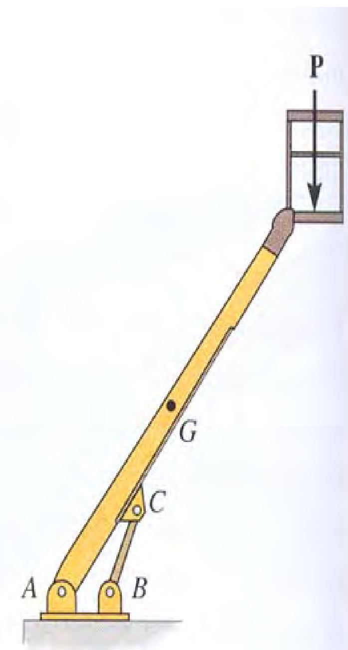
What is mechanics?

- Study of what happens to a "thing" (the technical name is "BODY") when FORCES are applied to it.
- Either the body or the forces could be large or small.



Branches of mechanics

- Mechanics
  - Rigid Bodies (Things that do not change shape)
    - Statics
    - Dynamics
  - Deformable Bodies (Things that do change shape)
  - Fluids
    - Incompressible
    - Compressible

An Overview of Mechanics

Mechanics: **the study of how bodies react to forces acting on them**

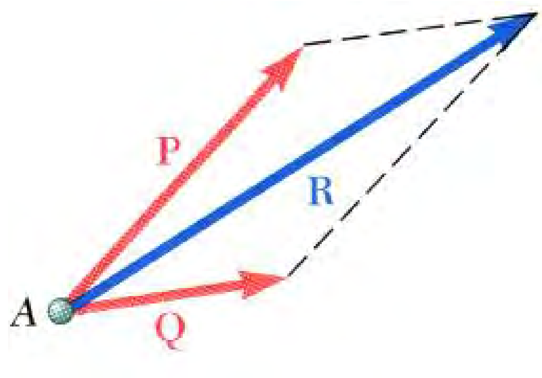
Statics: **the study of bodies in equilibrium**

Dynamics:

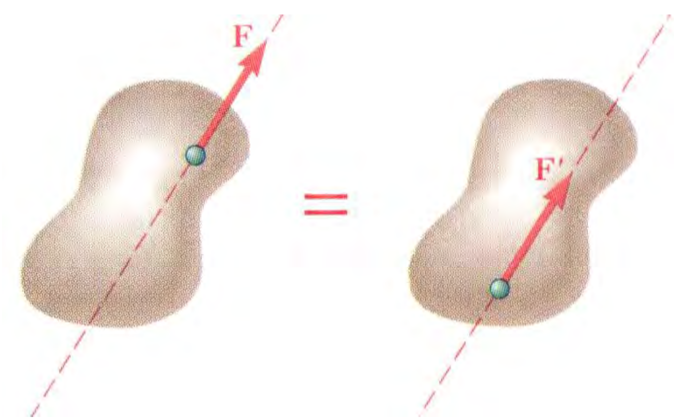
1. **Kinematics – concerned with the geometric aspects of motion**
2. **Kinetics - concerned with the forces causing the motion**

What may happen if statics is not applied properly?

Fundamental Principles



- *Parallelogram Law*



- *Principle of Transmissibility*

- *Newton's First Law*: If the resultant force on a particle is zero, the particle will remain at rest or continue to move in a straight line.

- *Newton's Second Law*: A particle will have an acceleration proportional to a nonzero resultant applied force.

$$\vec{F} = m\vec{a}$$

- *Newton's Third Law*: The forces of action and reaction between two particles have the same magnitude and line of action with opposite sense.

- *Newton's Law of Gravitation*: Two particles are attracted with equal and opposite forces,

$$F = G\frac{Mm}{r^2} \qquad W = mg, \quad g = \frac{GM}{R^2}$$

UNITS OF MEASUREMENT

- Four fundamental physical quantities.
  - Length
  - Mass
  - Time
  - Force
- Newton's 2^{nd} Law relates them: $F = m * a$
- We use this equation to develop systems of units.
- Units are arbitrary names we give to the physical quantities.

UNITS OF MEASUREMENT

- *Space* - associated with the notion of the position of a point P given in terms of three coordinates measured from a reference point or origin.
- *Time* - definition of an event requires specification of the time and position at which it occurred.
- *Mass* - used to characterize and compare bodies, e.g., response to earth's gravitational attraction and resistance to changes in translational motion.
- *Force* - represents the action of one body on another. A force is characterized by its point of application, magnitude, and direction, i.e., a force is a vector quantity.

In Newtonian Mechanics, space, time, and mass are absolute concepts, independent of each other. Force, however, is not independent of the other three. The force acting on a body is related to the mass of the body and the variation of its velocity with time.

Unit systems

- Force, mass and acceleration are called the base units.
- The fourth unit, time is derived from the acceleration term.
- We will work with two unit systems in statics:
    - International System (SI)
    - U.S. Customary (USCS)

Unit systems

Name	Length	Time	Mass	Force
International System of Units (SI)	meter (m)	second (s)	kilogram (kg)	newton* (N) $\left(\frac{kg \cdot m}{s^2}\right)$
U.S. Customary (FPS)	foot (ft)	second (s)	slug* $\left(\frac{lb \cdot s^2}{ft}\right)$	pound (lb)

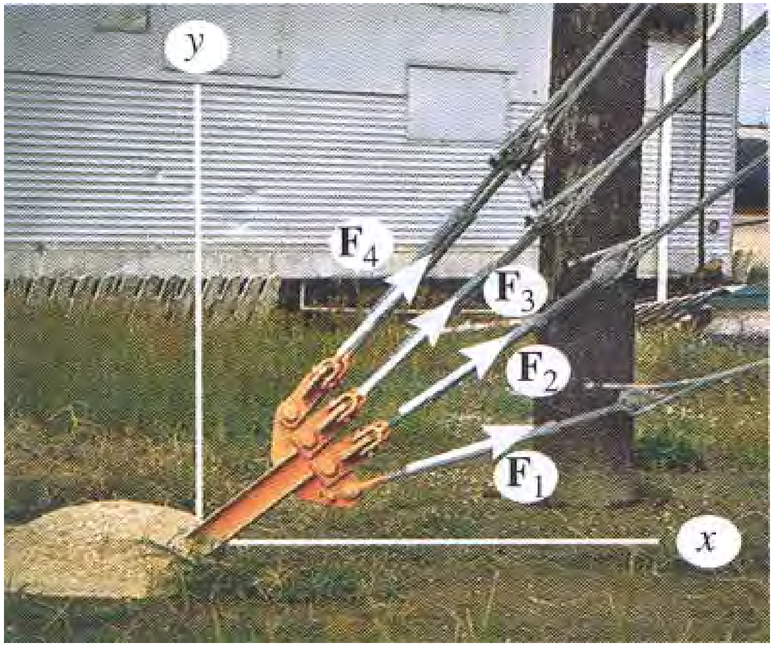
Common conversion factors

- Work problems in the units given unless otherwise instructed!
  - 1 ft = 0.3048 m
  - 1 lb = 4.4482 N
  - 1 slug = 14.5938 kg
  - Example: Convert a torque value of 47 in • lb into SI units.
    - Answer is 5.31026116 N • m?

The international system of units

- No Plurals (e.g., m = 5 kg not kgs)
- Separate Units with a • (e.g., meter second = m • s)
- Most symbols are in lowercase.
    - Some exceptions are **N**, **Pa, M** and **G.**
- Exponential powers apply to units, e.g., cm • cm = cm^2
- Compound prefixes should not be used.
- When writing exponential notation
    - 4.5×10^3 N <u>*NOT*</u> 4.5E3N

APPLICATION OF VECTOR ADDITION



There are four concurrent cable forces acting on the bracket.

How do you determine the resultant force acting on the bracket ?

Method of Problem Solution

- *Problem Statement*:
  Includes given data, specification of what is to be determined, and a figure showing all quantities involved.

- *Free-Body Diagrams*:
  Create separate diagrams for each of the bodies involved with a clear indication of all forces acting on each body.

- *Fundamental Principles*:
  The six fundamental principles are applied to express the conditions of rest or motion of each body. The rules of algebra are applied to solve the equations for the unknown quantities.

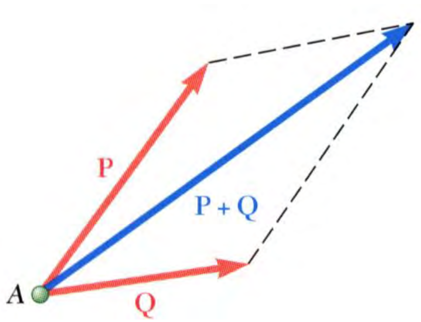
- *Solution Check*:
  - Test for errors in reasoning by verifying that the units of the computed results are correct,
  - test for errors in computation by substituting given data and computed results into previously unused equations based on the six principles,
  - **always** apply experience and physical intuition to assess whether results seem "reasonable"

SCALARS AND VECTORS

	Scalars	***Vectors***
Examples:	mass, volume	force, velocity
Characteristics:	It has a magnitude (positive or negative)	It has a magnitude and direction
Addition rule:	Simple arithmetic	Parallelogram law
Special Notation:	None	Bold font, a line, an arrow or a “carrot”

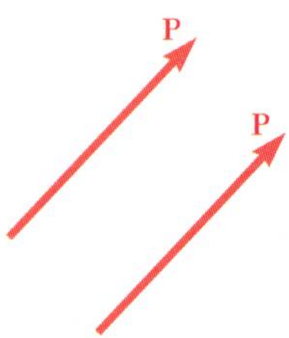
In the PowerPoint presentation vector quantity is represented ***Like this*** (in **bold**, *italics*).

