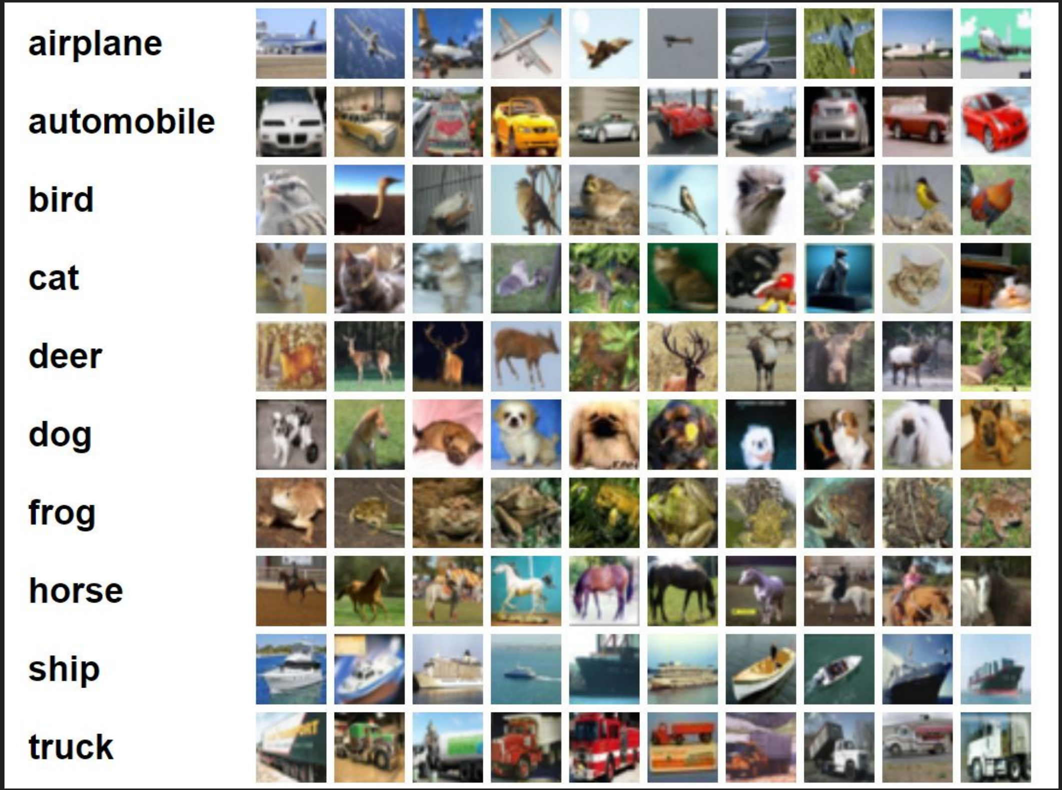


Learning from data

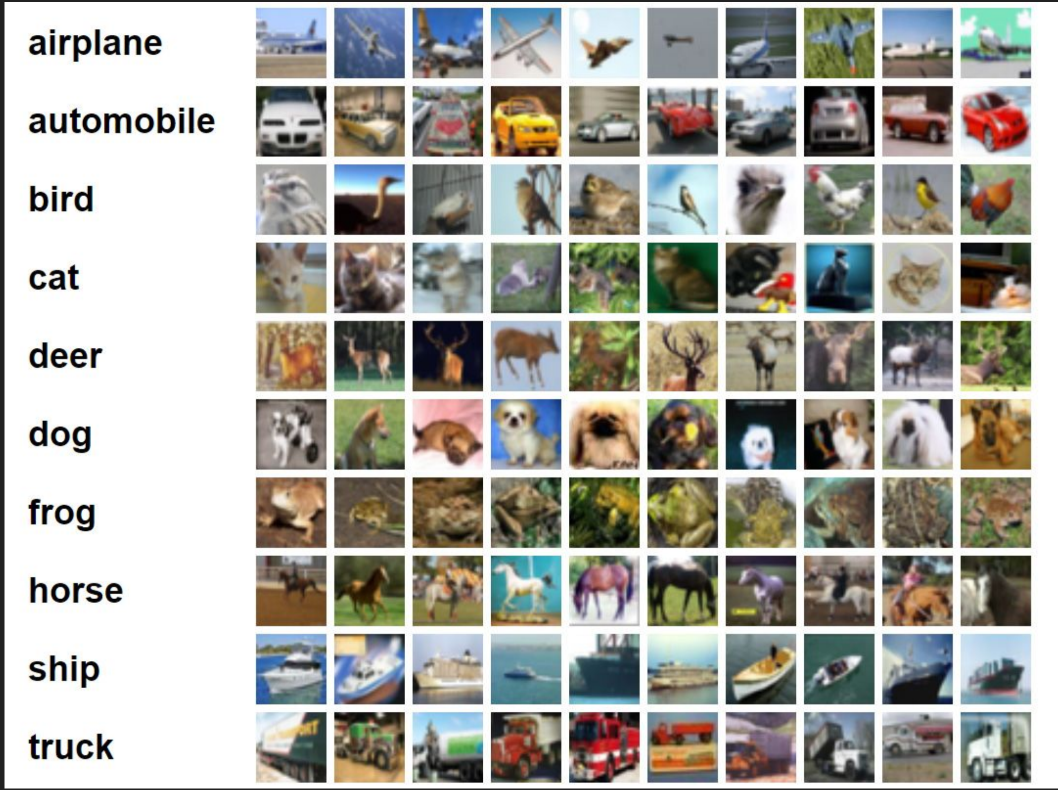
https://www.analyticsvidhya.com/blog/2018/03/comprehensive-collection-deep-learning-datasets/

