Probabilistic Graphical Models Midterm Exam	Instructor: B. Nasihatkon	دانشگاه صنعتی خواجه نصیرالدین طوسی K. N. TOOSI UNIVERSITY OF TECHNOLOGY
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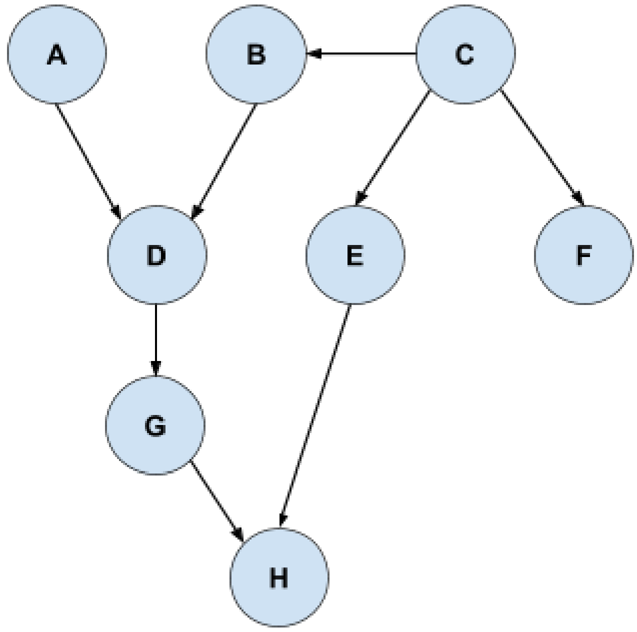
Q1- Bayesian nets / Independence (21 points)

Consider the following Bayesian network



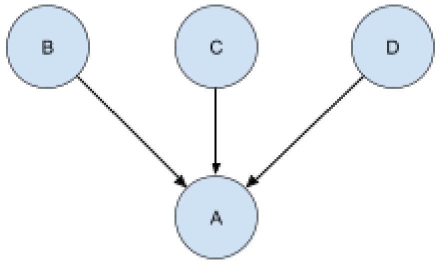
A) write the joint distribution in terms of conditional probability distributions (CPDs) **(5 points)**:

$P(A, B, C, D, E, F, G, H) =$

B) Which of the following statement are True, and which are False in general (i.e. can be false for some distribution with the above network) **(16 points)**.

	True/False			True/False
$A \perp C$			$A \perp B$	
$A \perp H$			$H \perp C \mid E$	
$A \perp E$			$A \perp B \mid D$	
$C \perp G$			$H \perp F \mid C$	
$D \perp H \mid A, B$			$C \perp A \mid H$	
$H \perp F$			$F \perp A \mid H$	
$H \perp F \mid E$			$F \perp A \mid B, H$	
$H \perp F \mid E, G$			$A \perp F \mid E, H$	

Q2- Bayesian nets / Context-specific independence (32 points)



Consider the conditional probability distribution $P(A \mid B, C, D)$ in the following Bayesian network, where $A, B, C, D \in \{0,1\}$ are all binary variables. The following piece of code contains a function that receives B,C and D as arguments and returns a 2-tuple representing the probability of A=0 and A=1 given B,C,D. Therefore, the return value is equal to **(**$P(A=0|B,C,D)$, $P(A=1|B,C,D)$**).**

```
function cpd_A_given_BCD(B,C,D)

    if B == 1
        return (.2, .8)
    else if C == 0
        return (.6, .4)
    else if D == 1
        return (.9, .1)
    else
        return (.3, .7)
```

A) Which of the following statements are True and which ones are False). Here $X \perp Y \mid Z$ means that X is independent of Y given Z **(14 Points)**.

	True/False			True/False
$A \perp B \mid C, D$			$A \perp D \mid B=0, C=0$	
$A \perp B \mid C=0$			$A \perp D \mid B=0, C=1$	
$A \perp B \mid C=1$			$A \perp D \mid B=1, C=0$	
$A \perp C \mid B=0$			$A \perp D \mid B=1, C=1$	
$A \perp C \mid B=1$			$B \perp C$	
$A \perp D \mid B=0$			$A \perp C \mid B=1,\ D=1$	
$A \perp D \mid B=1$			$A \perp C \mid B=0,\ D=1$	

B) How many parameters (at least) are needed to represent a CPD $P(A|B,C,D)$ with the above form? Why? **(3 points)**

C) How many parameters are needed to represent a ***general*** CPD $P(A|B,C,D)$ for binary variables A,B,C,D (not in the above form or any other special form). Why? **(3 points)**