Vectors

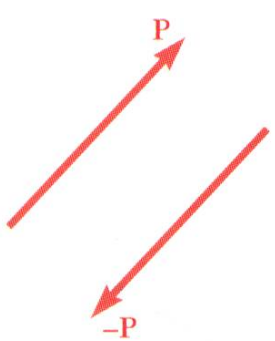


- *Vector*: parameter possessing magnitude and direction which add according to the parallelogram law. Examples: displacements, velocities, accelerations.

- *Scalar*: parameter possessing magnitude but not direction. Examples: mass, volume, temperature



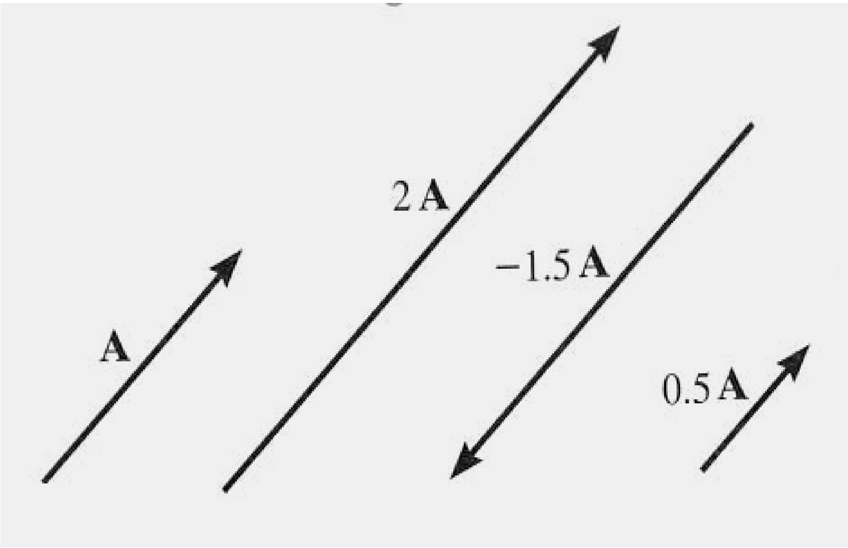
- Vector classifications:
  - *Fixed* or *bound* vectors have well defined points of application that cannot be changed without affecting an analysis.
  - *Free* vectors may be freely moved in space without changing their effect on an analysis.
  - *Sliding* vectors may be applied anywhere along their line of action without affecting an analysis.



- *Equal* vectors have the same magnitude and direction.

- *Negative* vector of a given vector has the same magnitude and the opposite direction.

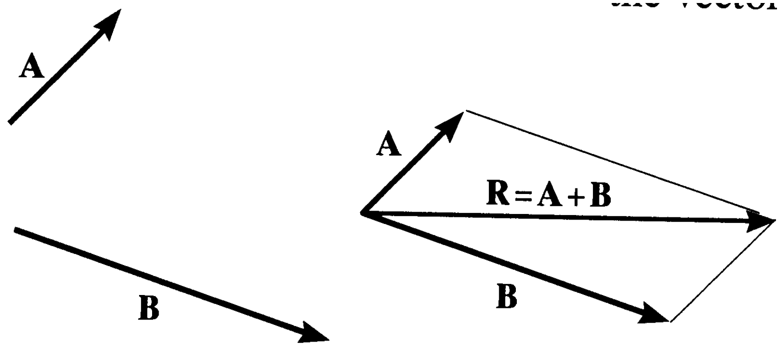
VECTOR OPERATIONS



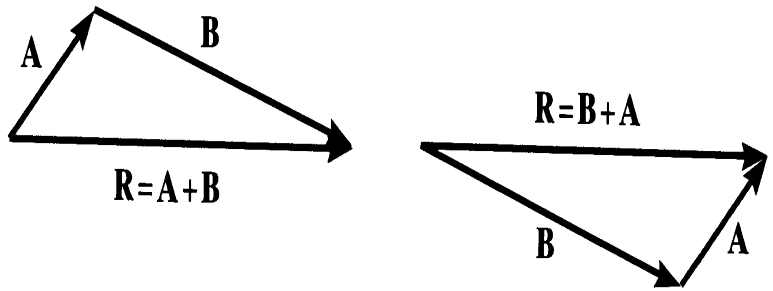
Scalar Multiplication and Division

VECTOR ADDITION
--PARALLELOGRAM LAW

Parallelogram Law:



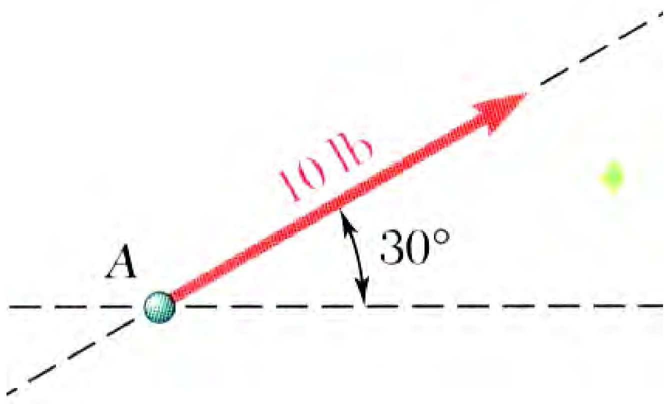
Triangle method (always 'tip to tail'):



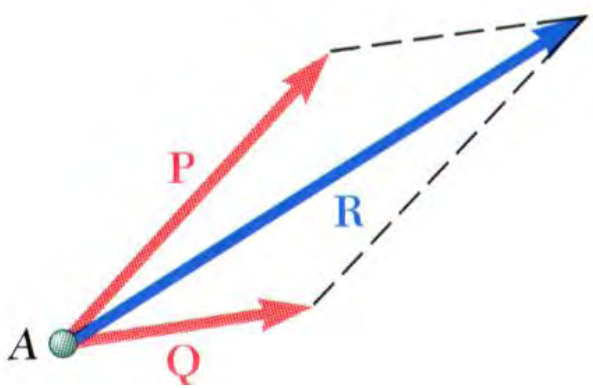
How do you subtract a vector?

How can you add more than two concurrent vectors graphically ?

Resultant of Two Forces

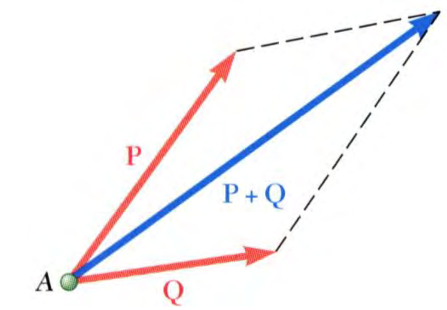


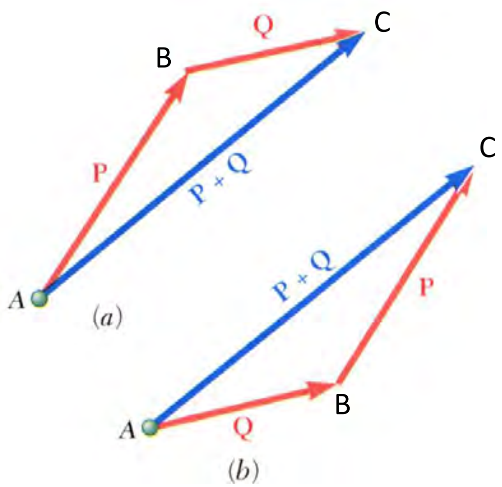
- force: action of one body on another; characterized by its *point of application*, *magnitude*, *line of action*, and *sense*.

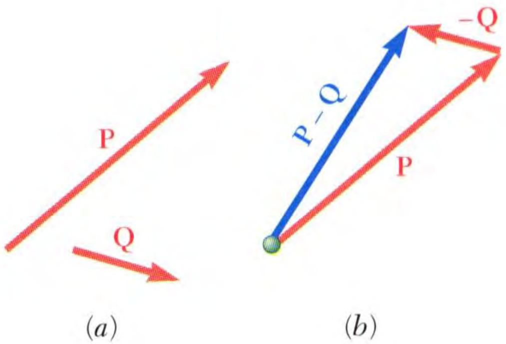


- Experimental evidence shows that the combined effect of two forces may be represented by a single *resultant* force.
- The resultant is equivalent to the diagonal of a parallelogram which contains the two forces in adjacent legs.
- Force is a *vector* quantity.