Supervised Learning



http://seansoleyman.com/effect-of-dataset-size-on-image-classification-accuracy/

Supervised Learning

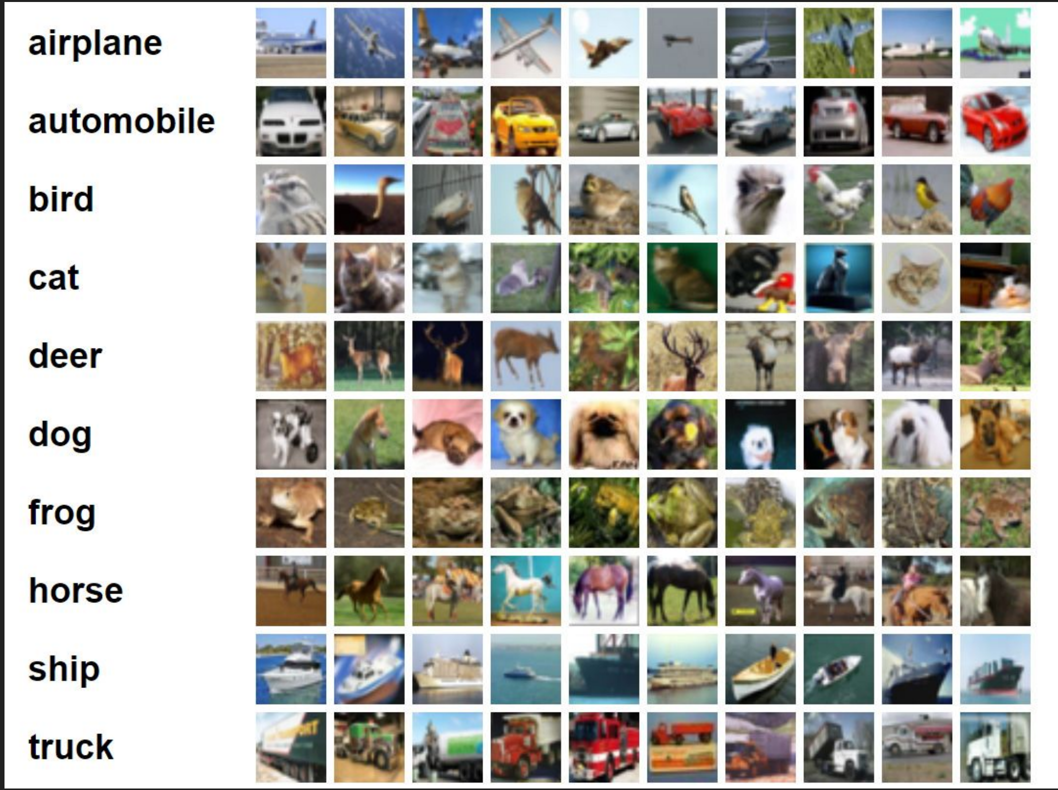


http://seansoleyman.com/effect-of-dataset-size-on-image-classification-accuracy/

Training data:

X_1, y_1
X_2, y_2
X_3, y_3
$\vdots$
X_n, y_n

Supervised Learning



http://seansoleyman.com/effect-of-dataset-size-on-image-classification-accuracy/

Training data:

