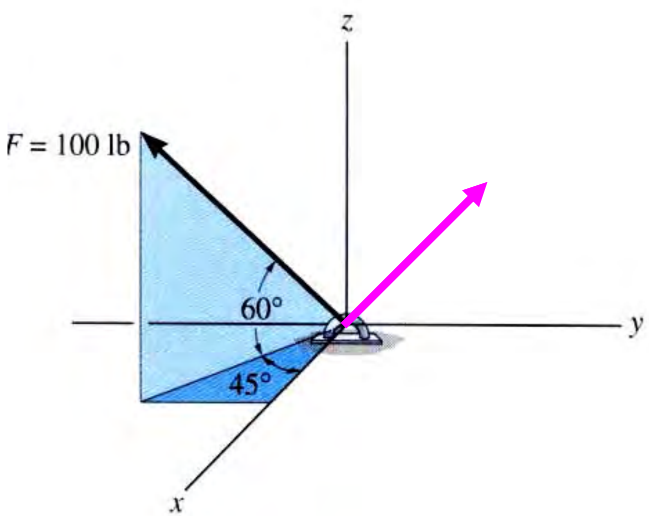
EXAMPLE



Given: Two forces ***F*** and ***G*** are applied to a hook. Force ***F*** is shown in the figure and it makes 60° angle with the X-Y plane. Force ***G*** is pointing up and has a magnitude of 80 lb with α = 111° and β = 69.3°.

Find: The resultant force in the Cartesian vector form.

Plan:

1) Using geometry and trigonometry, write ***F*** and ***G*** in the Cartesian vector form.
2) Then add the two forces.

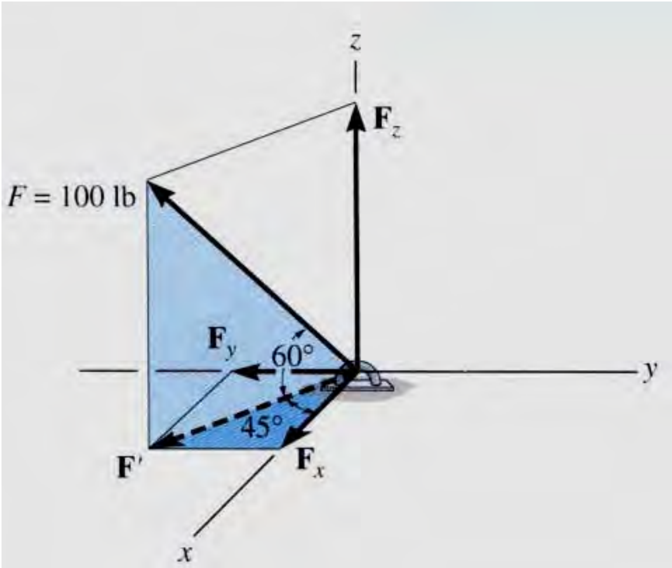
Solution : First, resolve force ***F***.

$F_z = 100 \sin 60° = 86.60$ lb

$F' = 100 \cos 60° = 50.00$ lb

$F_x = 50 \cos 45° = 35.36$ lb

$F_y = 50 \sin 45° = 35.36$ lb



Now, you can write:

F = {35.36 ***i*** – 35.36 ***j*** + 86.60 ***k***} lb

Now resolve force ***G***.

We are given only α and β. Hence, first we need to find the value of γ.

Recall the formula $\cos^2(\alpha) + \cos^2(\beta) + \cos^2(\gamma) = 1$.

Now substitute what we know. We have

$\cos^2(111°) + \cos^2(69.3°) + \cos^2(\gamma) = 1$.

Solving, we get $\gamma = 30.22°$ or $120.2°$. Since the vector is pointing up, $\gamma = 30.22°$

Now using the coordinate direction angles, we can get $\boldsymbol{U_{G}}$, and determine $\boldsymbol{G} = 80\ \boldsymbol{U_G}$ lb.

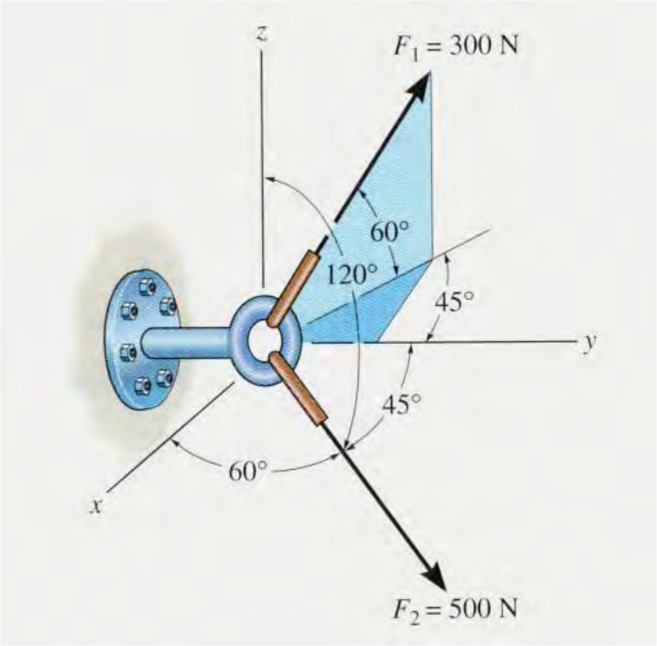
$\boldsymbol{G} = \{80\ (\cos(111°)\ \boldsymbol{i} + \cos(69.3°)\ \boldsymbol{j} + \cos(30.22°)\ \boldsymbol{k})\}$ lb