Addition of Vectors







- Trapezoid rule for vector addition
- Triangle rule for vector addition
- Law of cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$\vec{R} = \vec{P} + \vec{Q}$$

- Law of sines,

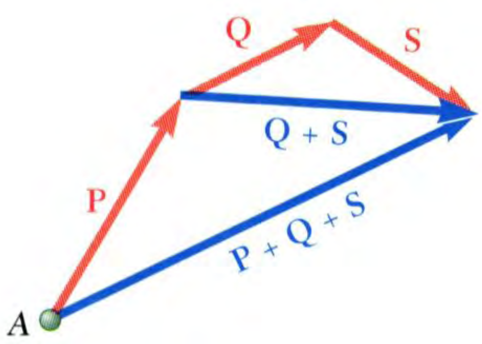
$$\frac{\sin A}{Q} = \frac{\sin B}{R} = \frac{\sin C}{A}$$

- Vector addition is commutative,

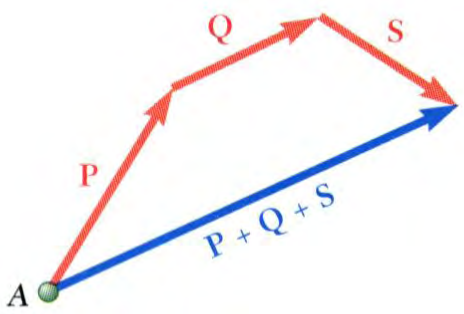
$$\vec{P} + \vec{Q} = \vec{Q} + \vec{P}$$

- Vector subtraction

Addition of Vectors

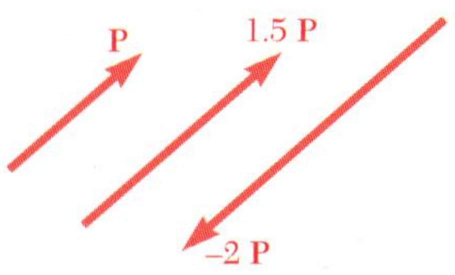


- Addition of three or more vectors through repeated application of the triangle rule



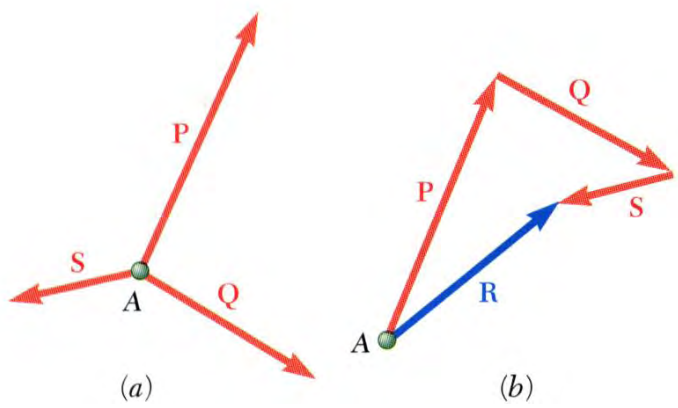
- The polygon rule for the addition of three or more vectors.
- Vector addition is associative,

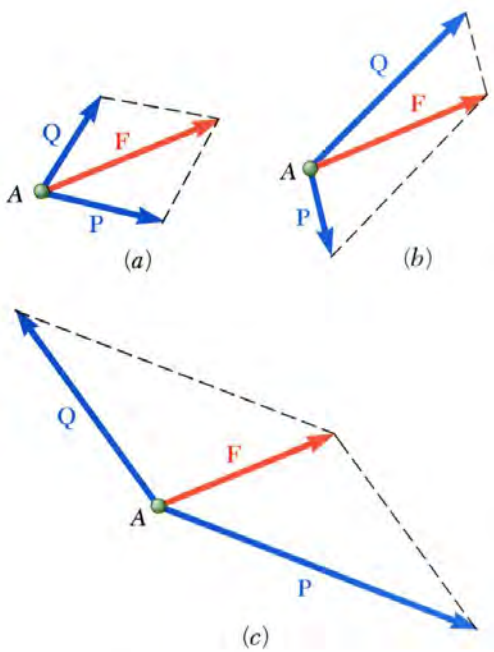
$$\vec{P} + \vec{Q} + \vec{S} = \left(\vec{P} + \vec{Q}\right) + \vec{S} = \vec{P} + \left(\vec{Q} + \vec{S}\right)$$



- Multiplication of a vector by a scalar

Resultant of Several Concurrent Forces



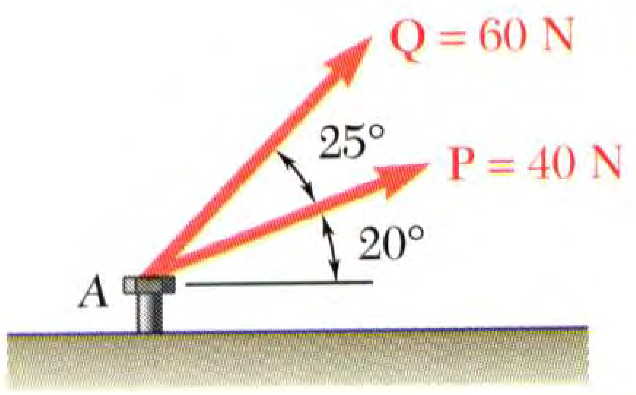


- *Concurrent forces*: set of forces which all pass through the same point.

  A set of concurrent forces applied to a particle may be replaced by a single resultant force which is the vector sum of the applied forces.

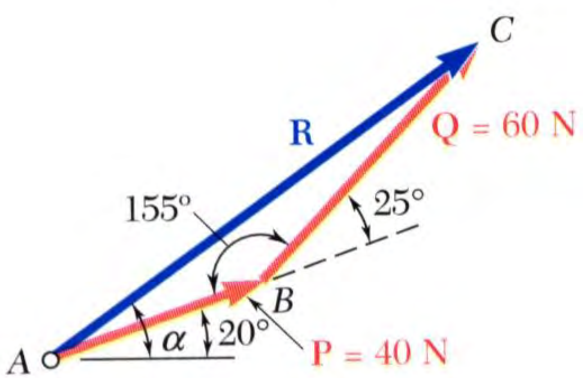
- *Vector force components*: two or more force vectors which, together, have the same effect as a single force vector.

Example



The two forces act on a bolt at *A*. Determine their resultant.

- Trigonometric solution - use the triangle rule for vector addition in conjunction with the law of cosines and law of sines to find the resultant.



- Trigonometric solution - Apply the triangle rule.

  From the Law of Cosines,

$$R^2 = P^2 + Q^2 - 2PQ\cos B$$
$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

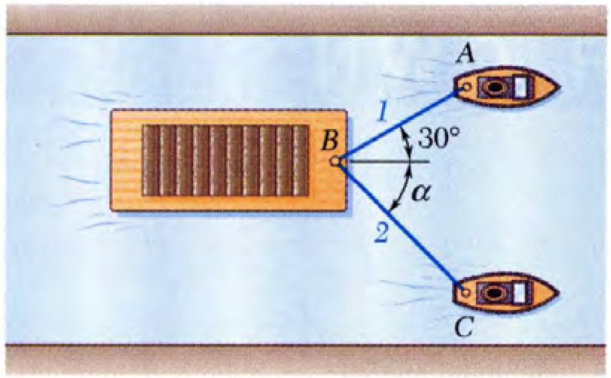
$$\boxed{R = 97.73\text{N}}$$

From the Law of Sines,

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$
$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$
$$A = 15.04°$$
$$\alpha = 20° + A$$
$$\boxed{\alpha = 35.04°}$$

Sample Problem



A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000 lbf directed along the axis of the barge, determine

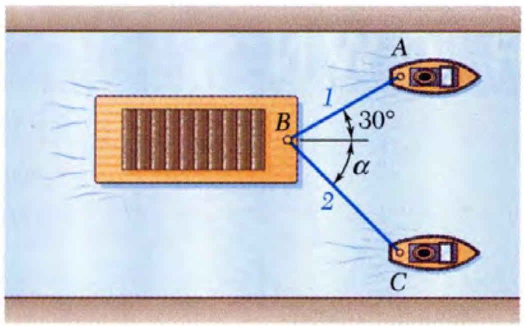
a) the tension in each of the ropes for $\alpha = 45^{o}$,

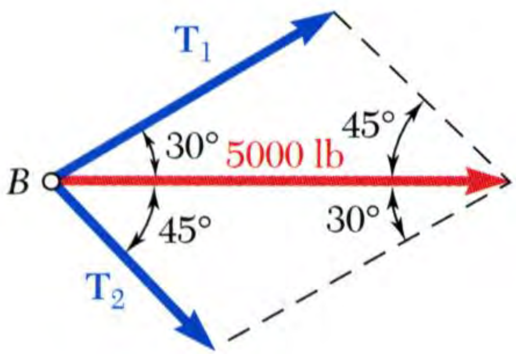
b) the value of α for which the tension in rope 2 is a minimum.

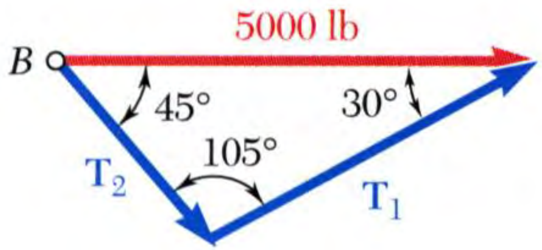
SOLUTION:

- Find a graphical solution by applying the Parallelogram Rule for vector addition. The parallelogram has sides in the directions of the two ropes and a diagonal in the direction of the barge axis and length proportional to 5000 lbf.
- Find a trigonometric solution by applying the Triangle Rule for vector addition. With the magnitude and direction of the resultant known and the directions of the other two sides parallel to the ropes given, apply the Law of Sines to find the rope tensions.
- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α.

Sample Problem







- Graphical solution - Parallelogram Rule with known resultant direction and magnitude, known directions for sides.