	Apple
	Apple
	Orange
⋮	
	Orange

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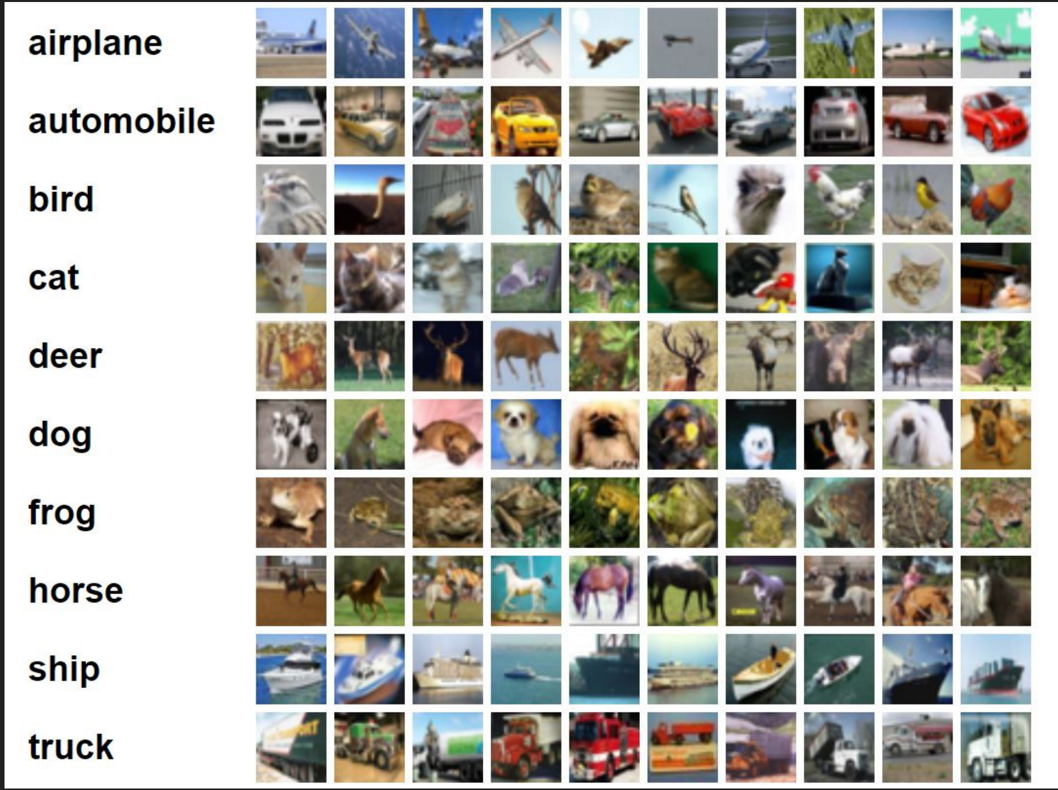
Supervised Learning



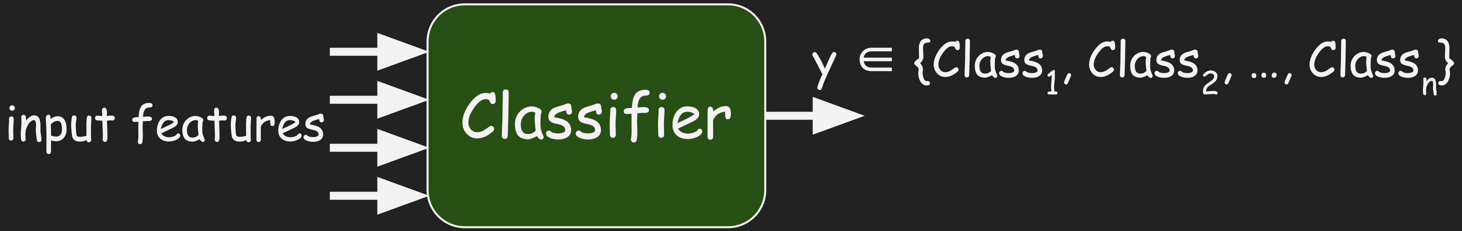
http://seansoleyman.com/effect-of-dataset-size-on-image-classification-accuracy/

Training data:

