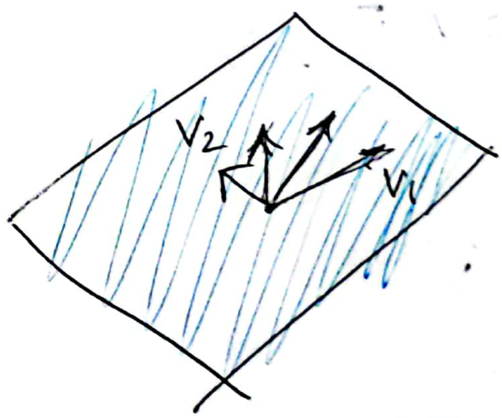
MA12 (1)



$A \quad \lambda_0 = 2$

$A v_1 = 2 v_1$

$A v_2 = 2 v_2$

$c = a + bi \qquad |c| = \sqrt{a^2+b^2} = \sqrt{c\bar{c}}$

$\bar{c} = a - bi$

$$\mathbf{V} = \begin{bmatrix} v_1 \\ v_2 \\ | \\ v_n \end{bmatrix} \in \mathbb{C}^n \qquad \|v\| = \sqrt{|v_1|^2+|v_2|^2+\cdots+|v_n|^2}$$

$$= \sqrt{\bar{v}^T v}$$

v^H

$$= \sqrt{v^* v} \qquad \overline{AB} = \bar{A}\,\bar{B}$$

$$\langle u, v\rangle = v^* u = \bar{v}^T u = \overline{v^T \bar{u}} = \overline{\bar{u}^T v} = \overline{u^* v} = \overline{\langle v, u\rangle}$$

$v \perp u \Rightarrow v^* u = \emptyset$

Orthogonal $\quad U \in \mathbb{R}^{n\times n} \qquad U^T U = U U^T = I$

$$U = \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \cdots & \vec{u}_n \end{bmatrix} \qquad \begin{bmatrix} - u_1^T - \\ - u_2^T - \\ \vdots \\ - u_n^T - \end{bmatrix} \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \cdots & \vec{u}_n \end{bmatrix} = I \qquad u_i^T u_j = \delta(i-j)$$

$U \in \mathbb{C}^{n\times n}$ Unitary

$$U^*U = UU^* = I$$

$A \in \mathbb{R}^{n\times n}$ $\quad A^T = A \quad \Rightarrow \quad \boxed{\bar{A}^T = A.}$

$$Av = \lambda v \Rightarrow v^*Av = \lambda v^*v = \lambda \underbrace{\|v\|^2}_{\text{real}}$$

$$\overline{v^*Av_1} = \overline{\bar{v}^TAv} = \underbrace{\overset{1\times n}{v^T}\,\overset{n\times n}{\bar{A}}\,\overset{n\times 1}{\bar{v}}}_{\substack{1\times 1\\ \in \mathbb{C}}} = \bar{v}^T\bar{A}^Tv = \bar{v}^TAv = v^*Av$$

$x \in \mathbb{C} \Rightarrow x = x^T$ $\qquad \Rightarrow v^*Av \in \mathbb{R}$

$$\underbrace{v^*Av}_{\text{real}} = \lambda \underbrace{\|v\|^2}_{\text{real}} \Rightarrow \lambda \in \mathbb{R}$$

$A \in \mathbb{C}^{n\times n}$ $\quad \bar{A}^T = A \quad (A^* = A)$

$$\begin{bmatrix} 2 & i & 4 \\ -i & 3 & 1+2i \\ 4 & 1-2i & 0 \end{bmatrix}$$

$\bar{A}^T = A$

$A^* = A$

$A^H = A$

$A \in \mathbb{C}^{n\times n}$

$A^* = A$ Hermitian Matrix

A real, $A^T = A \Rightarrow A^* = A$

$$\begin{cases} Av_1 = \lambda_1 v_1 \\ Av_2 = \lambda_2 v_2 \\ \lambda_1 \neq \lambda_2 \end{cases} \Rightarrow \begin{cases} v_2^*Av_1 = \lambda_1 v_2^*v_1 \\ v_1^*Av_2 = \lambda_2 v_1^*v_2 \xRightarrow{(\bar{\ })^T} v_2^*A^*v_1 = \bar{\lambda}_2 v_2^*v_1 \end{cases}$$