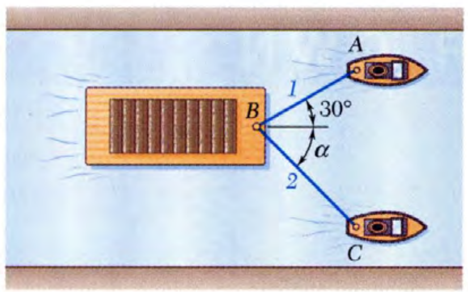
$$\boxed{T_1 = 3700\,\text{lbf} \quad T_2 = 2600\,\text{lbf}}$$

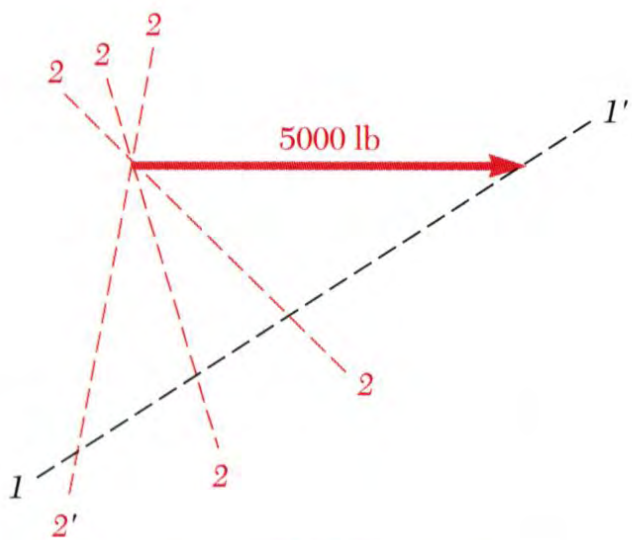
- Trigonometric solution - Triangle Rule with Law of Sines

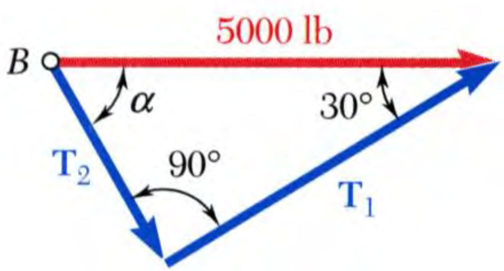
$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000\,\text{lbf}}{\sin 105°}$$

$$\boxed{T_1 = 3660\,\text{lbf} \quad T_2 = 2590\,\text{lbf}}$$

Sample Problem







- The angle for minimum tension in rope 2 is determined by applying the Triangle Rule and observing the effect of variations in α.

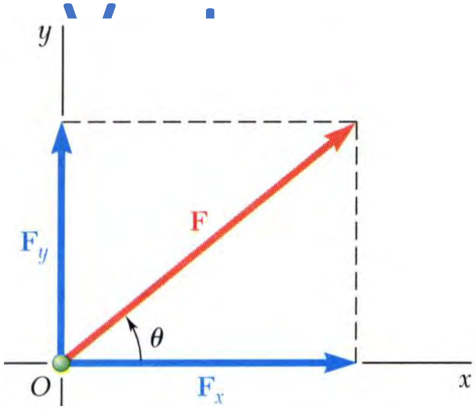
- The minimum tension in rope 2 occurs when $\mathbf{T_1}$ and $\mathbf{T_2}$ are perpendicular.

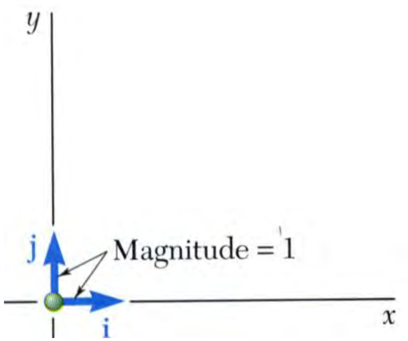
$$T_2 = (5000\,\text{lbf})\sin 30^\circ \qquad \boxed{T_2 = 2500\,\text{lbf}}$$

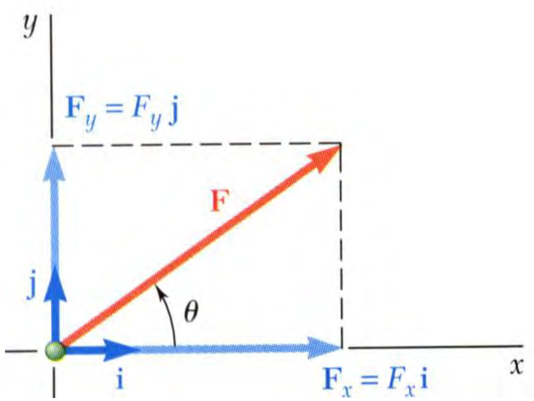
$$T_1 = (5000\,\text{lbf})\cos 30^\circ \qquad \boxed{T_1 = 4330\,\text{lbf}}$$

$$\alpha = 90^\circ - 30^\circ \qquad \boxed{\alpha = 60^\circ}$$

Rectangular Components of a Force: Unit







- May resolve a force vector into perpendicular components so that the resulting parallelogram is a rectangle. $\vec{F}_x$ and $\vec{F}_y$ are referred to as *rectangular vector components* and

$$\vec{F} = \vec{F}_x + \vec{F}_y$$

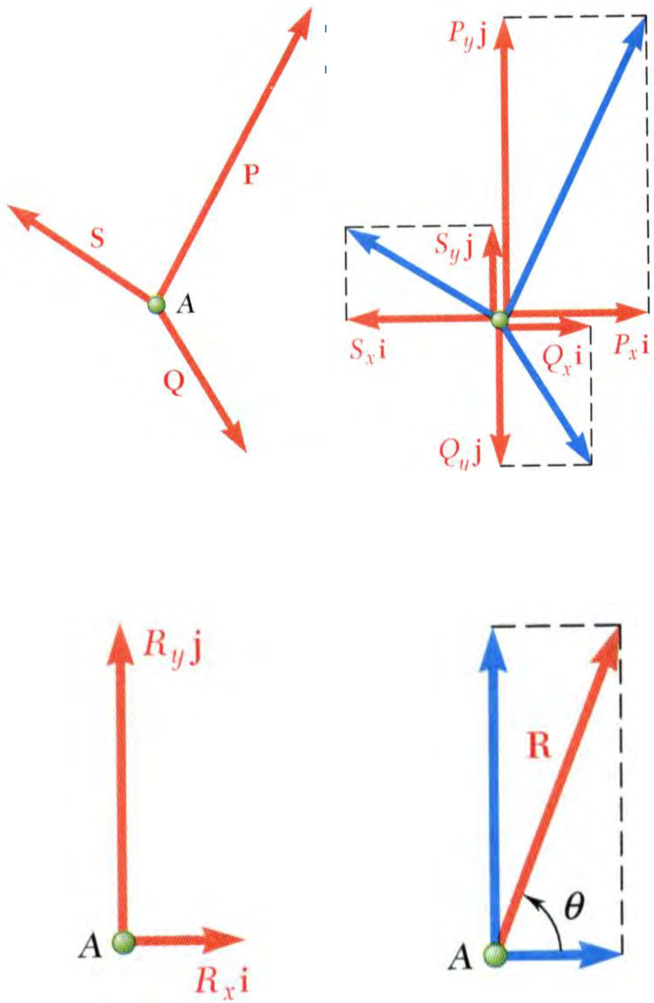
- Define perpendicular *unit vectors* $\vec{i}$ and $\vec{j}$ which are parallel to the *x* and *y* axes.

- Vector components may be expressed as products of the unit vectors with the scalar magnitudes of the vector components.

$$\vec{F} = F_x\vec{i} + F_y\vec{j}$$

F_x and F_y are referred to as the *scalar components* of $\vec{F}$

Addition of Forces by Summing



- Wish to find the resultant of 3 or more concurrent forces,

$$\vec{R} = \vec{P} + \vec{Q} + \vec{S}$$

- Resolve each force into rectangular components

$$\begin{aligned} R_x\vec{i} + R_y\vec{j} &= P_x\vec{i} + P_y\vec{j} + Q_x\vec{i} + Q_y\vec{j} + S_x\vec{i} + S_y\vec{j} \\ &= \left(P_x + Q_x + S_x\right)\vec{i} + \left(P_y + Q_y + S_y\right)\vec{j} \end{aligned}$$

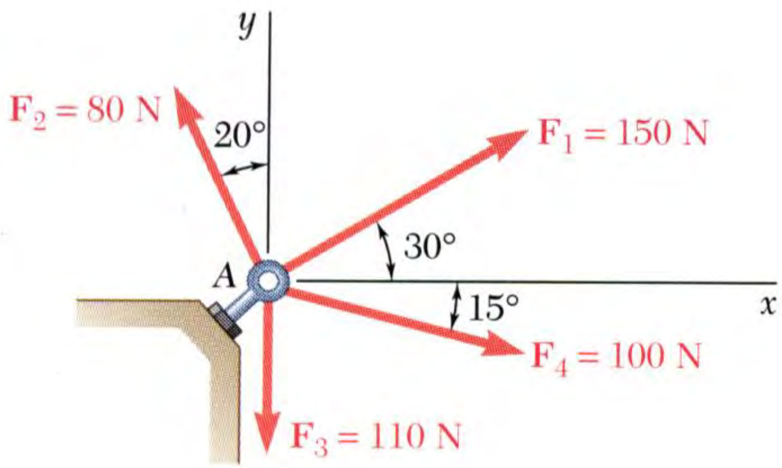
- The scalar components of the resultant are equal to the sum of the corresponding scalar components of the given forces.

$$R_x = P_x + Q_x + S_x = \sum F_x \qquad R_y = P_y + Q_y + S_y = \sum F_y$$

- To find the resultant magnitude and direction,

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$

Sample Problem

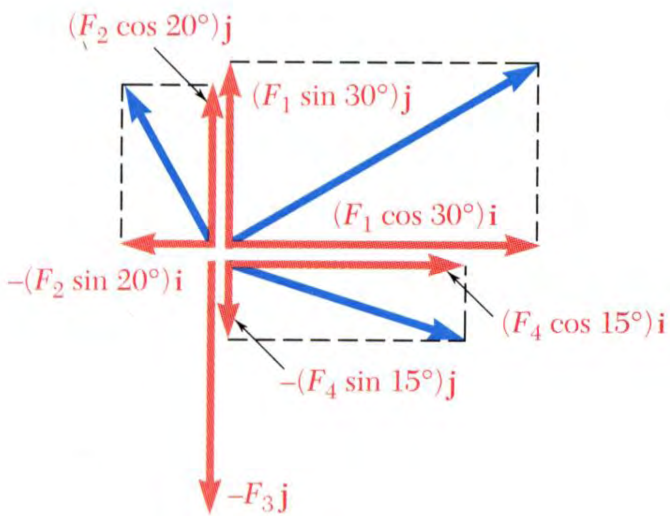


Four forces act on bolt *A* as shown. Determine the resultant of the force on the bolt.