	0
	0
	1
⋮	
	1

Supervised Learning

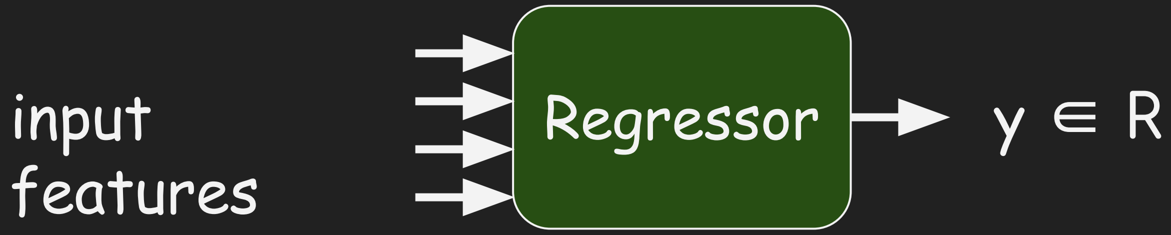
Classification



Classification

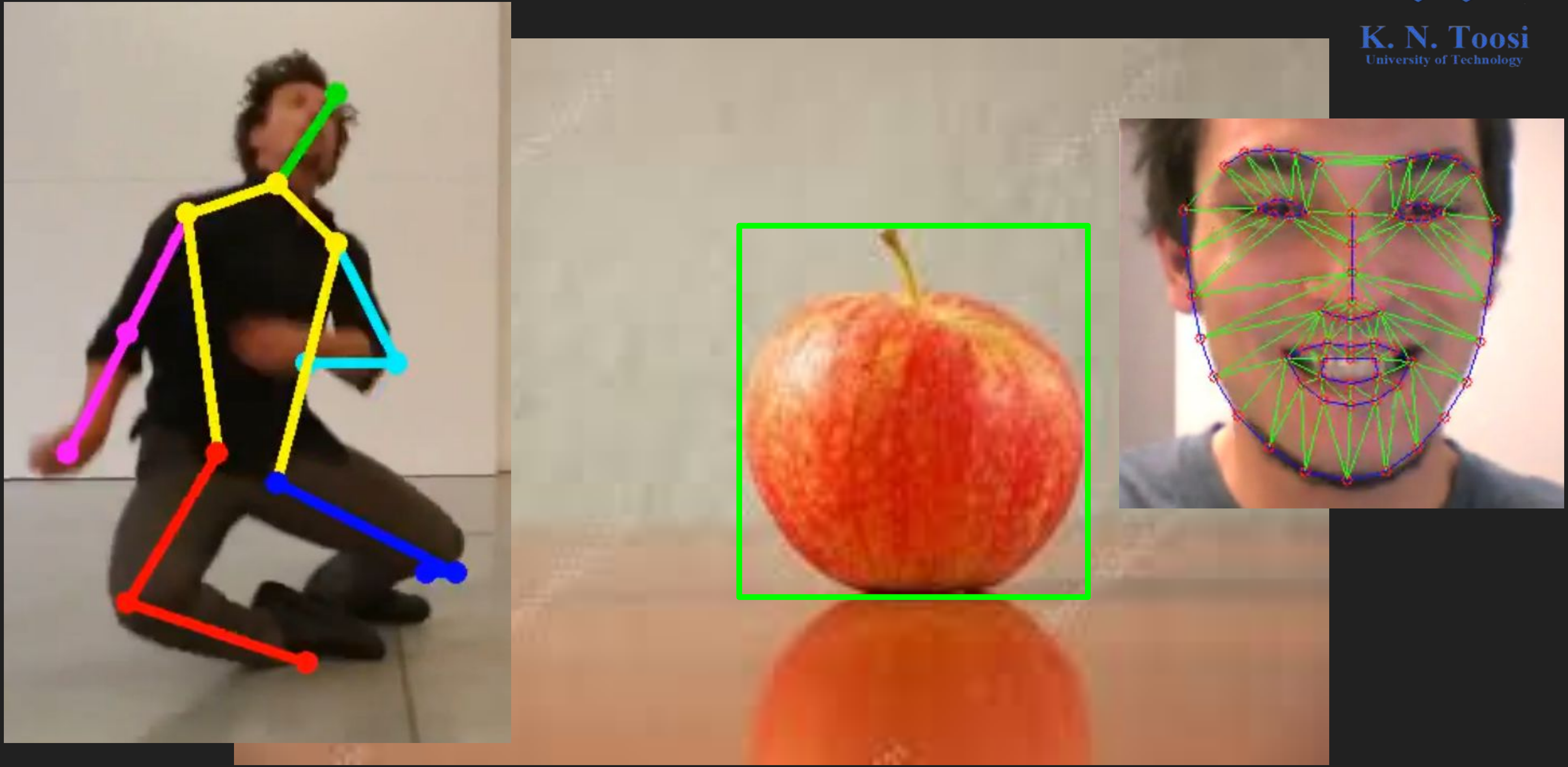
Classification

Regression



Regression

Regression



Learnable Models

Learnable Models: Example

Learnable Models: Example

Learnable Models: Input-output map

$$y = f(x)$$

$$f: R^m \rightarrow R^n$$

