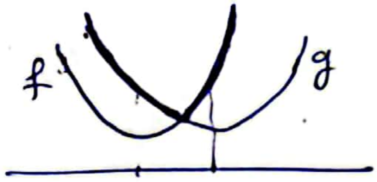


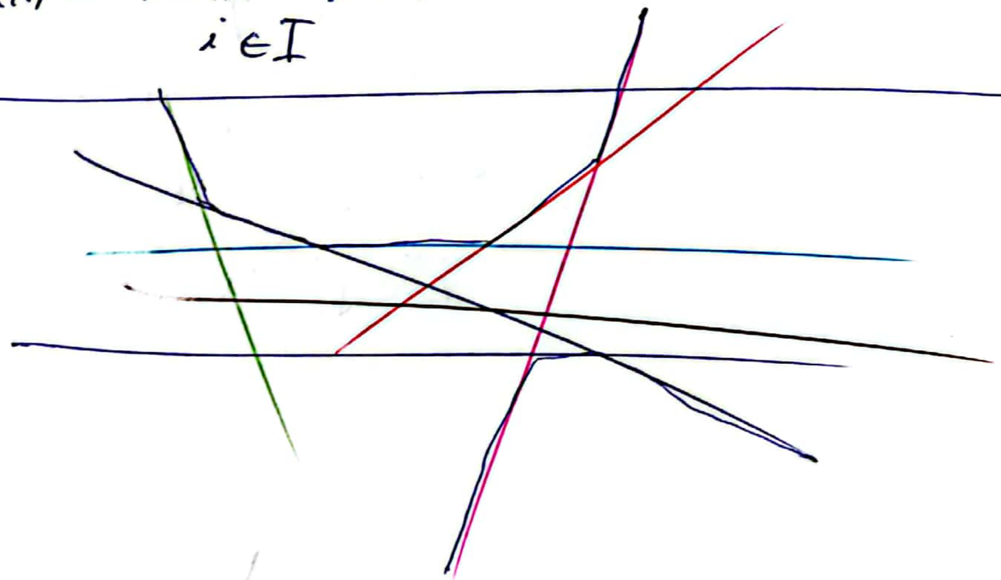
f, g convex

$$h(x) = \max(f(x), g(x))$$

h is convex

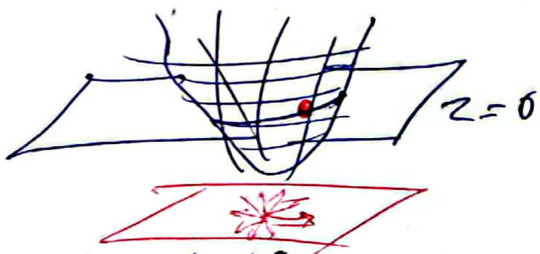
$f_i(x)$ convex $\quad i \in I$

$$g(x) = \max_{i \in I} f_i(x)$$

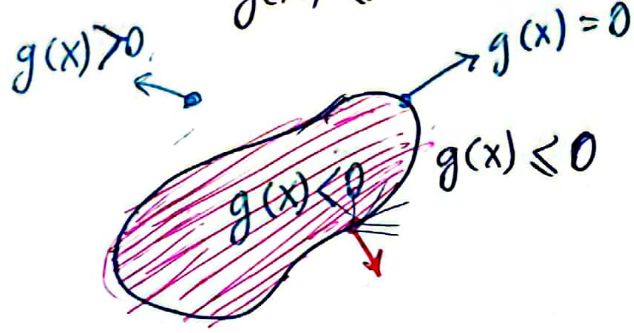


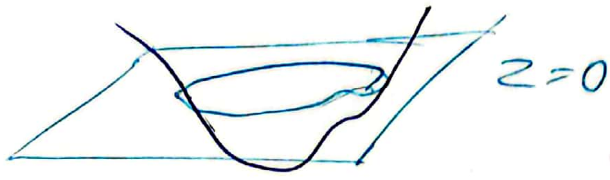
$f : \mathbb{R}^n \to \mathbb{R}$ $\qquad$ $g : \mathbb{R}^n \to \mathbb{R}$