SOLUTION:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

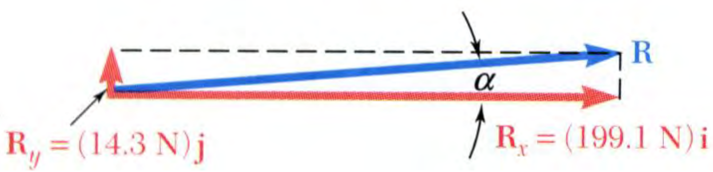
Sample Problem



SOLUTION:

- Resolve each force into rectangular components.

force	*mag*	$x-comp$	$y-comp$
$\vec{F}_1$	150	+129.9	+75.0
$\vec{F}_2$	80	−27.4	+75.2
$\vec{F}_3$	110	0	−110.0
$\vec{F}_4$	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$



- Determine the components of the resultant by adding the corresponding force components.

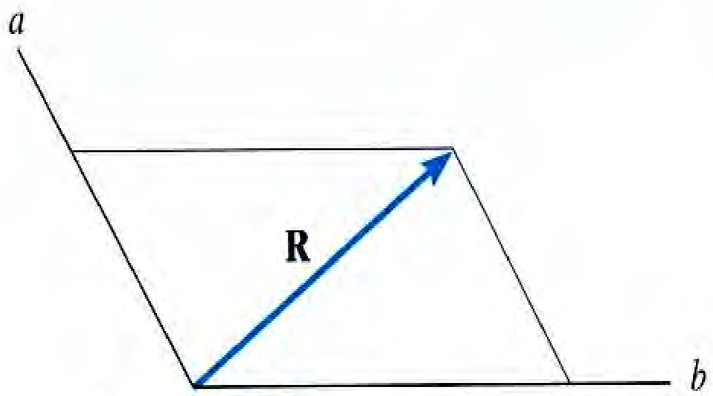
- Calculate the magnitude and direction.

$$R = \sqrt{199.1^2 + 14.3^2} \qquad \boxed{R = 199.6\,\mathrm{N}}$$

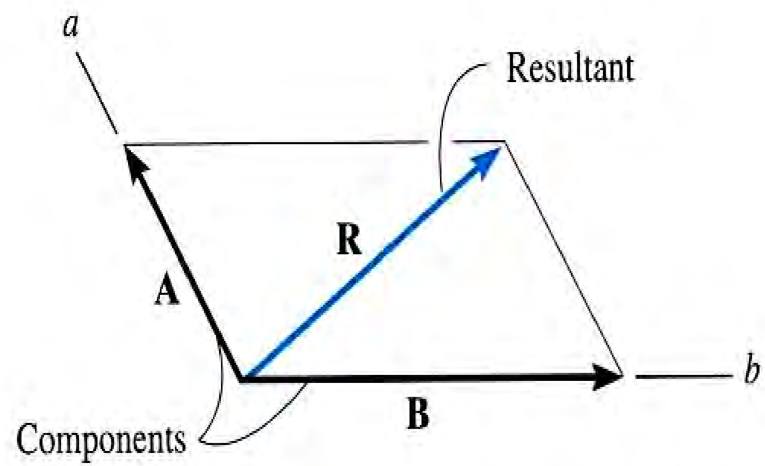
$$\tan\alpha = \frac{14.3\,\mathrm{N}}{199.1\,\mathrm{N}} \qquad \boxed{\alpha = 4.1°}$$

RESOLUTION OF A VECTOR

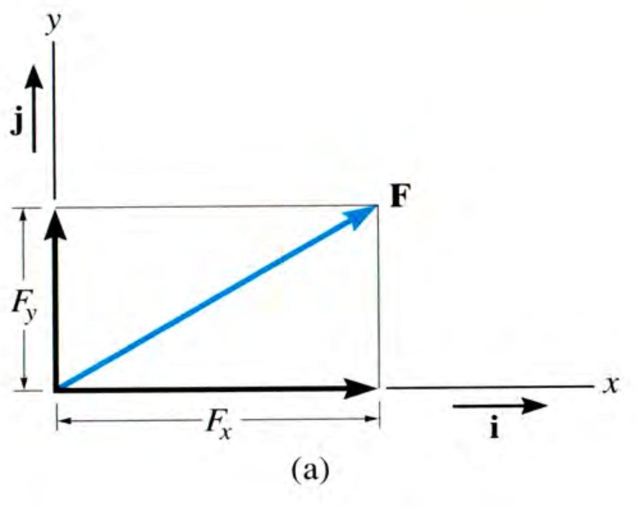
"Resolution" of a vector is breaking up a vector into components. It is kind of like using the parallelogram law in reverse.



Extend parallel lines from the head of **R** to form components



CARTESIAN VECTOR NOTATION

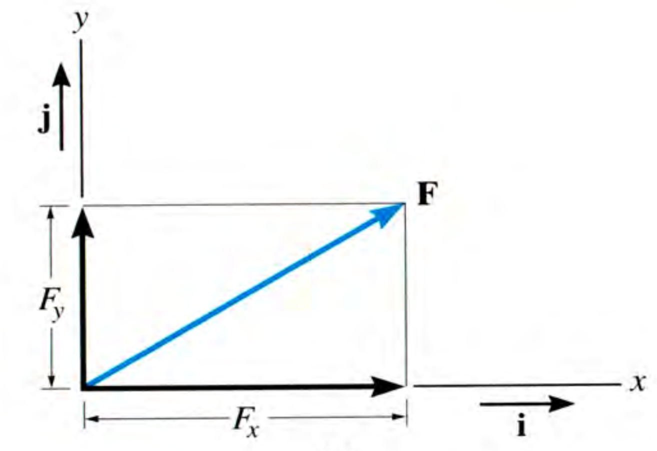


- We 'resolve' vectors into components using the x and y axes system.
- Each component of the vector is shown as a magnitude and a direction.

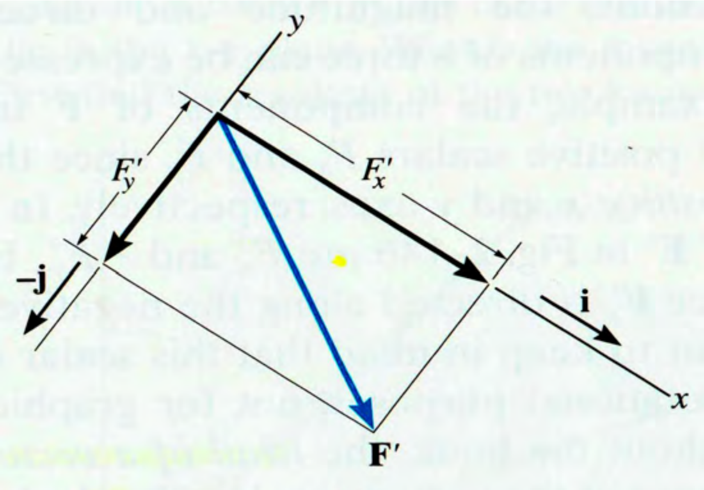
- The directions are based on the x and y axes. We use the "unit vectors" ***i*** and ***j*** to designate the x and y axes.

For example,

$\boldsymbol{F}$ = Fx $\boldsymbol{i}$ + Fy $\boldsymbol{j}$ ***or*** $\boldsymbol{F}$ = F'x $\boldsymbol{i}$ + F'y $\boldsymbol{j}$



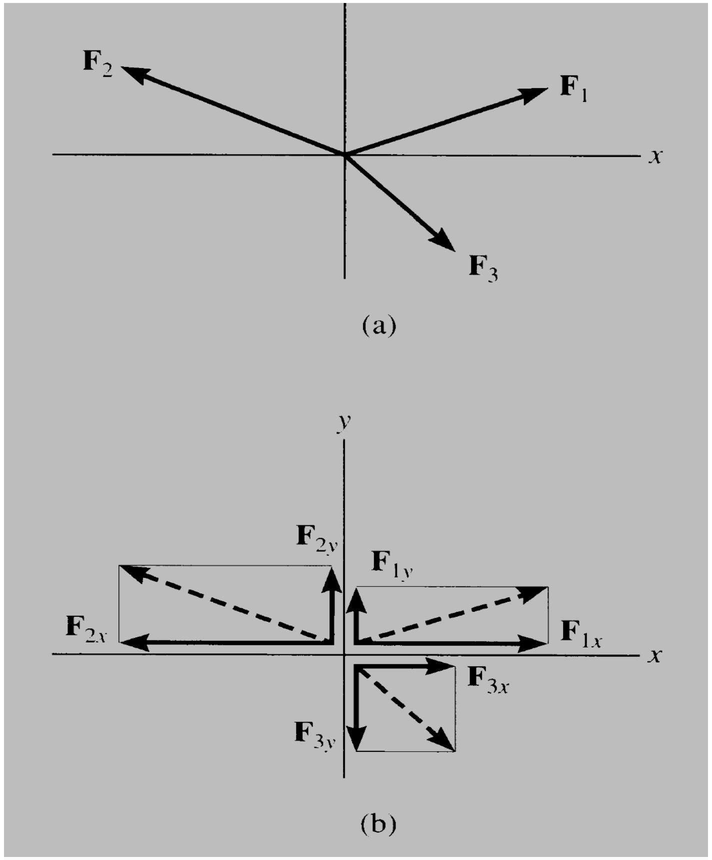
(a)



