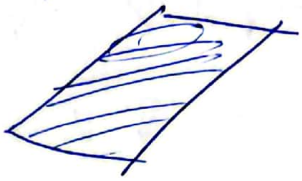
$$\min_x f(x) \qquad f:\mathbb{R}^n \to \mathbb{R}$$

$$\min_x f(x) \text{ subject to } \underline{\underline{x \in S}}$$

قید
constraint

$\{x \mid \|x\|=1\}$



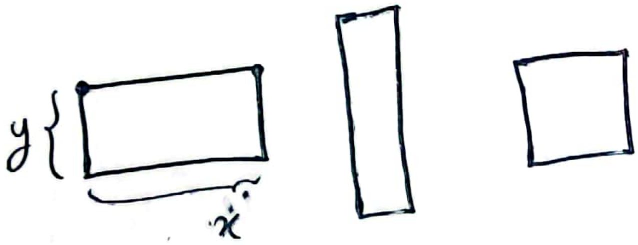
$\|x\| \leqslant 1$

$$\min_x f(x) \text{ subject to } h(x)=b$$

$$\min_x f(x) \text{ subject to } h(x)\leqslant b$$

constraned Optimization بهینه سازی مقید ، با محدودیت

1 meter



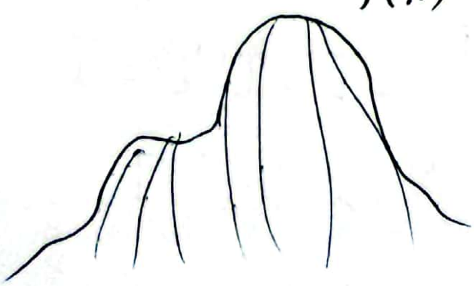
$$\max_{x,y} \; xy \qquad 2(x+y)=1$$

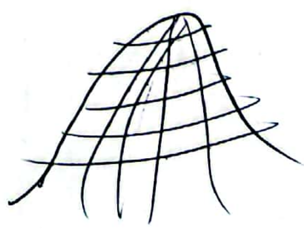
$$\max_{x,y} \; xy \quad \text{s.t.} \begin{cases} x+y=1/2 \\ x \geqslant 0 \\ y \geqslant 0 \end{cases}$$