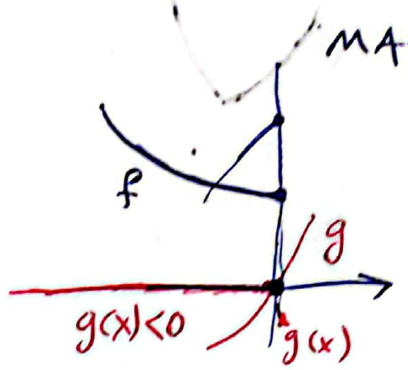
$$\nabla g = \begin{bmatrix} \frac{\partial g}{\partial x_1} \\ \frac{\partial g}{\partial x_2} \\ \vdots \\ \frac{\partial g}{\partial x_n} \end{bmatrix}$$

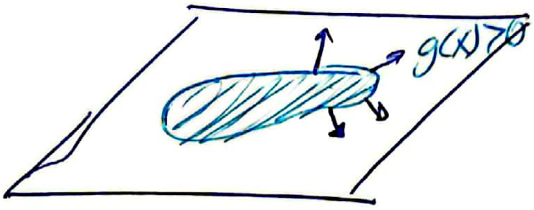


$$D[u]g(x_0) = u^T \nabla g(x_0)$$

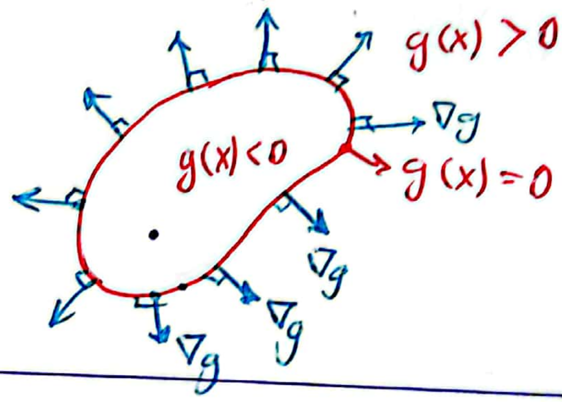


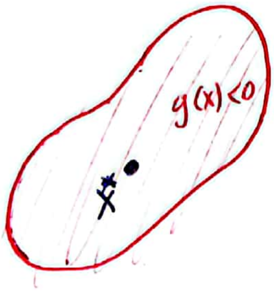






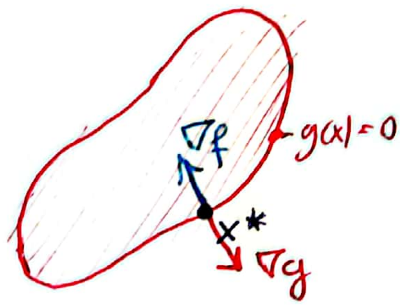
$$x^* = \underset{x}{\arg\min}\ f(x) \quad \text{s.t.} \quad g(x) \leq 0$$





$$g(x^*) < 0$$

$$\nabla f(x^*) = 0$$



$$g(x^*) = 0$$

$$\nabla f + \lambda \nabla g = 0$$

$$\lambda \geq 0$$

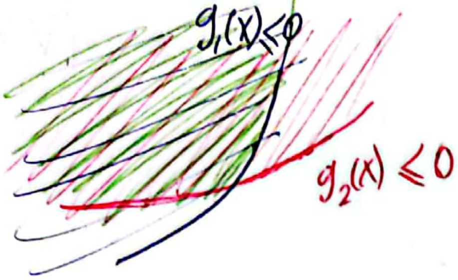
$$\nabla f(x) + \lambda \nabla g(x) = 0$$

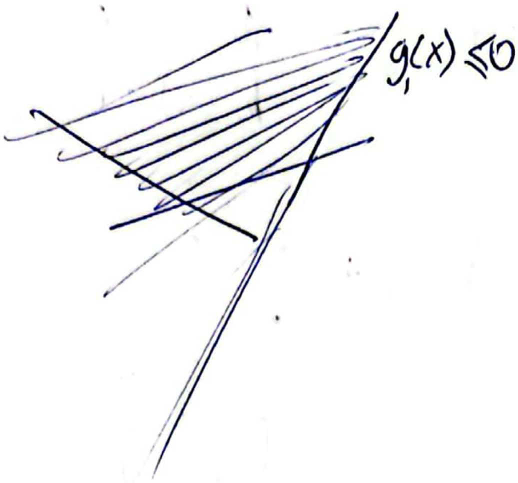
$$\lambda \geq 0$$

$$\lambda\, g(x) = 0$$

$$\boxed{g(x) < 0 \Rightarrow \lambda = 0}$$

$$\mathcal{L}(x,\lambda) = f(x) + \lambda\, g(x)$$





$$\min_x f(x) \quad \text{s.t.} \quad g_i(x) \leq 0 \quad i=1 \dots p$$
$$h_i(x) = 0 \quad i = 1 \dots q$$

