$$\begin{bmatrix} 3 & 0.5 & -2 & 4 \\ 7 & 6 & 8 & -2 \\ 4 & 1.2 & 7.8 & \sqrt{2} \end{bmatrix} \in \mathbb{R}^{3\times 4}$$

$$\begin{bmatrix} 2 & -2 \\ 1 & 100 \end{bmatrix} \in \mathbb{Z}^{2\times 2}$$

$$\begin{bmatrix} 1 & 2+3j \\ -j & 2 \\ e^{j} & 0 \end{bmatrix} \in \mathbb{C}^{3\times 2}$$

$(2, 3, -1)$

نمایش ستونی

$$\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

بردار ستونی colum vector

نمایش سطری

$$\begin{bmatrix} 2 & 3 & -1 \end{bmatrix} \in \mathbb{R}^{1\times 3}$$

بردار سطری row vector

$$\begin{bmatrix} a & b & c & d \\ x & y & z & t \\ p & q & r & s \end{bmatrix}_{3\times 4} \begin{bmatrix} -1 \\ 2 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} (-1)a + 2b + (-3)c + 4d \\ (-1)x + 2y + (-3)z + 4t \\ (-1)p + 2q + (-3)r + 4s \end{bmatrix} \in \mathbb{R}^3$$

$$= (-1)\begin{bmatrix} a \\ x \\ p \end{bmatrix} + 2\begin{bmatrix} b \\ y \\ q \end{bmatrix} + (-3)\begin{bmatrix} c \\ z \\ r \end{bmatrix} + 4\begin{bmatrix} d \\ t \\ s \end{bmatrix} \in \mathbb{R}^3$$

$\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n \in \mathbb{R}^m$

$$\text{span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n) = \{a_1\vec{u}_1 + a_2\vec{u}_2 + \dots + a_n\vec{u}_n \mid a_1, \dots a_n \in \mathbb{R}\}$$

$$U = \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \dots & \vec{u}_n \end{bmatrix} \in \mathbb{R}^{m\times n} \quad / \quad \vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \in \mathbb{R}^n$$

$$\text{span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n) = \left\{ \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \cdots & \vec{u}_n \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \mid a_1, a_2, \dots, a_n \in \mathbb{R} \right\}$$

MA3 (11)

$$= \left\{ \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \cdots & \vec{u}_n \end{bmatrix} \vec{a} \mid \vec{a} \in \mathbb{R}^n \right\}$$

$$U = \begin{bmatrix} \vec{u}_1 & \vec{u}_2 & \cdots & \vec{u}_n \end{bmatrix} \in \mathbb{R}^{m \times n} \qquad \vec{u}_i \in \mathbb{R}^m$$

column space of U = range of U $= C(U)$

U فضای ستونی $= \text{span}(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n) \subseteq \mathbb{R}^m$

$$= \{ U\vec{a} \mid \vec{a} \in \mathbb{R}^n \} = C(U)$$

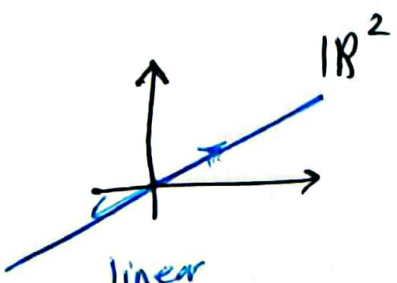
Linear Subspace

V a vector space.