Equlity constraint . محدودیت های تساوی

$$\min f(x) \text{ subject to } g(x)=b$$

$f:\mathbb{R}^n \to \mathbb{R}$
$g:\mathbb{R}^n \to \mathbb{R}$

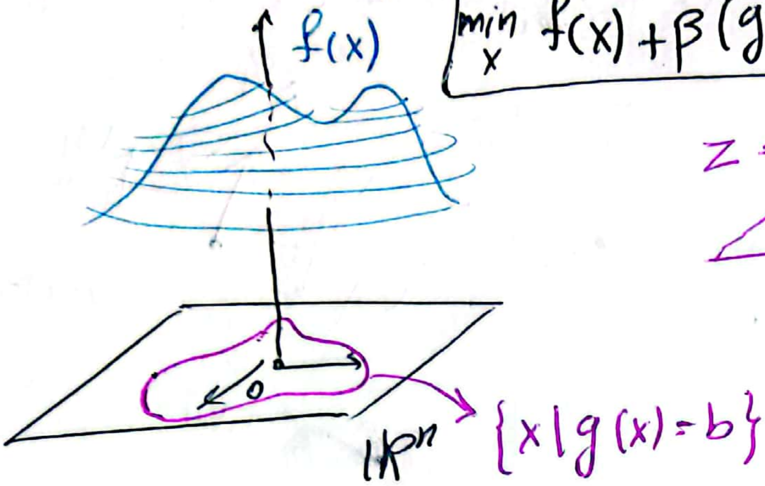


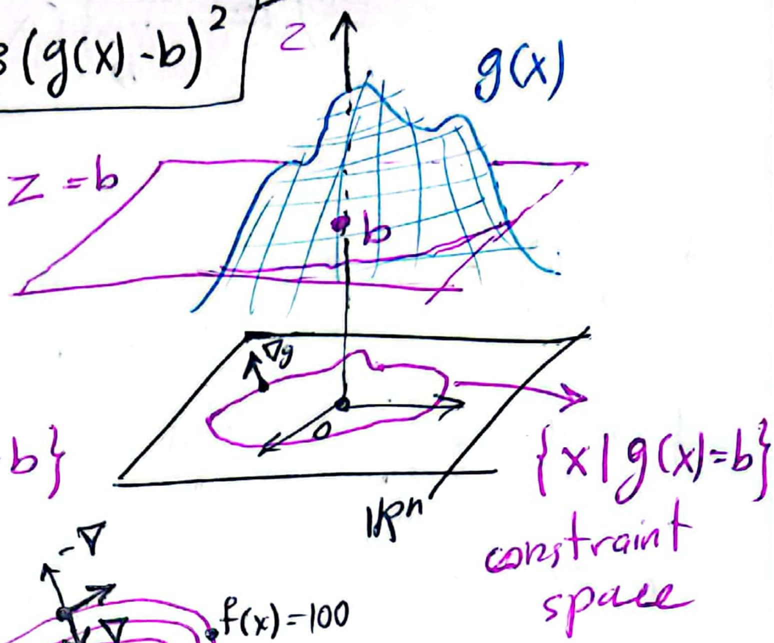


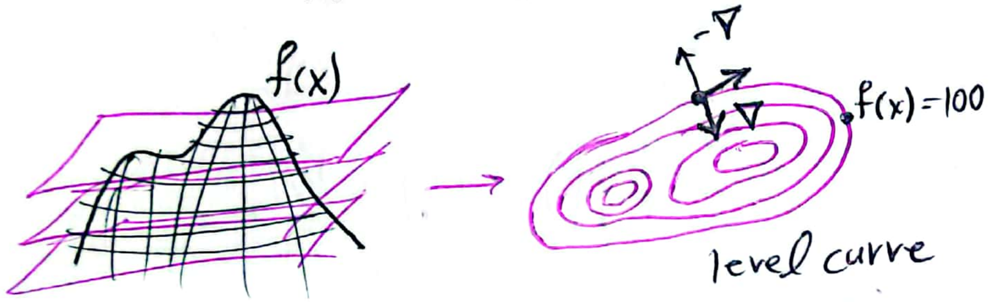
$f: \mathbb{R}^n \to \mathbb{R}$ $\qquad$ $g: \mathbb{R}^n \to \mathbb{R}$

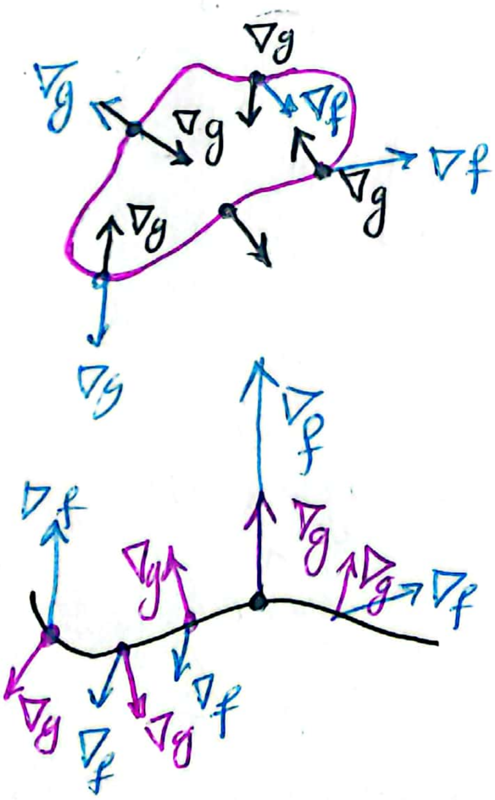
$$\min_x \; f(x) \quad \text{subject to} \quad g(x) = b$$