$\boldsymbol{G} = \{-28.67\ \boldsymbol{i} + 28.28\ \boldsymbol{j} + 69.13\ \boldsymbol{k}\}$ lb

Now, $\boldsymbol{R} = \boldsymbol{F} + \boldsymbol{G}$ **or**

$\boldsymbol{R} = \{6.69\ \boldsymbol{i} - 7.08\ \boldsymbol{j} + 156\ \boldsymbol{k}\}$ lb

Example



Given: The screw eye is subjected to two forces.

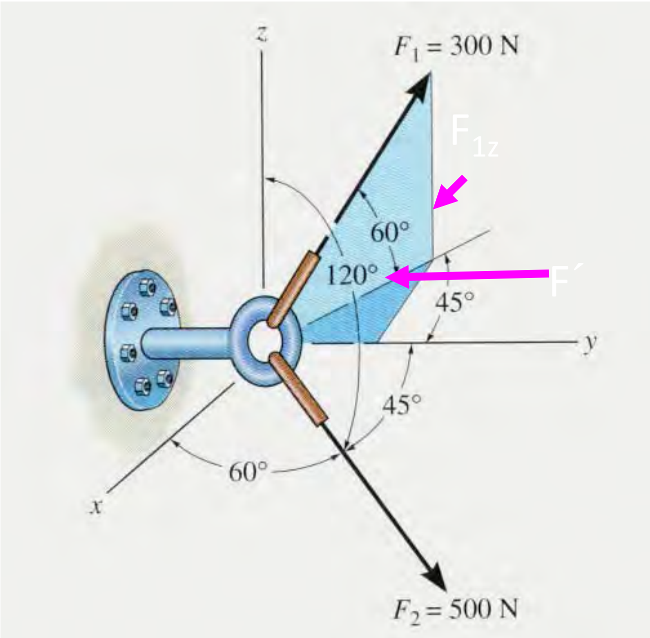
Find: The magnitude and the coordinate direction angles of the resultant force.

Plan:

1) Using the geometry and trigonometry, write $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ in the Cartesian vector form.

2) Add $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ to get $\boldsymbol{F_R}$.

3) Determine the magnitude and α, β, γ .



First resolve the force $\boldsymbol{F_1}$.

$F_{1z} = 300 \sin 60° = 259.8$ N

$F´ = 300 \cos 60° = 150.0$ N

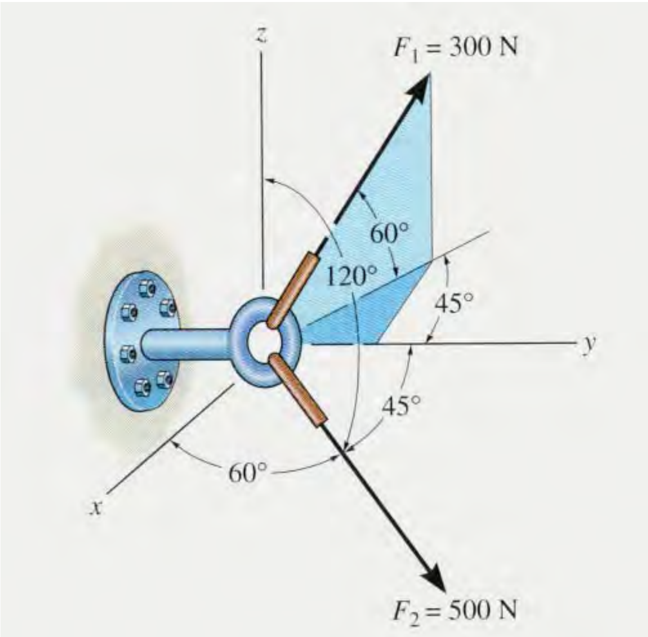
F’ can be further resolved as,

$F_{1x} = -150 \sin 45° = -106.1$ N

$F_{1y} = 150 \cos 45° = 106.1$ N

Now we can write :

$\boldsymbol{F_1} = \{-106.1\, \boldsymbol{i} + 106.1\, \boldsymbol{j} + 259.8\, \boldsymbol{k}\}$ N