Learnable Models: Input-output map

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$$y = f(x,\theta)$$

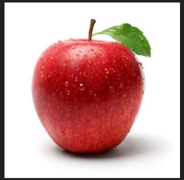
$$f: R^m \rightarrow R^n$$

Learnable Models: Input-output map

$$y = f(x, \theta) \qquad \theta \in R^k$$

$$f: R^m \times R^k \rightarrow R^n$$

Learnable Models: Example



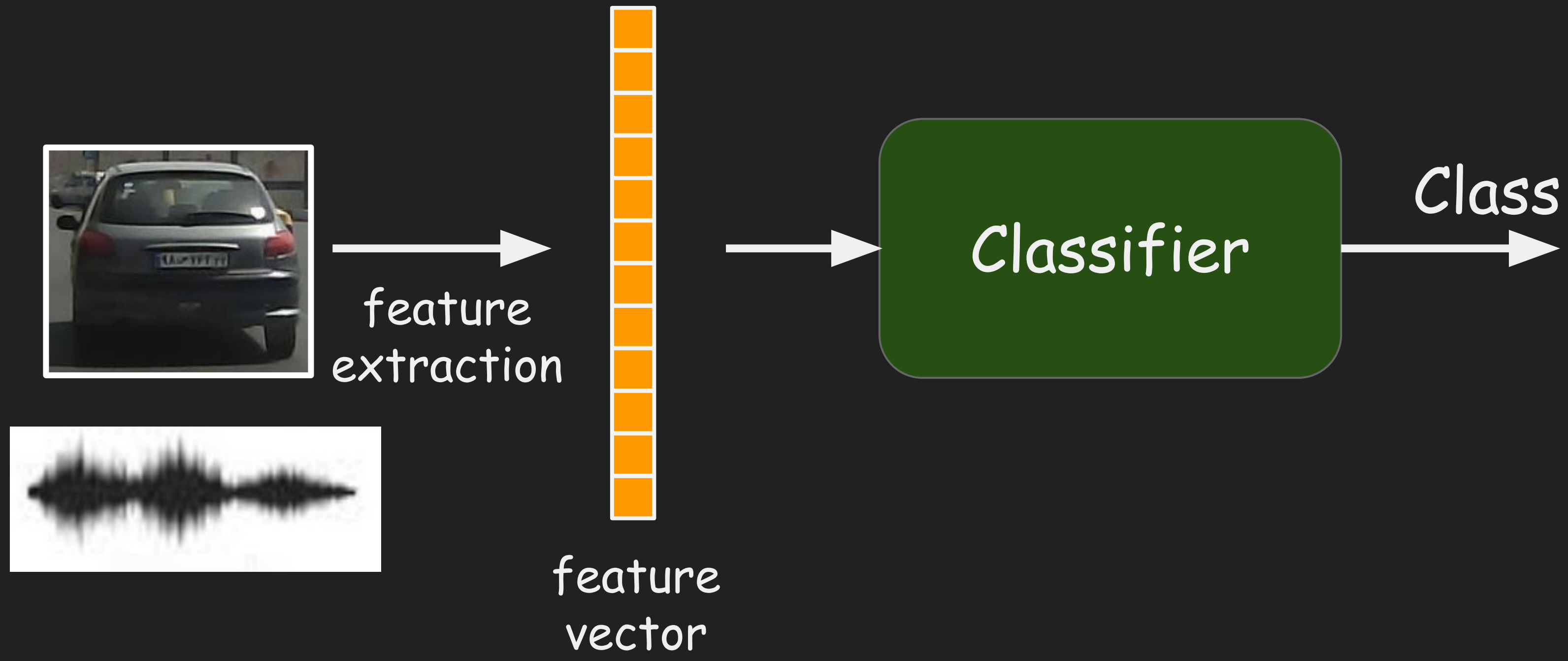