D) Assume that **P(B = 0) = 0.4** and **P(C = 0) = 0.7** and **P(D = 0) = 0.2**. For the above Bayesian network compute the following probabilities. You need to write the steps towards the final answer. Do not **just** write the final solution, and do not just write the formulae **(12 points)**:

P(A = 0 | B =0, C=1, D=0) =

P(A = 0, B =0, C=1, D=0) =

P(A = 0, B =0, C=1, D=1) =

P(A = 0, B =0, C=1) =

P(D=0 | A = 0, B =0, C=1) =

P(A = 0 | B = 1) =

Extra points! (3+3 points)

P(A = 0) =

P(A = 0 | B = 0) =

Q3- Markov Random Fields/Gibbs distribution (24 points)

Consider the following joint distribution for a Markov Random Field.
$P(A,B,C,D,E,F) = \frac{1}{Z}\phi_1(A,B)\,\phi_2(B,C,D,E)\,\phi_3(C,D,F)$

A) If $A \in \{0,1\}$, $B \in \{1,2,3\}$, $C \in \{1,2\}$, $D \in \{1,2,3,4\}$, $E \in \{0,1\}$, $F \in \{0,1\}$, how many parameters are needed in general to represent the above joint distribution? why? (assume ϕ_1, ϕ_2 and ϕ_3 do not have any specific forms) **(3 points)**

B) How many parameters were needed (at least) to parameterize an arbitrary joint distribution $P(A,B,C,D,E,F)$ (without any special structure) with the same variable domains? Why? **(2 points)**

C) Draw the MRF graph corresponding to the above distribution. **(5 points)**

D) Which of the following statements are true and which are false (for general potential functions ϕ_1, ϕ_2 and ϕ_3)? **(14 points)**

	True/False			True/False
$A \perp B \mid C,D,E,F$			$F \perp E \mid C$	
$A \perp C \mid B$			$A \perp D \mid B=2, C=1$	
$A \perp B \mid C$			$B \perp F \mid C, D$	
$A \perp F \mid C$			$B \perp E \mid C,D$	
$A \perp F \mid B,C$			$E \perp A \mid B,C$	
$A \perp F$			$E \perp F \mid B,C,D$	
$A \perp E$			$F \perp A \mid B$	

Q4- Energy Function / Graph Cuts

Consider the following energy function over the *binary* variables A, B, C, D

$E(A,B,C,D) = E_{AB}(A,B) + E_{BC}(B,C) + E_{CD}(C,D) + E_A(A) + E_B(B) + E_C(C) + E_D(D)$

where

$E_{AB}(0,0) = E_{AB}(1,1) = E_{BC}(0,0) = E_{BC}(1,1) = E_{CD}(0,0) = E_{CD}(1,1) = 0$

$E_{AB}(0,1) = E_{AB}(1,0) = 1$

$E_{BC}(0,1) = 2 \quad E_{BC}(1,0) = 3$

$E_{CD}(0,1) = 4 \quad E_{CD}(1,0) = 8$

$E_A(0) = 10 \quad E_A(1) = 12 \quad E_B(0) = 20 \quad E_B(1) = 22$

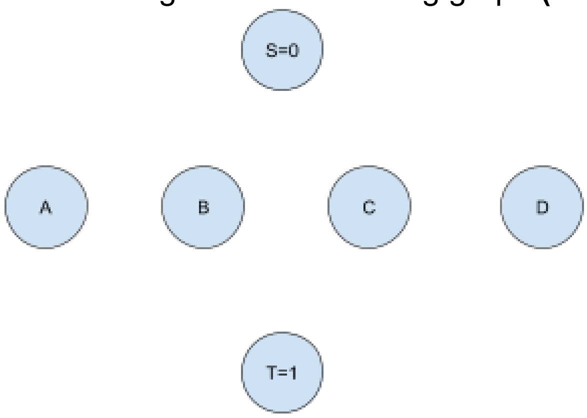
$E_C(0) = 40 \quad E_C(1) = 44 \quad E_D(0) = 80 \quad E_D(1) = 88$

1. Write the joint distribution in terms of the (unary and binary) energy terms ($E_{AB}(A,B)$, $E_{BC}(B,C)$, $E_{CD}(C,D)$, $E_A(A)$, $E_B(B)$, $E_C(C)$ and $E_D(D)$). You do not need to compute the partition function. **(5 points)**

$P(A,B,C,D) = \frac{1}{Z}$

2. Draw the corresponding MRF graph. **(3 points)**

3. Construct a min-cut/max-flow graph for the above energy function, by adding **directed** edges and their weights in the following graph **(10 points)**



4. Is $E(A,B,C,D)$ *submodular*? Why? **(5 points)**