The force $\boldsymbol{F_2}$ can be represented in the Cartesian vector form as:

$$\boldsymbol{F_2} = 500\{ \cos 60° \, \boldsymbol{i} + \cos 45° \, \boldsymbol{j} + \cos 120° \, \boldsymbol{k} \} \text{ N}$$

$$= \{ 250 \, \boldsymbol{i} + 353.6 \, \boldsymbol{j} - 250 \, \boldsymbol{k} \} \text{ N}$$

$$\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2}$$

$$= \{ 143.9 \, \boldsymbol{i} + 459.6 \, \boldsymbol{j} + 9.81 \, \boldsymbol{k} \} \text{ N}$$

$$F_R = (143.9^2 + 459.6^2 + 9.81^2)^{½} = 481.7 = 482 \text{ N}$$

$$\alpha = \cos^{-1} (F_{Rx} / F_R) = \cos^{-1} (143.9/481.7) = 72.6°$$

$$\beta = \cos^{-1} (F_{Ry} / F_R) = \cos^{-1} (459.6/481.7) = 17.4°$$

$$\gamma = \cos^{-1} (F_{Rz} / F_R) = \cos^{-1} (9.81/481.7) = 88.8°$$

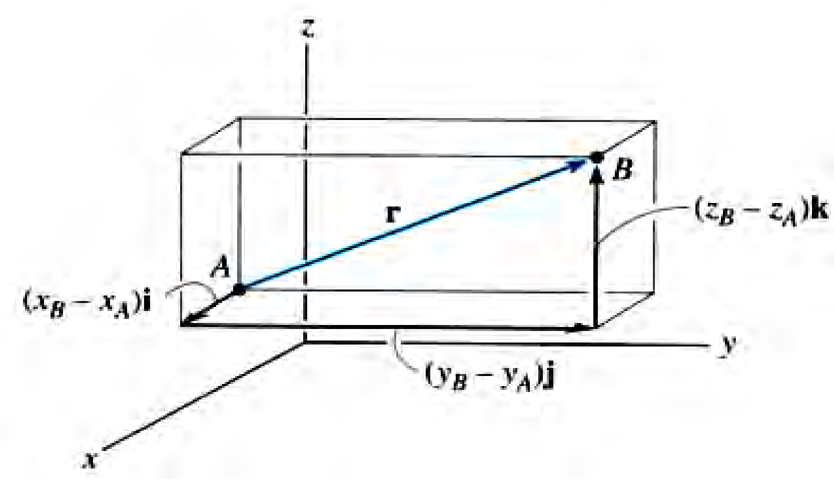
APPLICATIONS

Wing strut

How can we represent the force along the wing strut in a 3-D Cartesian vector form?

POSITION VECTOR

A position vector is defined as a fixed vector that locates a point in space relative to another point.



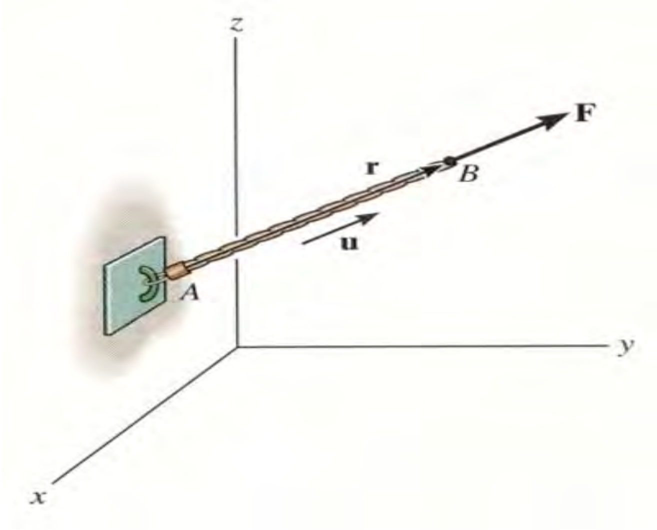
Consider two points, A & B, in 3-D space. Let their coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.

The position vector directed from A to B, $\boldsymbol{r}_{AB}$, is defined as

$\boldsymbol{r}_{AB} = \{(X_B - X_A)\,\boldsymbol{i} + (Y_B - Y_A)\,\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}\text{m}$

Please note that B is the ending point and A is the starting point. So ALWAYS subtract the “tail” coordinates from the “tip” coordinates!