$\bar{\lambda}_2 = \lambda_2$ real

$$\lambda_1 v_2^*v_1 = \lambda_2 v_2^*v_1 \Rightarrow \underline{(\lambda_1 - \lambda_2)}\langle v_1, v_2\rangle$$

$$\lambda_1 \neq \lambda_2 \Rightarrow v_1 \perp v_2$$

MA12 III

$A \in \underset{\mathbb{R}^{n\times n}}{\mathbb{C}^{n\times n}}$ has $\underline{n}$ lineary independent eigenvectors: $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n \in \underset{\mathbb{R}^n}{\mathbb{C}^n}$

(A has an Eigenbasis)

$$\Rightarrow V = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \end{bmatrix} \in \underset{\mathbb{C}^{n\times n}}{\mathbb{R}^{n\times n}} \text{ is nonsingular.}$$

$$AV = \begin{bmatrix} A\vec{v}_1 & A\vec{v}_2 & \cdots & A\vec{v}_n \end{bmatrix} = \begin{bmatrix} \lambda_1\vec{v}_1 & \lambda_2\vec{v}_2 & \cdots & \lambda_n\vec{v}_n \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \cdots & \vec{v}_n \end{bmatrix}}_{\substack{V \\ \text{modal matrix}}} \underbrace{\begin{bmatrix} \lambda_1 & & \emptyset \\ & \lambda_2 \ddots & \\ \emptyset & & \lambda_n \end{bmatrix}}_{\substack{\Lambda \\ \text{diagonal matrix} \\ \text{of eigenvalues}}}$$

$$AV = V\Lambda$$

eigen vectors are independent $\Rightarrow \underbrace{A = V\Lambda V^{-1}}_{\text{Eigendecomposition}}$

~~A has~~

$A \in \underset{\mathbb{C}^{n\times n}}{\mathbb{R}^{n\times n}}$ han $\underline{n}$ linearly independent eigen~~values~~vectors

$\Rightarrow$ A has an Eigendecomposition

$- : \cancel{\exists V \in \mathbb{R}^{n\times n} \text{ nonsingular}}$ s.t. $A = V\Lambda V^{-1}$

$\Rightarrow$ A is Diagonalizable قابل قطری سازی

$$AV = V\Lambda \Rightarrow A = V\Lambda V^{-1}$$

$$V^{-1}AV = \Lambda$$

$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ not diagonalizable

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Two matrices are similar if $\exists V \in \mathbb{C}^{n\times n}$
$A, B \in \mathbb{C}^{n\times n}$.

$$A = VBV^{-1}$$

A diagonalizable $\Rightarrow$ A is similar to some diagonal matrix

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$v_1, v_2, \ldots, v_n$ is an eigenbasis for $A \in \mathbb{R}^{n\times n}$.

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \vec{x} = y_1\vec{v_1} + y_2\vec{v_2} + \cdots + y_n\vec{v_n}$$

$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_2 \end{bmatrix}$ are coordinates of x in the eigenbasis.

$$A\vec{x} = y_1A\vec{v_1} + y_2A\vec{v_2} + \cdots + y_nA\vec{v_n}$$

$$= (y_1\lambda_1)\vec{v_1} + (y_2\lambda_2)v_2 + \cdots + (y_n\lambda_n)v_2$$

$$\begin{bmatrix} y_1\lambda_1 \\ y_2\lambda_2 \\ \vdots \\ y_n\lambda_n \end{bmatrix} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_2 \end{bmatrix}$$

coordinates of $A\vec{x}$ in Eigenbasis.

$$\vec{x} = y_1 \vec{v_1} + y_2 \vec{v_2} + \cdots + y_n \vec{v_n} = \begin{bmatrix} \vec{v_1} & \vec{v_2} & \cdots & \vec{v_n} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = V\vec{y}$$

