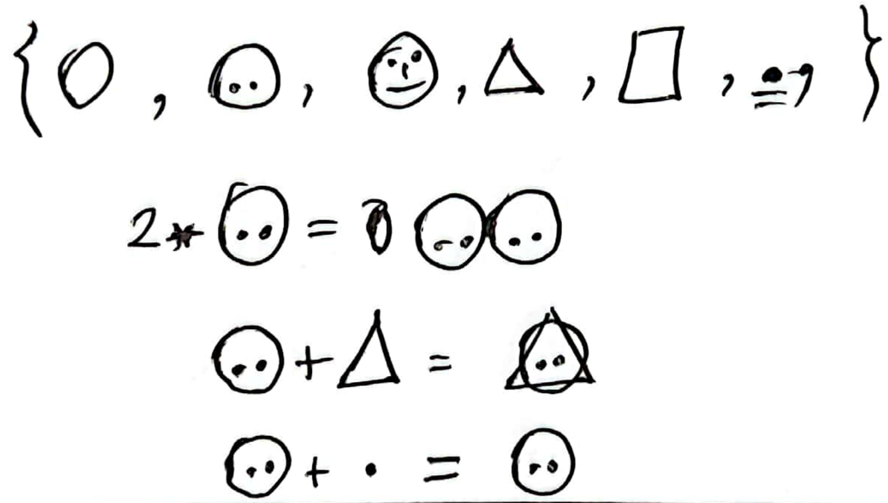
$$6 \cdot \begin{bmatrix} 2 \\ 3 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 12 \\ 18 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \\ -0.5 \end{bmatrix} + \begin{bmatrix} -4 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \\ 0.5 \end{bmatrix}$$

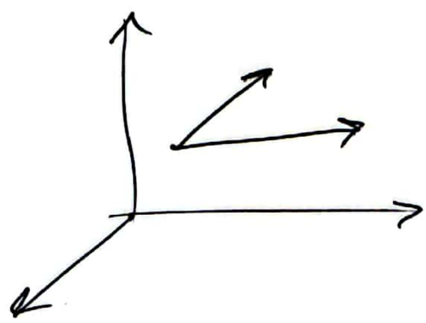


$$2\vec{x} + 3\vec{y}$$

$$a\vec{x} + b\vec{y} \qquad a, b \in \mathbb{R}$$

$$\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n \in V$$

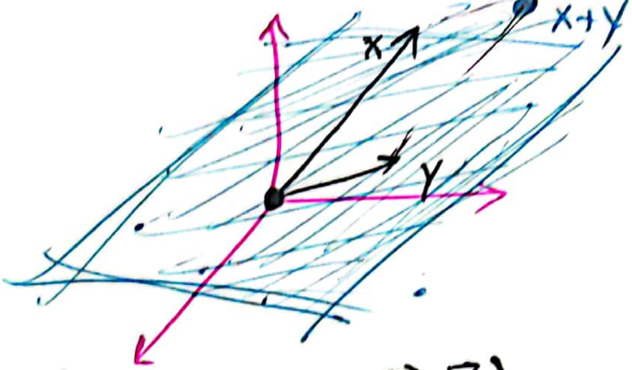
$$\sum_{i=1}^{n} a_i \vec{x}_i \qquad a_i \in \mathbb{R}$$



MA2 II

$\vec{x} + \vec{y}$

$-0.5x + 2y$



$$\{ax + by \mid a, b \in \mathbb{R}\} = \text{span}(\vec{x}, \vec{y})$$

$\vec{0} = 0\vec{x} + 0\vec{y}$ $\quad\quad$ $\vec{0} \in \text{span}(x, y)$

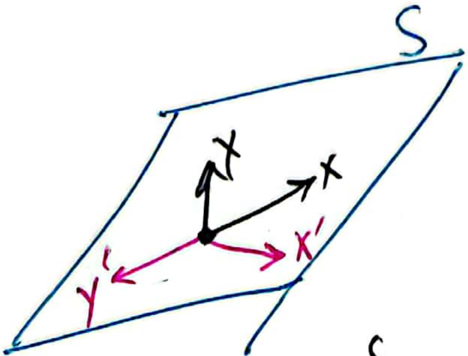
$\sin(x)$ $\quad\quad$ $\cos(x)$

$$\text{span}\left(\{\sin(an)\} \cup \{\cos(an)\}\right) = ?$$

$$\text{span}(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n) = \{a_1\vec{x}_1 + a_2\vec{x}_2 + \cdots, a_n\vec{x}_n \mid a_1, a_2, \ldots, a_n \in \mathbb{R}\}$$

$$= \left\{\sum_{i=1}^{n} a_i x_i \mid a_i \in \mathbb{R}\right\}$$

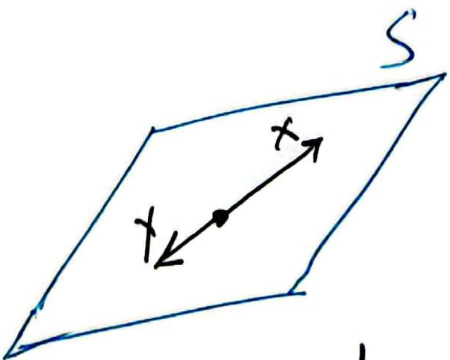
$$S \stackrel{?}{=} \text{span}(x_1, x_2, \ldots, x_n)$$