FORCE VECTOR DIRECTED ALONG A LINE



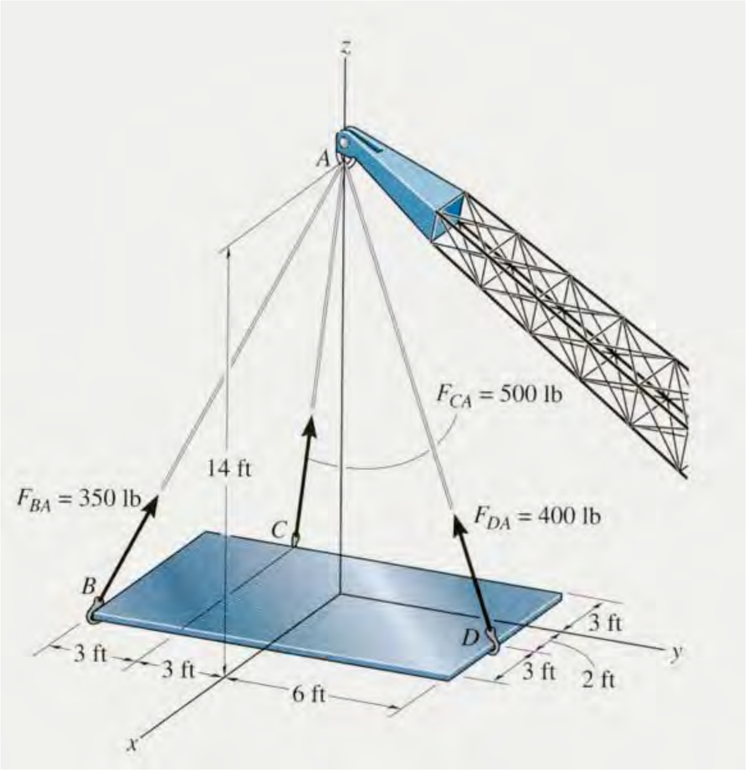
If a force is directed along a line, then we can represent the force vector in Cartesian Coordinates by using a unit vector and the force magnitude. So we need to:

a) Find the position vector, $\boldsymbol{r}_{AB}$, along two points on that line.

b) Find the unit vector describing the line's direction, $\boldsymbol{\lambda}_{AB} = (\boldsymbol{r}_{AB}/\mathrm{r}_{AB})$.

c) Multiply the unit vector by the magnitude of the force, $\boldsymbol{F} = \mathrm{F}\,\boldsymbol{\lambda}_{AB}$.

EXAMPLE



Given: 400 lb force along the cable DA.

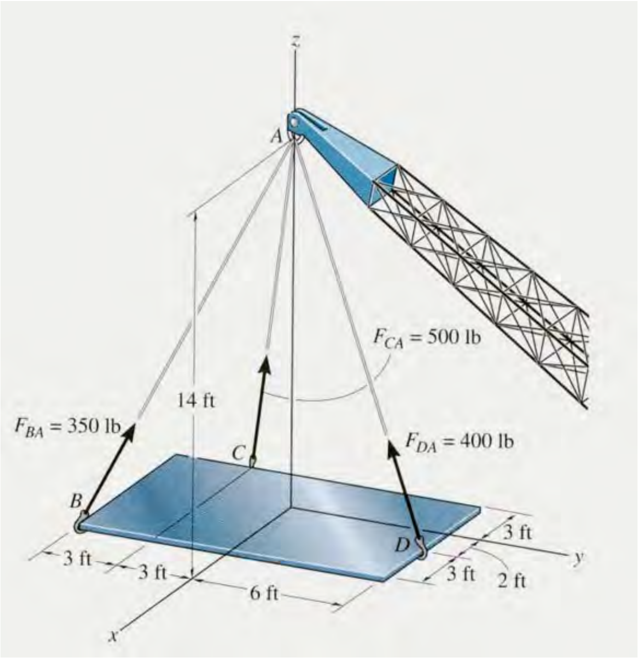
Find: The force $\boldsymbol{F_{DA}}$ in the Cartesian vector form.

<u>Plan</u>:

- Find the position vector $\boldsymbol{r_{DA}}$ and the unit vector $\boldsymbol{\lambda_{DA}}$.

2. Obtain the force vector as $\boldsymbol{F_{DA}} = 400 \text{ lb } \boldsymbol{\lambda_{DA}}$.

EXAMPLE (continued)



The figure shows that when relating D to A, we will have to go -2 ft in the x-direction, -6 ft in the y-direction, and +14 ft in the z-direction. Hence,

$\boldsymbol{r}_{DA}$ = {-2 $\boldsymbol{i}$ – 6 $\boldsymbol{j}$ + 14 $\boldsymbol{k}$} ft.

We can also find $\boldsymbol{r}_{DA}$ by subtracting the coordinates of D from the coordinates of A.