$$g : \mathbb{R}^n \to \mathbb{R}^p \qquad g(x) = \begin{bmatrix} g_1(x) \\ g_2(x) \\ \vdots \\ g_p(x) \end{bmatrix} \qquad h(x) = \begin{bmatrix} h_1(x) \\ h_2(x) \\ \vdots \\ h_q(x) \end{bmatrix}$$
$$h : \mathbb{R}^n \to \mathbb{R}^q$$

$$\min f(x) \quad \text{s.t.} \quad g(x) \leq \vec{0}_p$$
$$h(x) = \vec{0}_q$$

$$\mathcal{L}(x, \vec{\lambda}, \vec{\mu}) = f(x) + \sum_{i=1}^{p} \lambda_i g_i(x) + \sum_{i=1}^{q} \mu_i h_i(x)$$
$$= f(x) + \vec{\lambda}^T g(x) + \vec{\mu}^T h(x)$$

Any x such that

x feasible $\begin{cases} g_i(x) \leq 0 & i = 1 \dots p \\ h_i(x) = 0 & i = 1 \dots q \end{cases}$

Any λ such that

$\lambda \geq 0 \quad \{ \lambda_i \geq 0 \quad i = 1 \dots p$

For any feasible x

$\mu \in \mathbb{R}^q$

$\lambda \in \mathbb{R}^p \ \lambda \geq 0$

$$f(x) \geq \mathcal{L}(x, \lambda, \mu)$$

VI

$$\min f(x) \quad \text{s.t.} \quad \begin{array}{ll} g_i(x) \leqslant 0 & i=1 \cdots p \\ h_i(x) = 0 & j=1 \cdots q \end{array}$$

$$\mathcal{L}(x, \vec{\lambda}, \vec{\mu}) = f(x) + \sum_{i=1}^{p} \lambda_i g_i(x) + \sum_{i=1}^{q} \mu_i h_i(x)$$

$$\left.\begin{array}{r} \lambda \geqslant 0 \\ g(x) \leqslant \vec{0} \\ h(x) = \vec{0} \end{array}\right\} \Rightarrow \mathcal{L}(x, \lambda, \mu) \leqslant f(x)$$

بدون قید

$$f^*(\vec{\lambda}, \vec{\mu}) = \min_{x} \mathcal{L}(x, \lambda, \mu)$$

Lagrangian Dual of f

for any $x \in \mathbb{R}^n$, $\lambda \in \mathbb{R}^p$, $\mu \in \mathbb{R}^q$ $\Rightarrow f^*(\vec{\lambda}, \vec{\mu}) \leqslant \mathcal{L}(x, \lambda, \mu)$

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for feasible $x \left\{ \begin{array}{l} g(x) \leqslant 0 \\ h(x) = 0 \end{array} \right.$, $\lambda \geqslant 0$

$$f^*(\vec{\lambda}, \vec{\mu}) \leqslant \mathcal{L}(x, \lambda, \mu) \leqslant f(x)$$

$$f^*(\vec{\lambda}, \mu) \leqslant \underline{f(x)}$$

$$x^* = \arg\min_{x} f(x) \quad \text{s.t.} \quad g_i(x) \leqslant 0 \quad i=1 \cdots p$$

$$f(x^*) = y^* = \min f(x) \quad \text{s.t.} \rightarrow \quad h_i(x) = 0 \quad i=1 \cdots q$$

$$\lambda \geqslant 0 \Rightarrow f^*(\vec{\lambda}, \mu) \leqslant f(x^*) = y^*$$

$\lambda \geqslant 0 \Rightarrow f^*(\vec{\lambda}, \mu)$ is a lower bound for $f(x^*) = y^*$

$$z^* = \max_{\vec{\lambda}, \vec{\mu}} f^*(\vec{\lambda}, \vec{\mu}) \leqslant f(x^*)$$

s.t. $\lambda_i > 0 \quad i = 1, 2, \ldots, p$