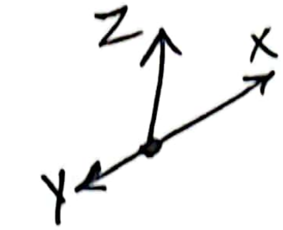
x, y span S

x', y' span S

x doesn't span S



x, y do not span S



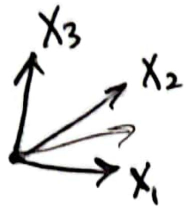
$x \in \mathrm{span}(y,z)$ → $x = -2y + 0z$

$y \in \mathrm{span}(x,z)$

$z \notin \mathrm{span}(x,y)$

$y = -0.5x$

x, y, z are linearly dependent

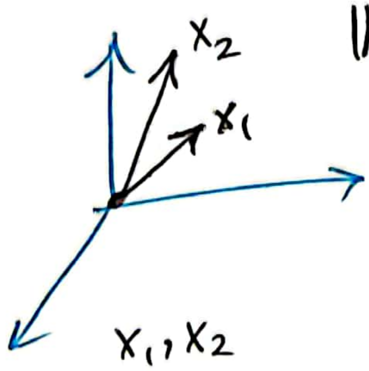


$$a_1\vec{x}_1 + a_2\vec{x}_2 + a_3\vec{x}_3 = \vec{0}$$
$$\Rightarrow a_1, a_2, a_3 = 0$$

$\vec{a}_1 \neq 0$

$$a_1\vec{x}_1 = (-a_2)\vec{x}_2 + (-a_3)\vec{x}_3$$
$$\vec{x}_1 = \left(-\frac{a_2}{a_1}\right)\vec{x}_2 + \left(-\frac{a_3}{a_1}\right)\vec{x}_3$$

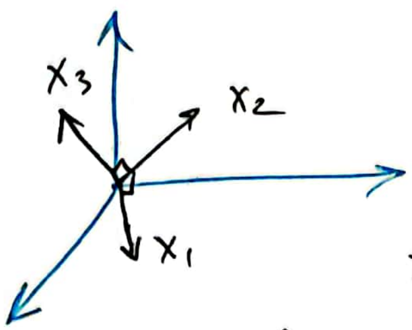
$\mathbb{R}^3$



x_1, x_2

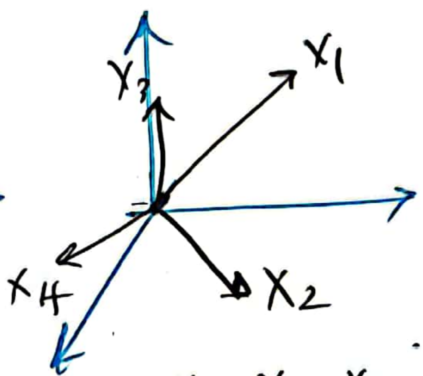
do not form a basis for $\mathbb{R}^3$

$\mathrm{span}(x_1, x_2) \neq \mathbb{R}^3$



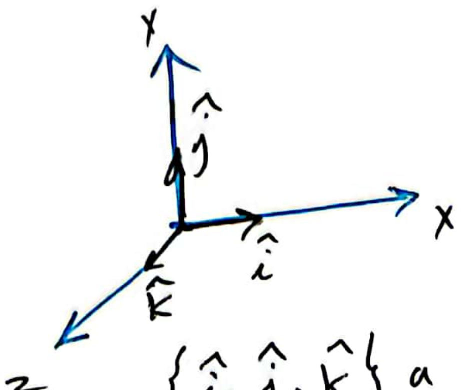
$\{x_1, x_2, x_3\}$

basis



x_1, x_2, x_3, x_4

not a basis



$\{\hat{i}, \hat{j}, \hat{k}\}$ a basis for $\mathbb{R}^3$

MA2 IV

V is a finite ~~d~~ dimensional vector space.

$\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$ form a basis for V.

Consider an arbitrary vector $\vec{x} \in V$.

$$\vec{x} \in V \Rightarrow \vec{x} \in \text{span}(\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n)$$

$$\Rightarrow \vec{x} = a_1\vec{v}_1 + a_2\vec{v}_2 + \dots + a_n\vec{v}_n \quad \text{for some } a_1, \text{—} a_n.$$

$$\vec{x} = b_1\vec{v}_1 + b_2\vec{v}_2 + \dots + b_n\vec{v}_n$$

$$(a_1-b_1)\vec{v}_1 + (a_2-b_2)\vec{v}_2 + \dots + (a_n-b_n)\vec{v}_n = \vec{0}$$

v_1 — v_n independent

$$\Rightarrow a_1 - b_1 = a_2 - b_2 = \dots = a_n - b_n = 0$$

$$a_1 = b_1,\ a_2 = b_2, \dots, a_n = b_n$$

$$X = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_n \end{bmatrix}$$

~~coo~~ coordinates of x