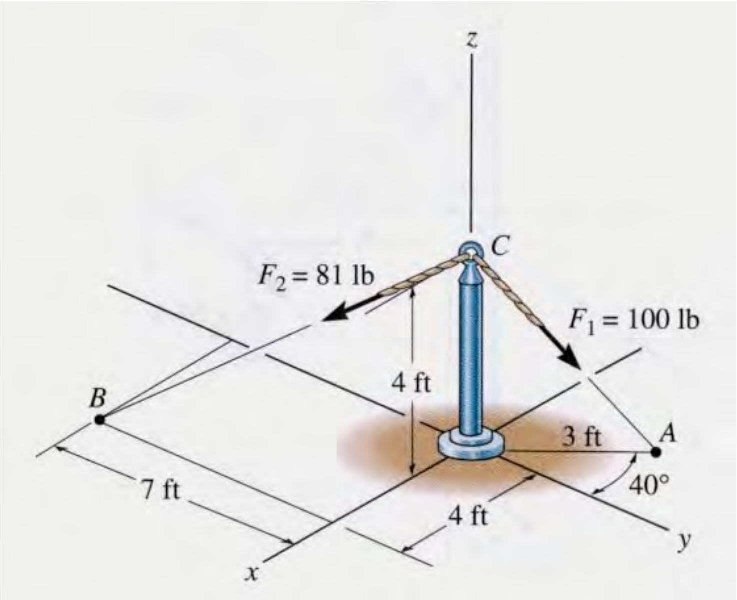
$r_{DA} = (2^2 + 6^2 + 14^2)^{0.5} = 15.36$ ft

$\boldsymbol{\lambda}_{DA} = \boldsymbol{r}_{DA}/r_{DA}$ and $\boldsymbol{F}_{DA} = 400\,\boldsymbol{\lambda}_{DA}$ lb

$\boldsymbol{F}_{DA} = 400\{(-2\,\boldsymbol{i} - 6\,\boldsymbol{j} + 14\,\boldsymbol{k})/15.36\}$ lb

$= \{-52.1\,\boldsymbol{i} - 156\,\boldsymbol{j} + 365\,\boldsymbol{k}\}$ lb

GROUP PROBLEM SOLVING



Given: Two forces are acting on a pipe as shown in the figure.

Find: The magnitude and the coordinate direction angles of the resultant force.

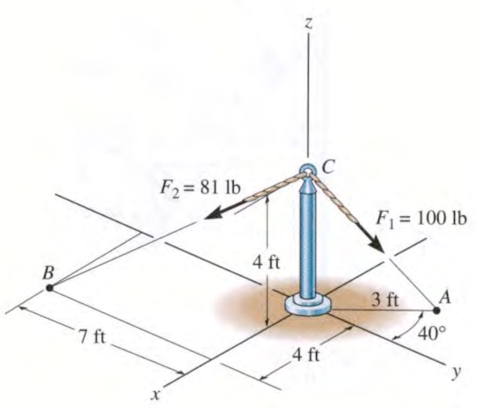
<u>Plan</u>:

- Find the forces along CA and CB in the Cartesian vector form.

2) Add the two forces to get the resultant force, $\boldsymbol{F_R}$.

3) Determine the magnitude and the coordinate angles of $\boldsymbol{F_R}$.

GROUP PROBLEM SOLVING (continued)



$\boldsymbol{F_{CA}}$ = 100 lb{$\boldsymbol{r_{CA}}/r_{CA}$}

$\boldsymbol{F_{CA}}$ = 100 lb(-3 sin 40° $\boldsymbol{i}$ + 3 cos 40° $\boldsymbol{j}$ – 4 $\boldsymbol{k}$)/5

$\boldsymbol{F_{CA}}$ = {-38.57 $\boldsymbol{i}$ + 45.96 $\boldsymbol{j}$ – 80 $\boldsymbol{k}$} lb

$\boldsymbol{F_{CB}}$ = 81 lb{$\boldsymbol{r_{CB}}/r_{CB}$}

$\boldsymbol{F_{CB}}$ = 81 lb(4 $\boldsymbol{i}$ – 7 $\boldsymbol{j}$ – 4 $\boldsymbol{k}$)/9

$\boldsymbol{F_{CB}}$ = {36 $\boldsymbol{i}$ – 63 $\boldsymbol{j}$ – 36 $\boldsymbol{k}$} lb

$\boldsymbol{F_R}$ = $\boldsymbol{F_{CA}}$ + $\boldsymbol{F_{CB}}$ = {-2.57 $\boldsymbol{i}$ – 17.04 $\boldsymbol{j}$ – 116 $\boldsymbol{k}$} lb

F_R = $(2.57^2 + 17.04^2 + 116^2)$ = 117.3 lb = 117 lb

$\alpha = \cos^{-1}(-2.57/117.3) = 91.3°$, $\beta = \cos^{-1}(-17.04/117.3) = 98.4°$

$\gamma = \cos^{-1}(-116/117.3) = 172°$

QUIZ

1. Two points in 3 – D space have coordinates of P (1, 2, 3) and Q (4, 5, 6) meters. The position vector $\boldsymbol{r_{QP}}$ is given by