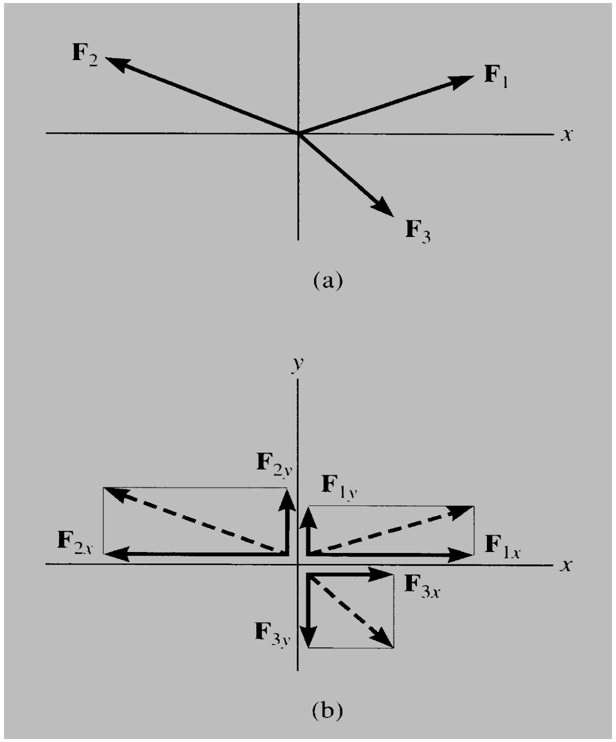
(b)

The x and y axes are always perpendicular to each other. Together,they can be directed at any inclination.

ADDITION OF SEVERAL VECTORS



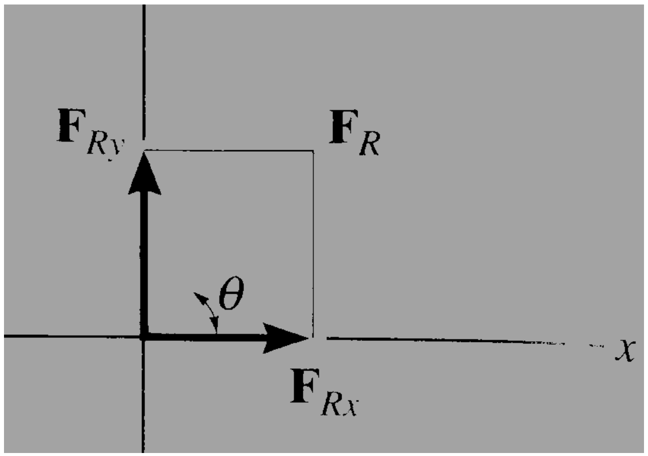
- Step 1 is to resolve each force into its components
- Step 2 is to add all the x components together and add all the y components together. These two totals become the resultant vector.
- Step 3 is to find the magnitude and angle of the resultant vector.



Example of this process,

$$
\begin{aligned}
\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\
&= F_{1x}\mathbf{i} + F_{1y}\mathbf{j} - F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{3x}\mathbf{i} - F_{3y}\mathbf{j} \\
&= (F_{1x} - F_{2x} + F_{3x})\mathbf{i} + (F_{1y} + F_{2y} - F_{3y})\mathbf{j} \\
&= (F_{Rx})\mathbf{i} + (F_{Ry})\mathbf{j}
\end{aligned}
$$

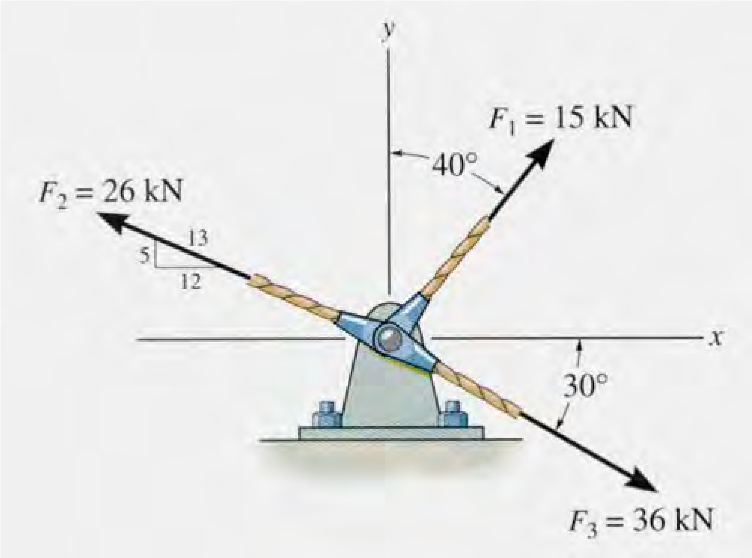
You can also represent a 2-D vector with a magnitude and angle.



$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

$$\theta = \tan^{-1}\left|\frac{F_{Ry}}{F_{Rx}}\right|$$

EXAMPLE



Given: Three concurrent forces acting on a bracket.

Find: The magnitude and angle of the resultant force.

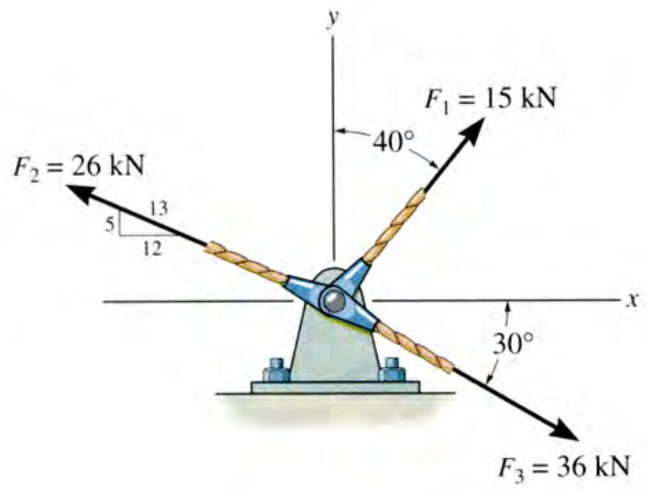
<u>Plan:</u>

a) Resolve the forces in their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

EXAMPLE (continued)



F1 = { 15 sin 40° ***i*** + 15 cos 40° ***j*** } kN

= { 9.642 ***i*** + 11.49 ***j*** } kN

F2 = { -(12/13)26 ***i*** + (5/13)26 ***j*** } kN

= { -24 ***i*** + 10 ***j*** } kN

F3 = { 36 cos 30° ***i*** – 36 sin 30° ***j*** } kN

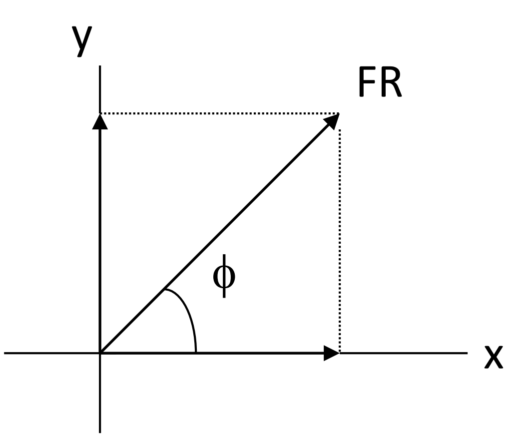
= { 31.18 ***i*** – 18 ***j*** } kN

EXAMPLE (continued)

Summing up all the ***i*** and ***j*** components respectively, we get,

FR = { (9.642 – 24 + 31.18) ***i*** + (11.49 + 10 – 18) ***j*** } kN

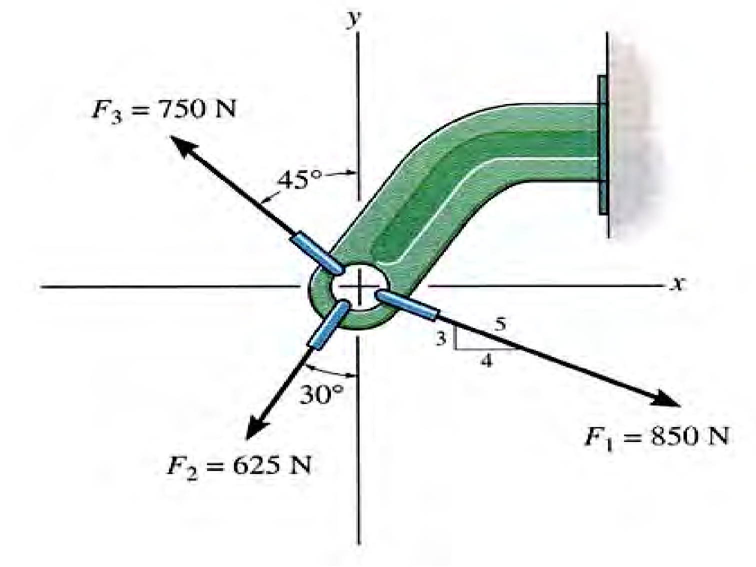
= { 16.82 ***i*** + 3.49 ***j*** } kN



$FR = ((16.82)^2 + (3.49)^2)^{1/2} = 17.2$ kN

$\phi = \tan^{-1}(3.49/16.82) = 11.7°$

Example



Given: Three concurrent forces acting on a bracket

Find: The magnitude and angle of the resultant force.

