# PGM - Homework 4

Due Date: Monday, Esfand 9, 1395 at 23:30

February 23, 2017

An undirected graphical model with set of nodes $G$ and the neighbourhood system $\mathcal{N}$ is called a Markov Random Field if

$$p(X_i \mid X_{G\setminus\{i\}}) = p(X_i \mid X_{\mathcal{N}_i}), \tag{1}$$

where $G$ is the set of nodes of the graph, $G \setminus \{i\}$ represents all graph nodes except node $i$, and $\mathcal{N}_i$ denotes the neighbours of $i$.

An undirected graphical model is called a Gibbs Random Field (and its joint distribution a Gibbs distribution) if the corresponding joint distribution can be factorized as the product of functions over cliques (= fully connected subgraphs) of the graph

$$p(X_G) = \frac{1}{Z} \prod_{C \in \mathcal{C}} \phi_C(X_C) \tag{2}$$

where $\mathcal{C}$ is the set of all cliques.

The Hammersley-Clifford theorem states that if the joint distribution $p(X_G)$ is nonzero for all $X_G$, then an undirected graphical model is a Markov random field if and only if it is a Gibbs random field. In other words, the two models are equivalent.

Your task in this howework is to prove the backward direction of the proof, that is, any Gibbs distribution with property (2) for which $\phi_C(X_C) > 0$ for all cliques $C$, will satisfy the Markov property (1).

**How to submit?** You must upload a file in the *courses* website. You need to either submit a pdf file created by latex or a scanned copy of your handwritten answers. Your homework will not get corrected if you simply take pictures with your phone. Use the "Cam Scanner" app if using your phone. You will get extra point by writing in latex.