A) {3 $\boldsymbol{i}$ + 3 $\boldsymbol{j}$ + 3 $\boldsymbol{k}$} m

B) {- 3 $\boldsymbol{i}$ – 3 $\boldsymbol{j}$ – 3 $\boldsymbol{k}$} m

C) {5 $\boldsymbol{i}$ + 7 $\boldsymbol{j}$ + 9 $\boldsymbol{k}$} m

D) {- 3 $\boldsymbol{i}$ + 3 $\boldsymbol{j}$ + 3 $\boldsymbol{k}$} m

E) {4 $\boldsymbol{i}$ + 5 $\boldsymbol{j}$ + 6 $\boldsymbol{k}$} m

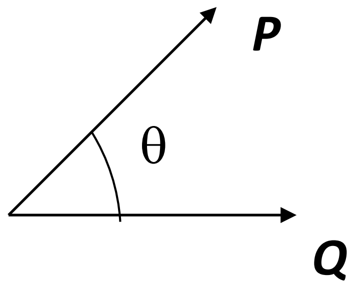
2. Force vector, ***F,*** directed along a line PQ is given by

A) $(\boldsymbol{F}/ F)\, \boldsymbol{r_{PQ}}$ B) $\boldsymbol{r_{PQ}}/r_{PQ}$

C) $F(\boldsymbol{r_{PQ}}/r_{PQ})$ D) $F(r_{PQ}/\boldsymbol{r_{PQ}})$

QUIZ

1. The dot product of two vectors ***P*** and ***Q*** is defined as



A) P Q cos θ B) P Q sin θ

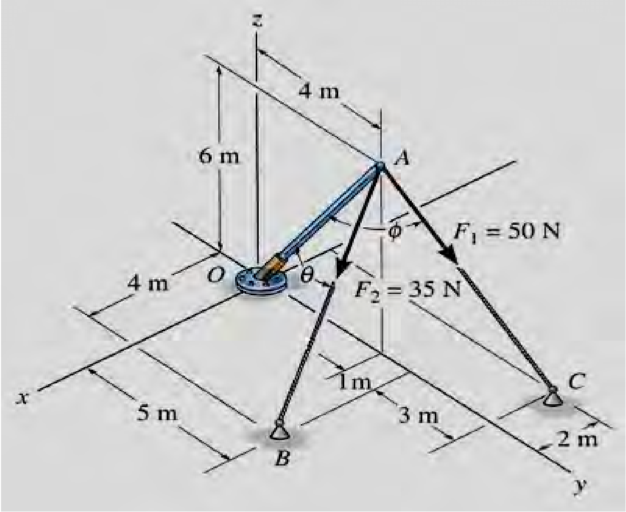
C) P Q tan θ D) P Q sec θ

2. The dot product of two vectors results in a __________ quantity.

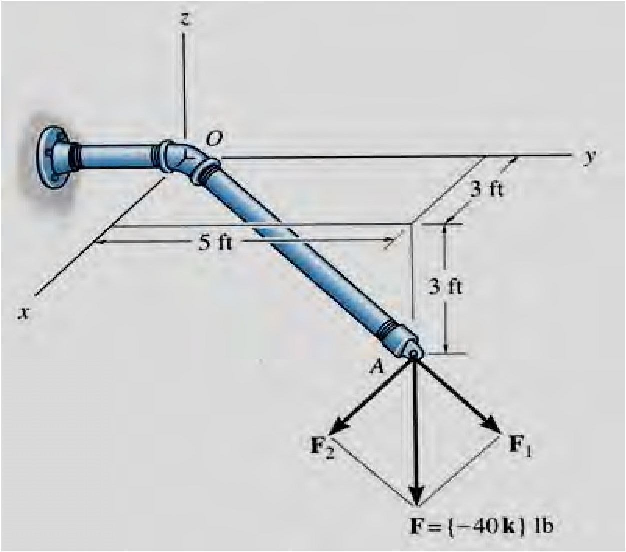
A) scalar B) vector

C) complex D) zero

DOT PRODUCT

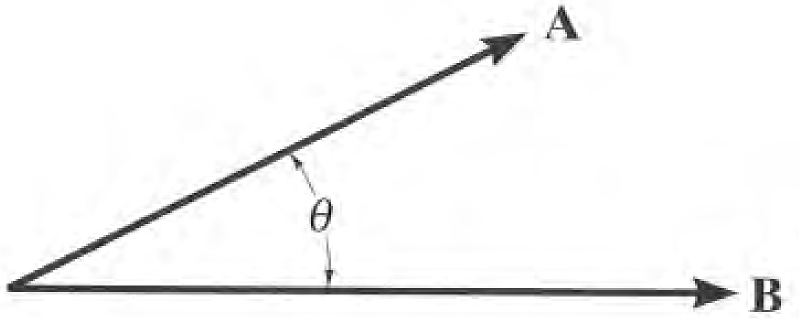


For this geometry, can you determine angles between the pole and the cables?



For force ***F*** at Point A, what component of it (F_1) acts along the pipe OA? What component (F_2) acts perpendicular to the pipe?

DEFINITION



The dot product of vectors ***A*** and ***B*** is defined as ***A•B*** = A B cos θ.

Angle θ is the smallest angle between the two vectors and is always in a range of 0º to 180º.