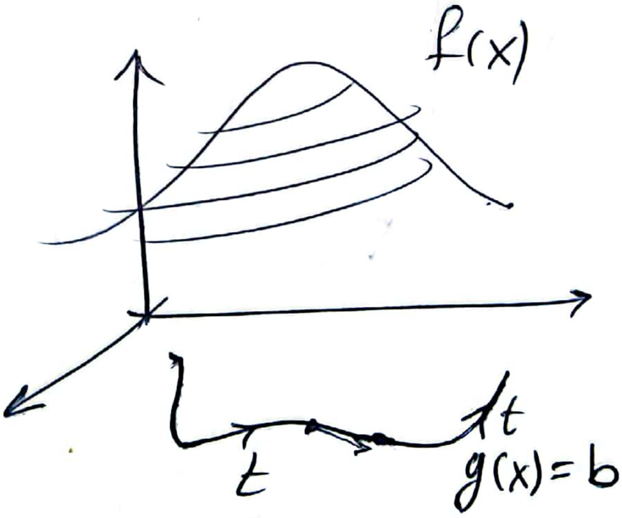
$$\boxed{\min_x \; f(x) + \beta\,(g(x) - b)^2} \to \beta \to \infty$$











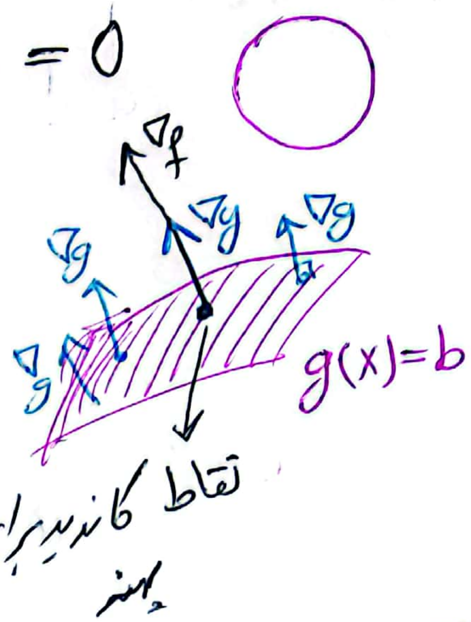
$\cancel{\nabla g}$ $\quad \boxed{\nabla f = \lambda \nabla g}$

MA III
23

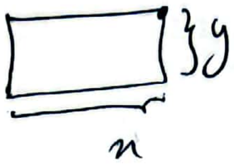
$$\min_x f(x) \quad \text{s.t.} \quad g(x)=b$$
$$\max_x$$

$\mathbb{R}^3 \to \mathbb{R}^{\circ}$

$$\nabla f(x) + \lambda \nabla g(x) = 0$$



$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = x_1 x_2$$



$$f(x,y) = f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = xy$$

$$\max_{x,y} \; xy \quad \text{s.t.} \quad x+y=1/2$$

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = xy \qquad g\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x+y$$

$$\nabla f = \begin{bmatrix} \partial f/\partial x \\ \dfrac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} \qquad \nabla g = \begin{bmatrix} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\nabla f + \lambda \nabla g = \vec{0} \Rightarrow \begin{bmatrix} y \\ x \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

دو معادله سه مجهول

$$x + y = 1/2$$

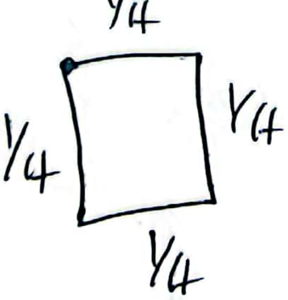
سه معادله سه مجهول

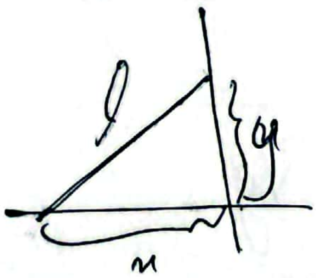
$$\Downarrow$$

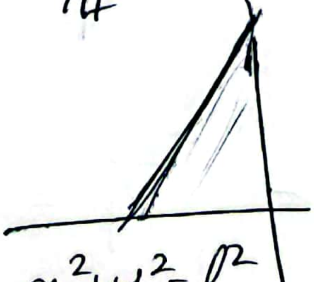
$$\left.\begin{array}{l} y+\lambda=0 \\ x+\lambda=0 \end{array}\right\} \; \underline{x=y}$$