I

x = features(I) → f → y = 0

$$y = f(x, \theta)$$

$$f: R^m \times R^k \rightarrow R^n$$

Features



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Feature space

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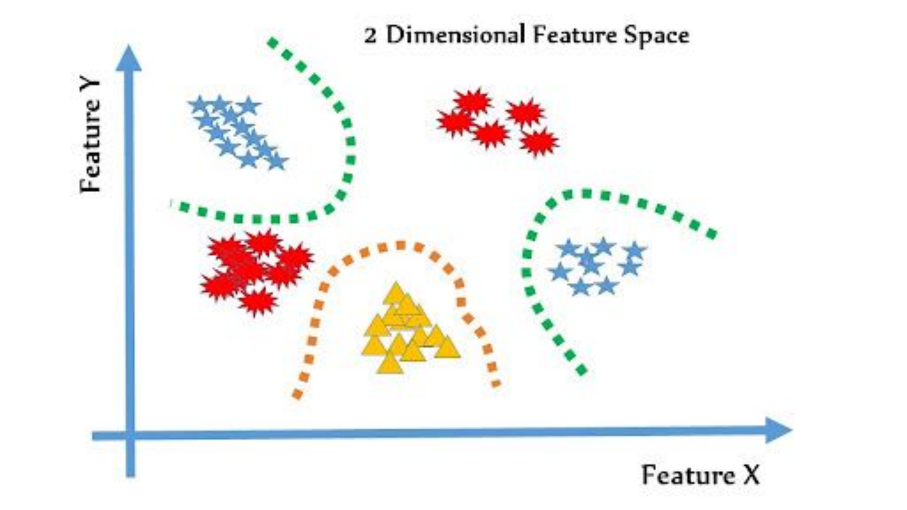
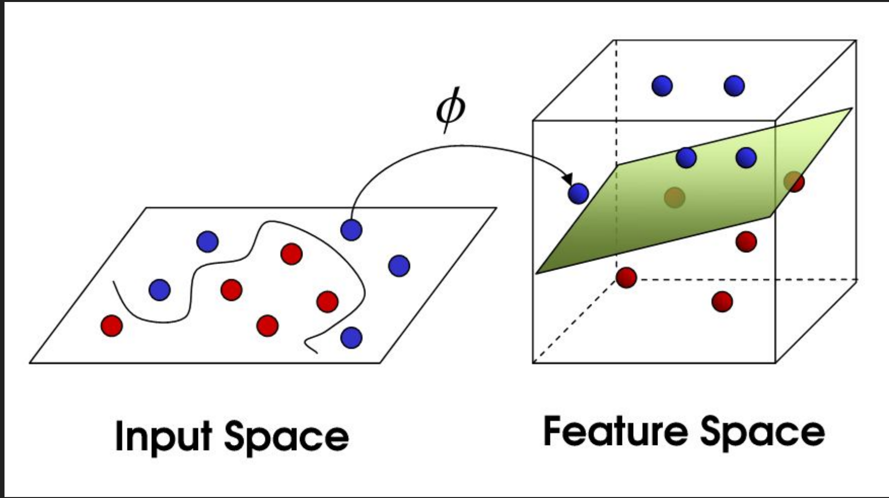
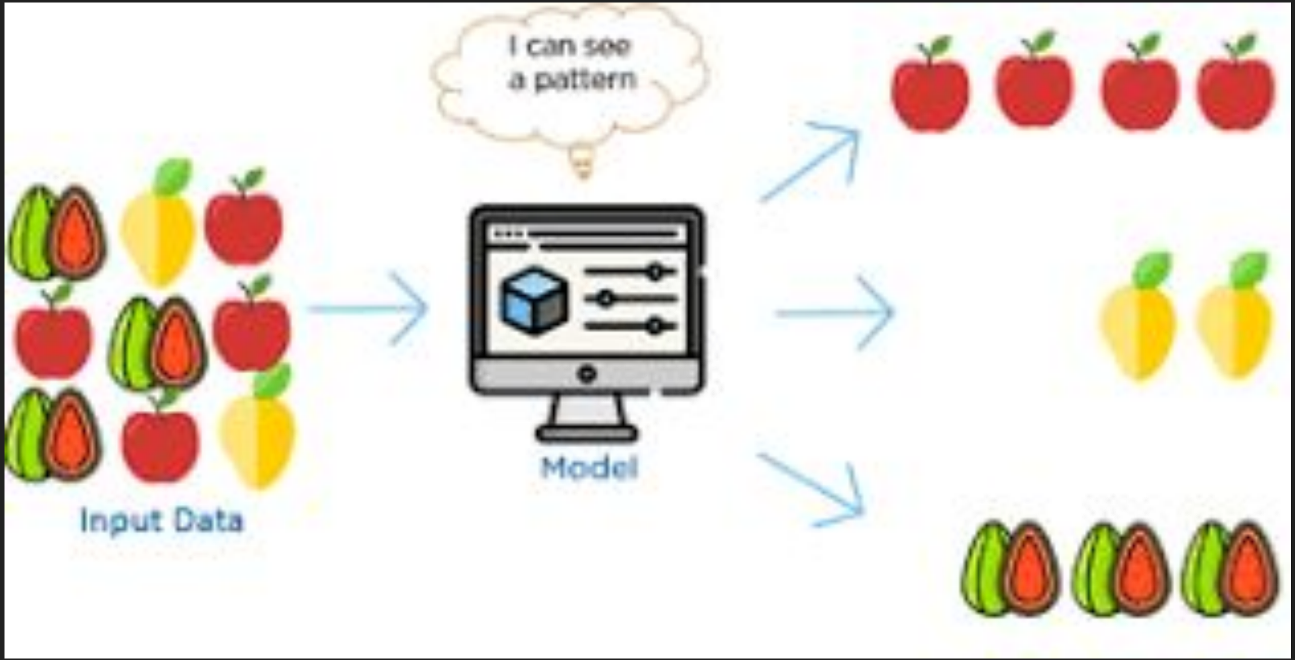