Dot Product Characteristics:

1. The result of the dot product is a scalar (a positive or negative number).
2. The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors.

DEFINITION

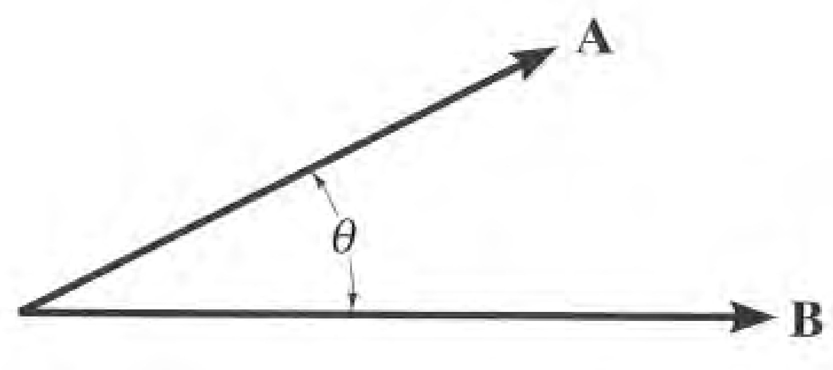
Examples: $\boldsymbol{i} \bullet \boldsymbol{j} = 0$

$\boldsymbol{i} \bullet \boldsymbol{i} = 1$

$$\boldsymbol{A} \bullet \boldsymbol{B} = (A_x \boldsymbol{i} + A_y \boldsymbol{j} + A_z \boldsymbol{k}) \bullet (B_x \boldsymbol{i} + B_y \boldsymbol{j} + B_z \boldsymbol{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

USING THE DOT PRODUCT TO DETERMINE THE ANGLE BETWEEN TWO VECTORS



For the given two vectors in the Cartesian form, one can find the angle by

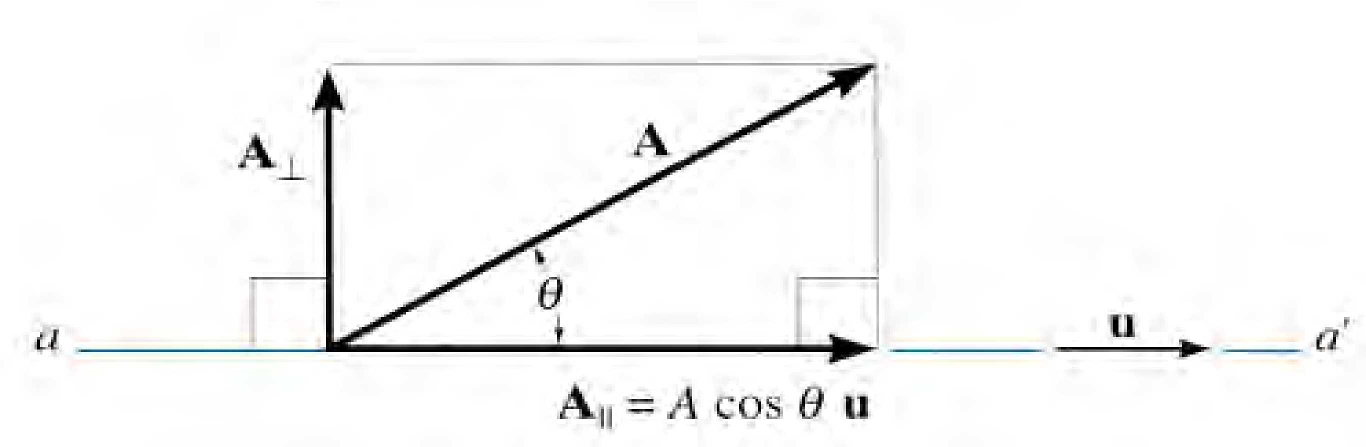
a) Finding the dot product, $\boldsymbol{A} \bullet \boldsymbol{B} = (A_x B_x + A_y B_y + A_z B_z)$,

b) Finding the magnitudes (A & B) of the vectors ***A*** & ***B***, and

c) Using the definition of dot product and solving for θ, i.e.,

$\theta = \cos^{-1} [(\boldsymbol{A} \bullet \boldsymbol{B})/(A\ B)]$, where $0^\circ \leq \theta \leq 180^\circ$.

DETERMINING THE PROJECTION OF A VECTOR



You can determine the components of a vector parallel and perpendicular to a line using the dot product.