MA 12 $\vec{V}$

$$\vec{y} = V^{-1}x$$

$$\cancel{Ax = V\begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \lambda_n \end{bmatrix} y} \qquad Ax = (\lambda_1 y_1)\vec{v_1} + (\lambda_2 y_2) v_2 + \cdots + \lambda_n y_n \vec{v_n}$$

$$Ax = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \ddots \lambda_n \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$$Ax = \underbrace{V \Lambda \underbrace{V^{-1} x}}$$

$$A^n x = V \Lambda^n V^{-1} x$$

$$A^n = \underbrace{A\,A \cdots A}_{n \text{ times}} = V\Lambda \underbrace{V^{-1} V}_{I} \Lambda V^{-1} \cdots V \Lambda V^{-1}$$

$$= V \underbrace{\Lambda \Lambda \cdots \Lambda}_{n} V^{-1} = V \Lambda^n V^{-1}$$

$A, B \in \mathbb{C}^{n\times n}$ ($\mathbb{R}^{n \times n}$) are jointly diagonalizable.

$$\exists\, V \in \mathbb{R}^{n\times n} \quad \begin{cases} A = V \Lambda_1 V^{-1} \\ B = V \Lambda_2 V^{-1} \end{cases}$$

$$AB = V \Lambda_1 \Lambda_2 V^{-1} = V \Lambda_2 \Lambda_1 V^{-1} = V \Lambda_2 V^{-1} V \Lambda_1 V^{-1} = BA$$

A, B commute.

MA (VI)
12

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$ signal

$w_1 \quad w_2 \quad w_3$ filter

$y_1, y_2, \ldots, y_5$

$w_1 x_1 + w_2 x_2 + w_3 x_3$ $\quad w_1 x_2 + w_2 x_3 + w_3 x_4$ $\quad w_1 x_5 + w_2 x_1 + w_3 x_2$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} w_1 & w_2 & w_3 & 0 & 0 \\ 0 & w_1 & w_2 & w_3 & 0 \\ 0 & 0 & w_1 & w_2 & w_3 \\ w_3 & 0 & 0 & w_1 & w_2 \\ w_2 & w_3 & 0 & 0 & w_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

circulant

$$A = V \Lambda^n V^{-1}$$

$A \in \mathbb{R}^{n \times n}$ diagonalizable

real eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$

$$\lim_{n\to\infty} A^n = \lim_{n\to\infty} V \Lambda^n V^{-1} \Rightarrow V\left(\lim_{n\to\infty} \Lambda^n V^{-1}\right)$$

$$V \begin{bmatrix} \lambda_1^n & & \\ & \lambda_2^n & \\ & & \ddots & \\ & & & \lambda_n^n \end{bmatrix} V^{-1}$$

$1 > \lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \quad \lim_{n\to\infty} A^n = 0_{n\times n}$

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~~$A \in \mathbb{C}^{n\times n}$ Hermitian A~~

MA 12
VII

$A \in \mathbb{R}^{n\times n}$ symmetric $A^T = A$

~~A~~ $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ can be chosen ~~orthonor~~ to be orthonormal

$\Rightarrow V = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \in \mathbb{R}^{n\times n}$ will be an orthogonal matrix. $V^{-1} = V^T$

eigen basis is orthonormal

$A \in \mathbb{R}^{n\times n}$ symmetric $\Longrightarrow$ A is diagonalizable

$$A = V \Lambda V^{-1} = V \Lambda V^T$$

$$= \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} \begin{bmatrix} v_1^T \\ v_2^T \\ \vdots \\ v_n^T \end{bmatrix}$$

$$= \lambda_1 \underbrace{v_1 v_1^T}_{n\times n} + \lambda_2 v_2 v_2^T + \dots + \lambda_n v_n v_n^T$$

$$A\vec{x} = \sum \lambda_i v_i (v_i^T x)$$

$A \in \mathbb{C}^{n\times n}$ Hermitian

$$A = V \Lambda V^{-1} = V \Lambda V^*$$

Normal matrix $A^* A = A A^*$