https://www.petersincak.com/news/why-i-do-not-believe-in-error-backpropagation/



https://towardsdatascience.com/the-kernel-trick-c98cdbcaeb3f

Unsupervised Learning



https://towardsdatascience.com/machine-learning-types-and-algorithms-d8b79545a6ec

Neural Networks

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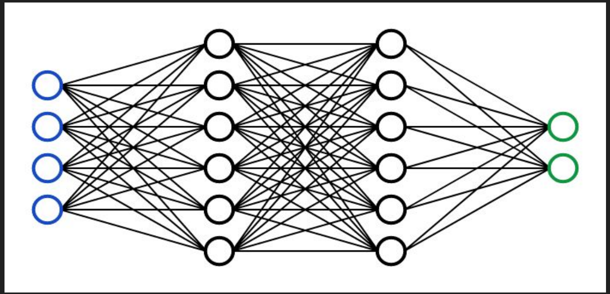
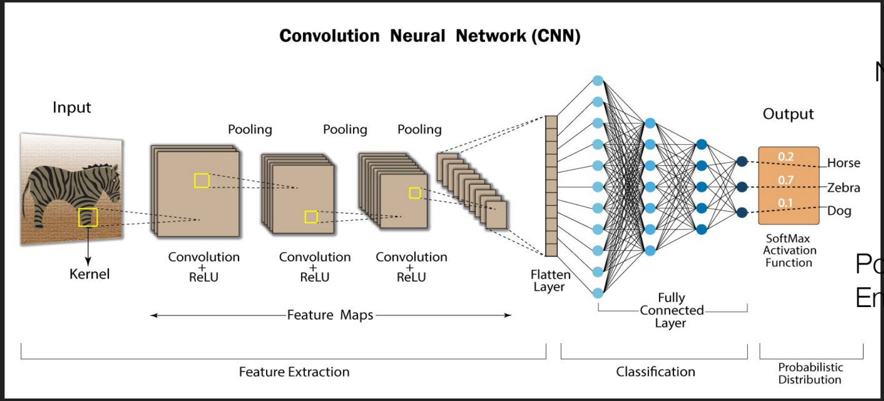
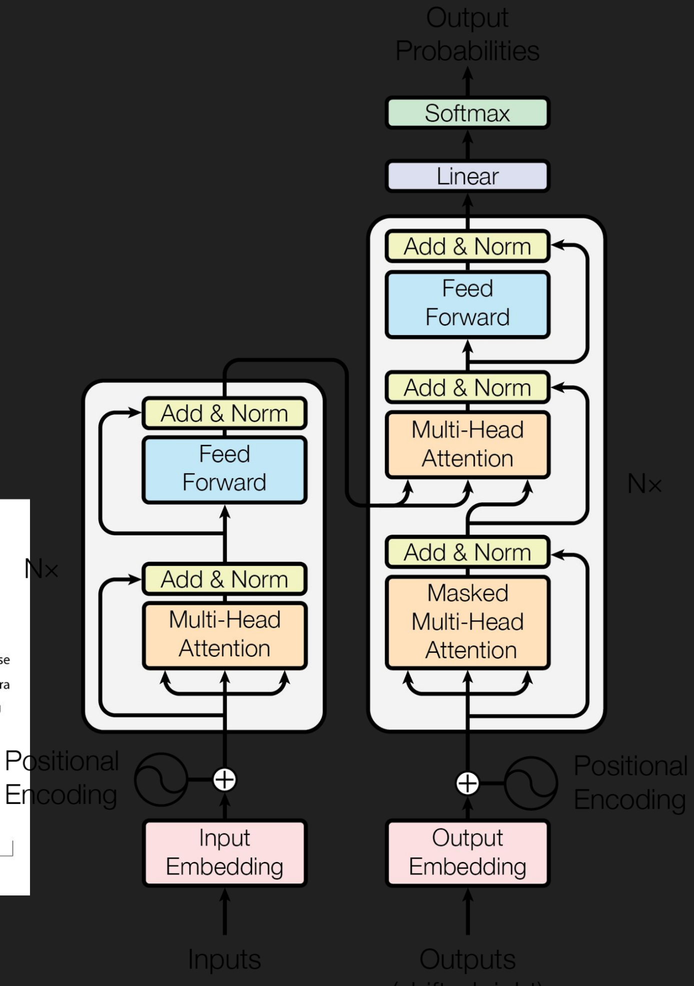