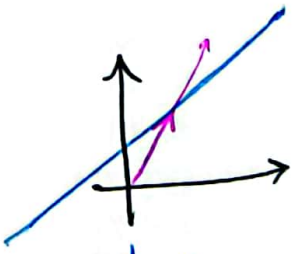
$S \subseteq V$ is a linear subspace of V if

$$\vec{x} \in S \Rightarrow a\vec{x} \in S \quad \text{for all } a \in \mathbb{R}$$

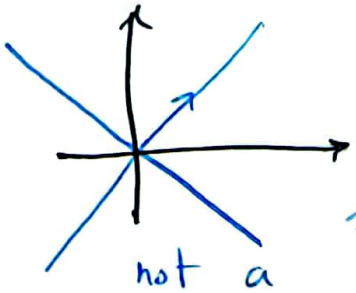
$$\vec{x}, \vec{y} \in S \Rightarrow \vec{x} + \vec{y} \in S$$

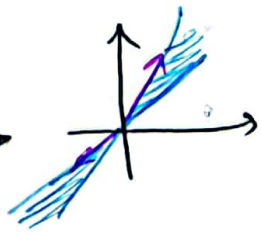


linear subspace

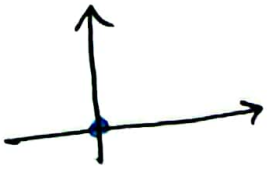


not a linear subspace

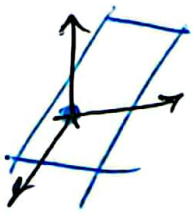


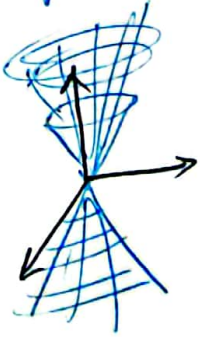


not a linear subspace



a linear subspace

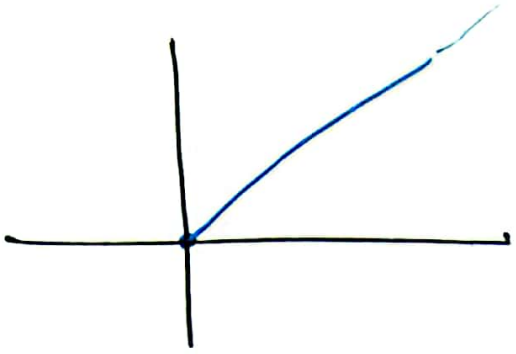


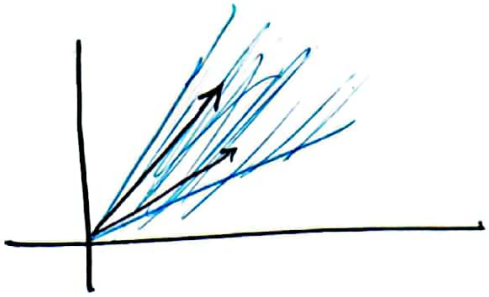


$\vec{x}, \vec{y} \in S \Rightarrow$

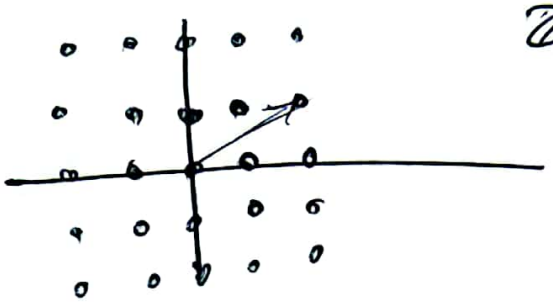
$a\vec{x} + b\vec{y} \in S$

for all $a, b \in \mathbb{R}$

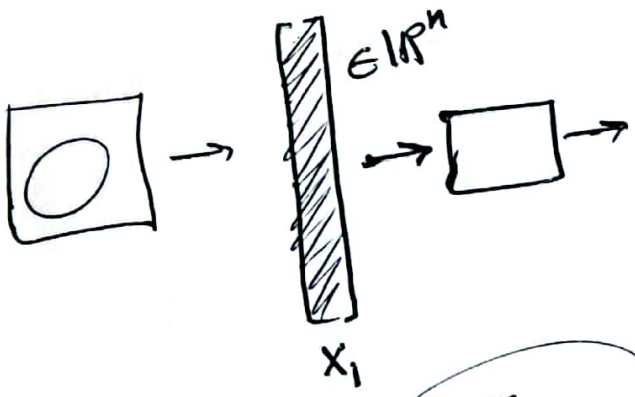


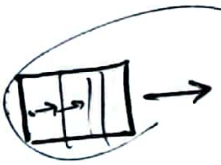


$\mathbb{Z}^n \subseteq \mathbb{R}^n$



$\mathbb{Z}^2$