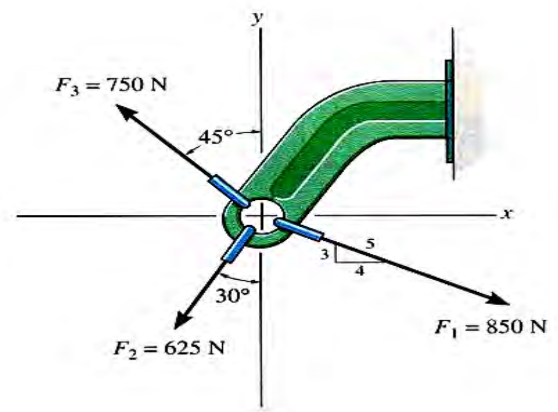
Plan:

a) Resolve the forces in their x-y components.

b) Add the respective components to get the resultant vector.

c) Find magnitude and angle from the resultant components.

Example (continued)



$\boldsymbol{F1} = \{ (4/5)\,850\, \boldsymbol{i} - (3/5)\,850\, \boldsymbol{j} \}$ N

$= \{ 680\, \boldsymbol{i} - 510\, \boldsymbol{j} \}$ N

$\boldsymbol{F2} = \{ -625 \sin(30°)\, \boldsymbol{i} - 625 \cos(30°)\, \boldsymbol{j} \}$ N

$= \{ -312.5\, \boldsymbol{i} - 541.3\, \boldsymbol{j} \}$ N

$\boldsymbol{F3} = \{ -750 \sin(45°)\, \boldsymbol{i} + 750 \cos(45°)\, \boldsymbol{j} \}$ N

$\{ -530.3\, \boldsymbol{i} + 530.3\, \boldsymbol{j} \}$ N

Example (continued)

Summing up all the ***i*** and ***j*** components respectively, we get,

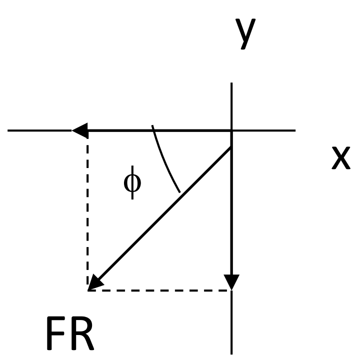
FR = { (680 – 312.5 – 530.3) ***i*** + (-510 – 541.3 + 530.3) ***j*** }N

= { - 162.8 ***i*** - 521 ***j*** } N

FR = $((162.8)^2 + (521)^2)^{½}$ = 546 N

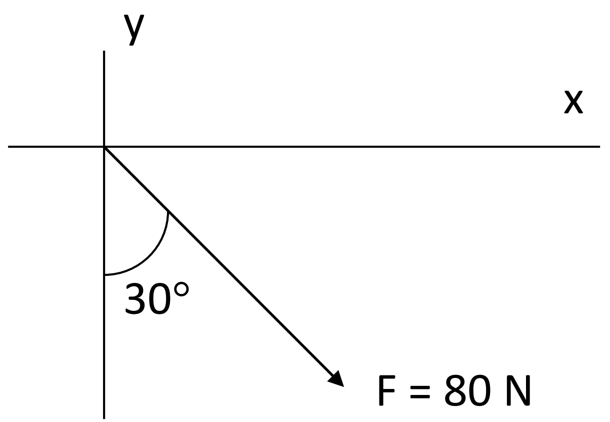
ϕ= $\tan^{-1}(521/162.8) = 72.64°$ or

From Positive x axis θ = 180 + 72.64 = 253 °



QUIZ

1. Resolve ***F*** along x and y axes and write it in vector form. ***F*** = { ______________ } N



A) 80 cos (30°) ***i*** - 80 sin (30°) ***j***

B) 80 sin (30°) ***i*** + 80 cos (30°) ***j***

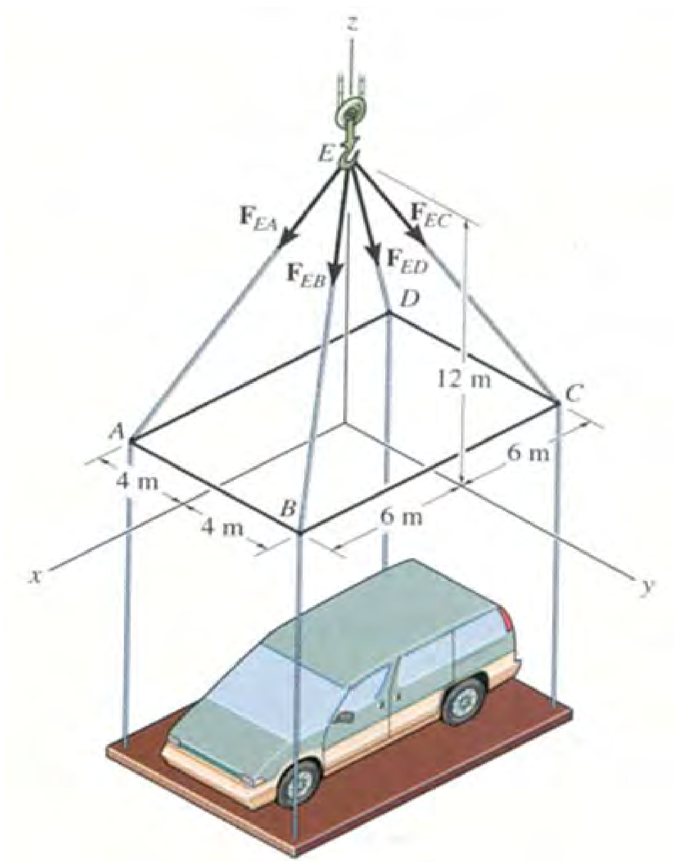
C) 80 sin (30°) ***i*** - 80 cos (30°) ***j***

D) 80 cos (30°) ***i*** + 80 sin (30°) ***j***

2. Determine the magnitude of the resultant (***F1*** + ***F2***) force in N when ***F1*** = { 10 ***i*** + 20 ***j*** } N and ***F2*** = { 20 ***i*** + 20 ***j*** } N .

A) 30 N B) 40 N C) 50 N

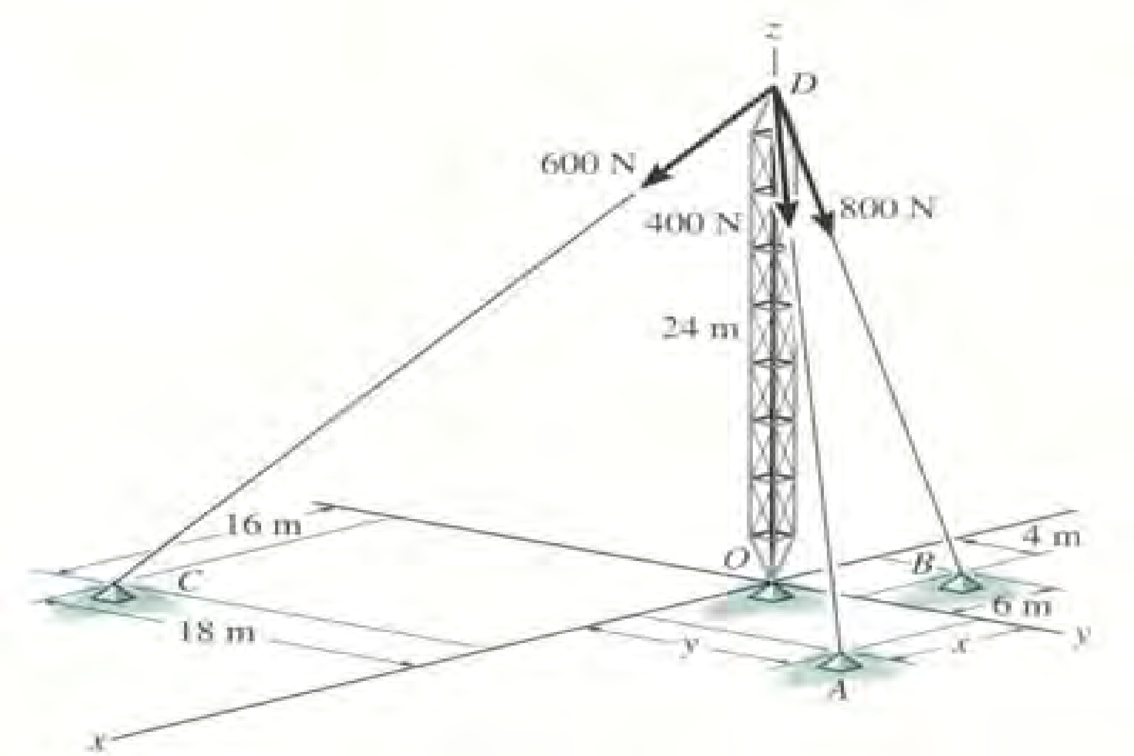
D) 60 N E) 70 N



Many problems in real-life involve 3-Dimensional Space.

How will you represent each of the cable forces in Cartesian vector form?