Natural Language Processing

ChatGPT

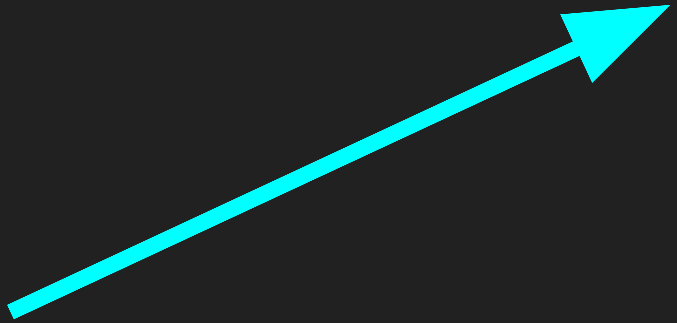
What are the applications of mathematics in AI?

Mathematics forms the foundation of many key concepts in AI, providing the theoretical frameworks, algorithms, and models that drive AI systems. Here are the main areas of mathematics and their applications in AI:

1. Linear Algebra

- **Applications**: Linear algebra is essential for representing and manipulating data in AI models. Vectors, matrices, and tensors are widely used to represent inputs, outputs, and transformations in algorithms like neural networks.
- **Specific Use Cases**:

Vectors



Learning from data

- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i

Learning from data

- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
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Cost function

Learning from data

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- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i
  - cost function:

$$C(\theta) = \Sigma_{i=1..n}\, d(\, f(\theta, x_i), y_i\,)$$

Learning from data

- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i
  - cost function:

$$C(\theta) = \Sigma_{i=1..n}\, d(\, f(\theta, x_i), y_i\,)$$

data output

Learning from data

- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i
  - cost function:

$$C(\theta) = \Sigma_{i=1..n}\, d(\, f(\theta, x_i), y_i\,)$$

model output given x_i

Learning from data

- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i
  - cost function:

$$C(\theta) = \Sigma_{i=1..n}\, d(\, f(\theta, x_i), y_i\,)$$

distance

Learning from data

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- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i
  - cost function:

$$C(\theta) = \Sigma_{i=1..n} \| f(\theta, x_i) - y_i \|^2$$

distance

Learning from data

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- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i
  - cost function:

$$C(\theta) = \Sigma_{i=1..n}\, d(\, f(\theta, x_i), y_i\,)$$

Learning from data

- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i
  - cost function:

$$C(\theta) = \Sigma_{i=1..n}\, d(\, f(\theta, x_i), y_i \,)$$

choose θ such that $C(\theta)$ is small

Learning from data

- Training data $(x_1, y_1), (x_2, y_2), \ldots, (x_N, y_N)$
  - choose θ such that $f(\theta, x_i)$ is close to y_i
  - cost function:

$$C(\theta) = \Sigma_{i=1..n}\, d(\, f(\theta, x_i), y_i\,)$$

$$\theta^* = \text{argmin}_\theta\, C(\theta)$$

Learning from data

What if we need to model to also give us a measure of uncertainty?

Learning from data

How to choose the cost function?

$$C(\theta) = \Sigma_{i=1..n} \| f(\theta, x_i) - y_i \|^2$$

why not $C(\theta) = \Sigma_{i=1..n} \| f(\theta, x_i) - y_i \|$?

Learning from data

Sometimes we need a **probabilistic framework** to choose the right model/cost function.

Course Overview

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- Part I: Linear Algebra and Matrix Analysis
- Part II: Multivariate Calculus
- Part III: Optimization
- Part IV: Probability and Statistics