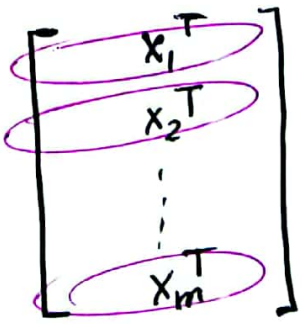
$x_1, x_2, -, x_m \rightarrow$  $\rightarrow$

$x_1, x_2, -, x_m \in \mathbb{R}^n$

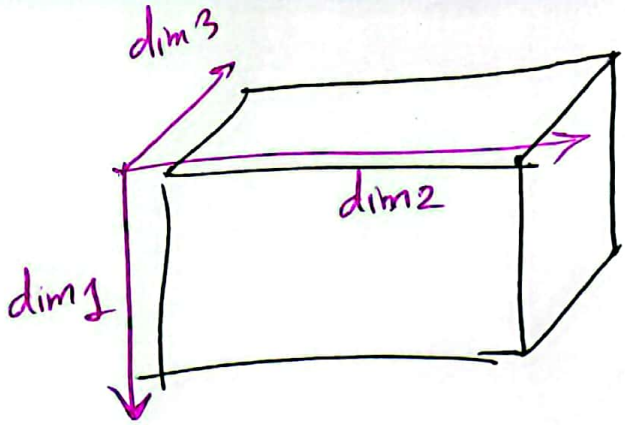
~~v^T~~ $\vec{v} \in \mathbb{R}^n$

Data as matrices

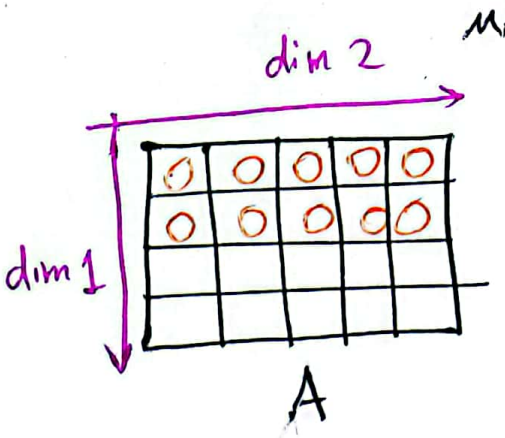
$$\begin{bmatrix} x_1 & x_2 & -- & x_m \end{bmatrix} \in \mathbb{R}^{n \times m}$$



$\in \mathbb{R}^{m \times n}$



$A[i,j,k]$



A

$A[i,j]$

$U[i,:] = U[i]$

$U[:,i] = W$

$V = U[i]$

$V[0,0] \mathrel{+}= 1$

$U\vec{a}$

$a^T U$

$$U = \begin{bmatrix} r_1^T \\ r_2^T \\ \vdots \\ r_m^T \end{bmatrix}$$

$$[a_1\ a_2\ \cdots\ a_m] \begin{bmatrix} \vec{r}_1^T \\ \vec{r}_2^T \\ \vdots \\ \vec{r}_m^T \end{bmatrix}$$

$$= a_1 \vec{r}_1^T + a_2 \vec{r}_2^T + \cdots + a_m \vec{r}_m^T$$

---

row space $(U) = R(U) = R\left(\begin{bmatrix} \vec{r}_1^T \\ \vec{r}_2^T \\ \vdots \\ \vec{r}_m^T \end{bmatrix}\right)$

$= \text{span}(\vec{r}_1, \vec{r}_2, \cdots, \vec{r}_m)$

$= C(U^T)$