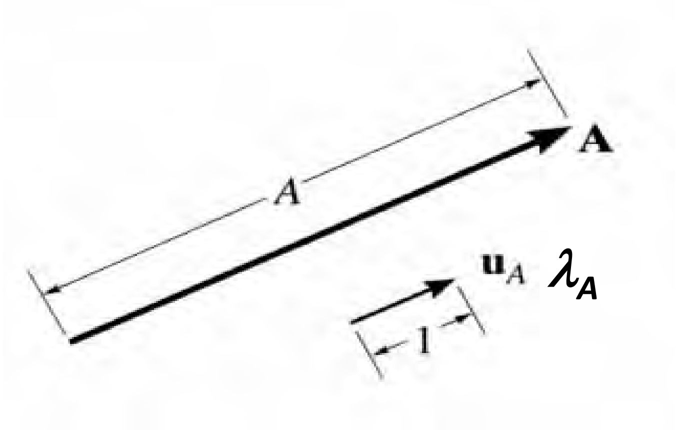
Given the forces in the cables, how will you determine the resultant force acting at D, the top of the tower?



A UNIT VECTOR

For a vector ***A*** with a magnitude of A, an unit vector is defined as

$\boldsymbol{\lambda_A} = \boldsymbol{U}_A = \boldsymbol{A}$ / A .



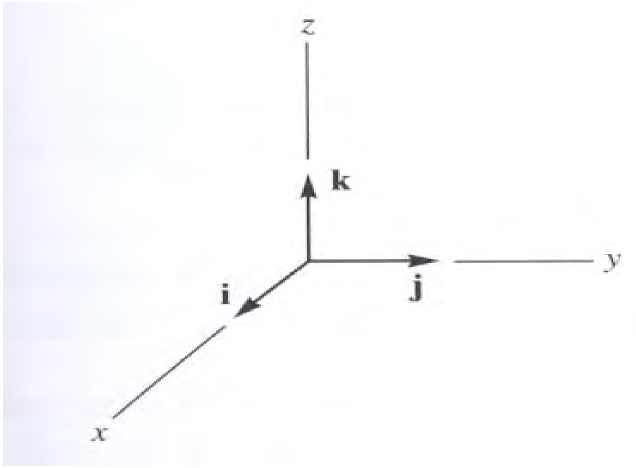
Characteristics of a unit vector:

a) Its magnitude is 1.

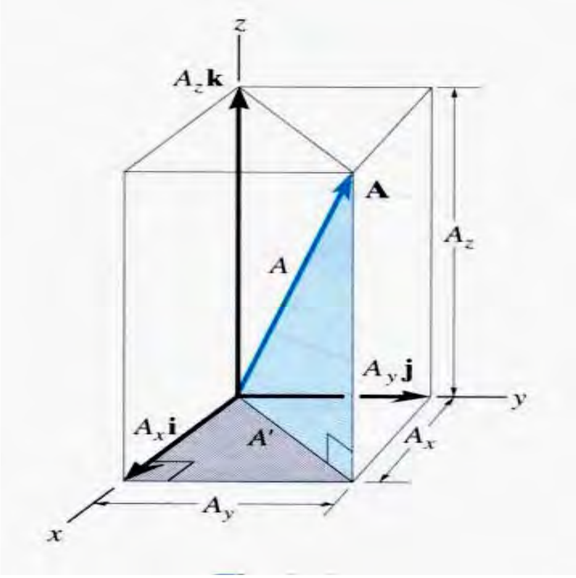
b) It is dimensionless.

c) It points in the same direction as the original vector (***A***).

The unit vectors in the Cartesian axis system are ***i***, ***j***, and ***k***. They are unit vectors along the positive x, y, and z axes respectively.



3-D CARTESIAN VECTOR TERMINOLOGY



Consider a box with sides A_X, A_Y, and A_Z meters long.

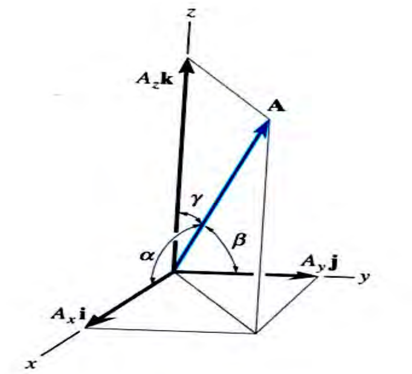
The vector ***A*** can be defined as ***A* = (**A_X ***i*** **+** A_Y ***j*** **+** A_Z ***k*)** m

The projection of the vector ***A*** in the x-y plane is A´. The magnitude of this projection, A´, is found by using the same approach as a 2-D vector: $A' = (A_X^2 + A_Y^2)^{1/2}$.

The magnitude of the position vector ***A*** can now be obtained as

$$A = ((A')^2 + A_Z^2)^{½} = (A_X^2 + A_Y^2 + A_Z^2)^{½}$$

3-D CARTESIAN VECTOR TERMINOLOGY (continued)



The direction or orientation of vector ***A*** is defined by the angles α, β, and γ.

These angles are measured between the vector and the positive X, Y and Z axes, respectively. Their range of values are from 0° to 180°

Using trigonometry, "direction cosines" are found using the formulas

$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

These angles are not independent. They must satisfy the following equation.

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

This result can be derived from the definition of a coordinate direction angles and the unit vector. Recall, the formula for finding the unit vector of any position vector:

$$\lambda_A = \mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\mathbf{i} + \frac{A_y}{A}\mathbf{j} + \frac{A_z}{A}\mathbf{k}$$

or written another way, $\lambda_A = u_A = \cos\alpha\, \boldsymbol{i} + \cos\beta\, \boldsymbol{j} + \cos\gamma\, \boldsymbol{k}$.

ADDITION/SUBTRACTION OF VECTORS

Once individual vectors are written in Cartesian form, it is easy to add or subtract them. The process is essentially the same as when 2-D vectors are added.

For example, if

$\boldsymbol{A} = A_X\, \boldsymbol{i} + A_Y\, \boldsymbol{j} + A_Z\, \boldsymbol{k}$ and

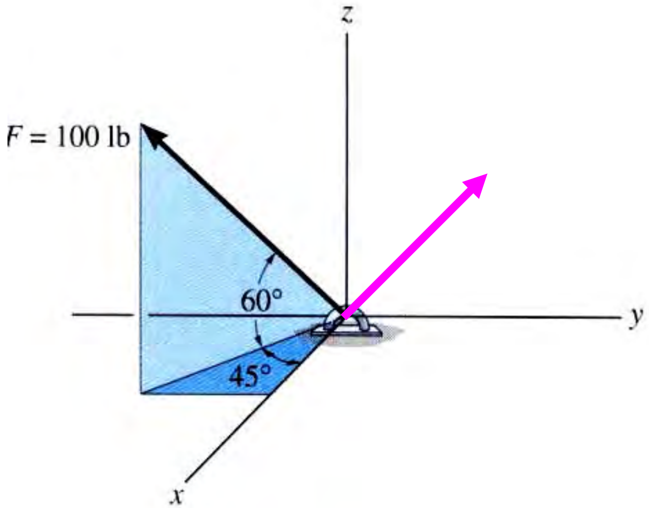
$\boldsymbol{B} = B_X\, \boldsymbol{i} + B_Y\, \boldsymbol{j} + B_Z\, \boldsymbol{k}$, then

$$\boldsymbol{A} + \boldsymbol{B} = (A_X + B_X)\, \boldsymbol{i} + (A_Y + B_Y)\, \boldsymbol{j} + (A_Z + B_Z)\, \boldsymbol{k}$$

or

$$\boldsymbol{A} - \boldsymbol{B} = (A_X - B_X)\, \boldsymbol{i} + (A_Y - B_Y)\, \boldsymbol{j} + (A_Z - B_Z)\, \boldsymbol{k}\ .$$

EXAMPLE



Given: Two forces ***F*** and ***G*** are applied to a hook. Force ***F*** is shown in the figure and it makes 60° angle with the X-Y plane. Force ***G*** is pointing up and has a magnitude of 80 lb with α = 111° and β = 69.3°.

Find: The resultant force in the Cartesian vector form.

Plan:

1) Using geometry and trigonometry, write ***F*** and ***G*** in the Cartesian vector form.
2) Then add the two forces.

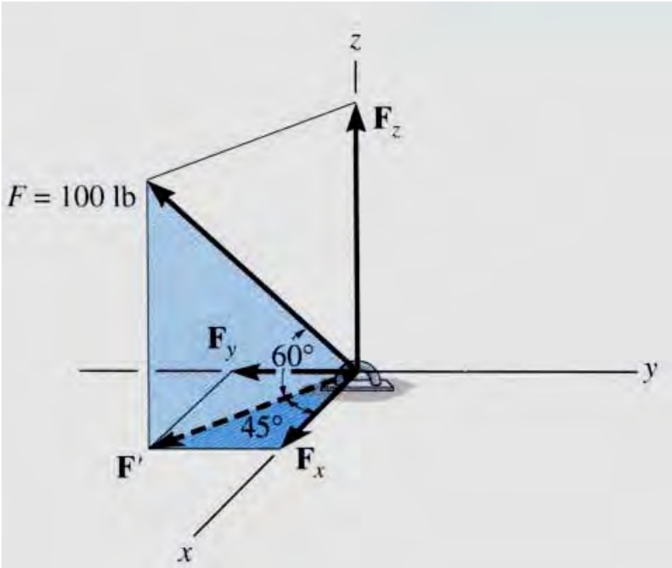
Solution : First, resolve force ***F***.

$F_z = 100 \sin 60° = 86.60$ lb

$F' = 100 \cos 60° = 50.00$ lb

$F_x = 50 \cos 45° = 35.36$ lb

$F_y = 50 \sin 45° = 35.36$ lb



Now, you can write:

$\boldsymbol{F} = \{35.36\,\boldsymbol{i} - 35.36\,\boldsymbol{j} + 86.60\,\boldsymbol{k}\}$ lb