$$x+y=1/2$$

$$x+x=1/2 \Rightarrow x=1/4 \quad y=1/4$$







$$\min_{x} f(x) \quad \text{s.t.} \quad g(x)=b \qquad f:\mathbb{R}^n \to \mathbb{R}$$
max

$$\left.\begin{array}{l}\nabla f + \lambda \nabla g = 0\\ g(x)=b\end{array}\right\} \begin{array}{l}n \text{ معادلة}\\ + \\ 1 \text{ معادلة}\end{array} \qquad \left.\begin{array}{l}x\\ \lambda\end{array}\right\} \begin{array}{l}n+1\\ \text{مجهول}\end{array}$$

Lagrangian $\quad \mathcal{L}(x,\lambda) = f(x) + \lambda(g(x)-b)$

$$\nabla_x L(x,\lambda) = \nabla f(x) + \lambda \nabla g(x) = 0$$

$$\frac{\partial L}{\partial \lambda} = \nabla_\lambda L(x,\lambda) = g(x) - b \quad = 0$$

Ex

$$\min_{x} f(x) \quad \text{s.t.} \quad g(x)=b$$

$$h(x) = g(x) - b$$

$$\min_{x} f(x) \quad \text{s.t.} \quad h(x)=0$$

$$\mathcal{L}(x,\lambda) = f(x) + \lambda h(x)$$

A symmetrix

$$f(x) = x^T A x$$

$$\min \ x^T A x \quad \text{s.t.} \quad \|x\|=1$$

$A \in \mathbb{R}^n \quad A^T = A$

mA 23 (V)

$\max \; x^T A x \quad \text{s.t.} \quad x^T x = 1$

$$\mathcal{L}(x,\lambda) = x^T A x + \lambda (x^T x - 1)$$

$$\nabla_x L = 2Ax + \lambda 2x = 0$$

$$Ax + \lambda x = 0$$

$$A\underline{x} = \underbrace{-\lambda}_{\alpha} x$$

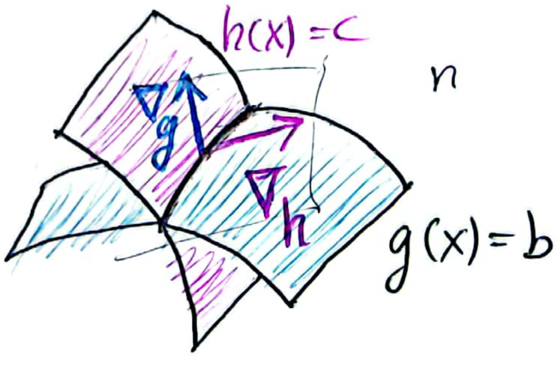
$x^T A x \quad \boxed{Av = \alpha v}$

(v, α) an eigenpair of $A \quad \|v\| = 1$

$$v^T A v = v^T(\alpha v) = \alpha v^T v = \alpha$$

$\max \; f(x) \quad \text{s.t.} \quad g(x) = b \quad , h(x) = c$

$\nabla f \in \text{span}(\nabla g, \nabla h)$



$\nabla f = \alpha \nabla g + \beta \nabla h$

$\nabla f + \alpha \nabla g + \beta \nabla h = 0$

$\mathcal{L}(x, \alpha, \beta) = f(x) + \alpha(g(x) - b) + \beta(h(x) - c)$

$f(x) + \lambda^T (p(x) - d)$

$\begin{cases} g(x) = b \\ h(x) = c \end{cases} \quad p(x) = \begin{bmatrix} g(x) \\ h(x) \end{bmatrix} \quad p: \mathbb{R}^n \to \mathbb{R}^2 \quad d = \begin{bmatrix} b \\ c \end{bmatrix}$

$p(x) = \vec{d} \quad \lambda = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$