Steps:

1. Find the unit vector, $\boldsymbol{U}_{aa'}$ along line aa´

2. Find the scalar projection of $\boldsymbol{A}$ along line aa´ by

$$A_{\parallel} = \boldsymbol{A} \bullet \boldsymbol{U} = A_x U_x + A_y U_y + A_z U_z$$

DETERMINING THE PROJECTION OF A VECTOR (continued)

3. If needed, the projection can be written as a vector, $\boldsymbol{A}_{||}$, by using the unit vector $\boldsymbol{U}_{aa'}$ and the magnitude found in step 2.

$$\boldsymbol{A}_{||} = A_{||}\,\boldsymbol{U}_{aa'}$$

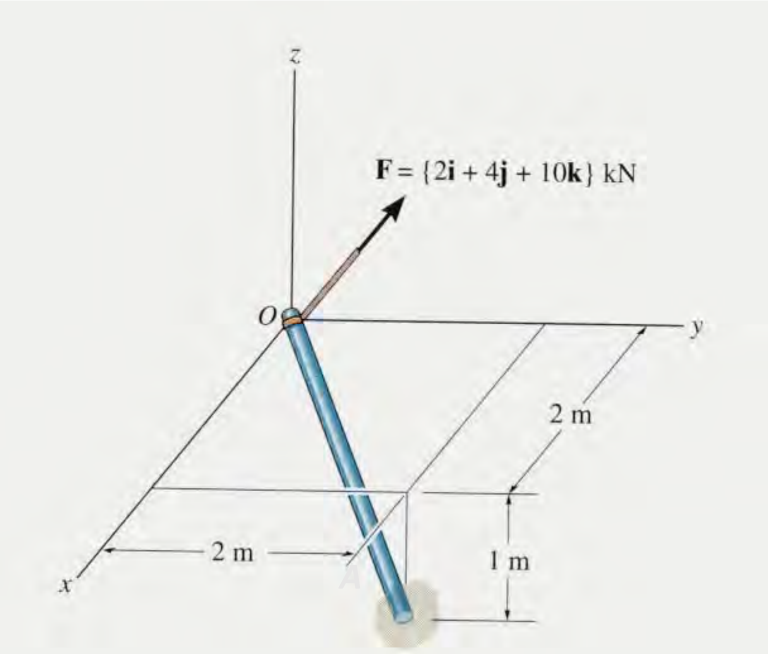
4. The scalar and vector forms of the perpendicular component can easily be obtained by

$$A_{\perp} = (A^2 - A_{||}^{\,2})^{1/2} \text{ and}$$

$$\boldsymbol{A}_{\perp} = \boldsymbol{A} - \boldsymbol{A}_{||}$$

(rearranging the vector sum of $\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{||}$)

EXAMPLE



Given: The force acting on the pole

Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole OA.

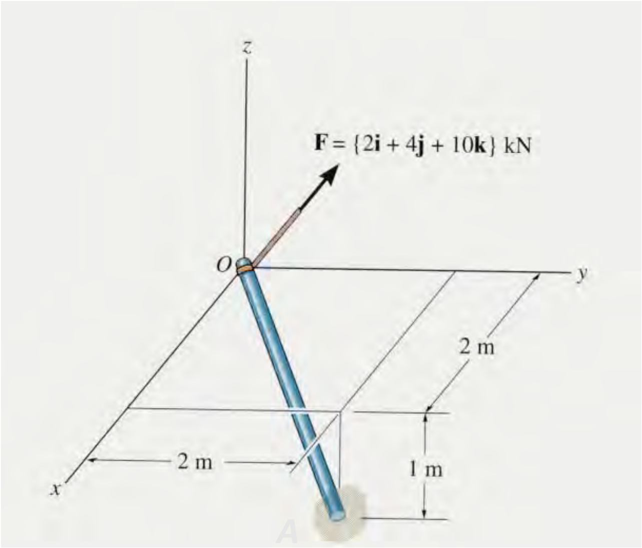
Plan:

1. Get $\boldsymbol{r_{OA}}$

2. $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{OA}})/(F \ r_{OA})\}$

3. $F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u_{OA}}$ or $F \cos \theta$

EXAMPLE (continued)



$\boldsymbol{r_{OA}} = \{2\,\boldsymbol{i} + 2\,\boldsymbol{j} - 1\,\boldsymbol{k}\}$ m

$r_{OA} = (2^2 + 2^2 + 1^2)^{1/2} = 3$ m

$\boldsymbol{F} = \{2\,\boldsymbol{i} + 4\,\boldsymbol{j} + 10\,\boldsymbol{k}\}$kN

$F = (2^2 + 4^2 + 10^2)^{1/2} = 10.95$ kN

$\boldsymbol{F} \bullet \boldsymbol{r_{OA}} = (2)(2) + (4)(2) + (10)(-1) = 2$ kN·m

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{OA}})/(F\, r_{OA})\}$

$\theta = \cos^{-1}\{2/(10.95 * 3)\} = 86.5°$

$\boldsymbol{u_{OA}} = \boldsymbol{r_{OA}}/r_{OA} = \{(2/3)\,\boldsymbol{i} + (2/3)\,\boldsymbol{j} - (1/3)\,\boldsymbol{k}\}$

$F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u_{OA}} = (2)(2/3) + (4)(2/3) + (10)(-1/3) = 0.667$ kN

Or $F_{OA} = F \cos\theta = 10.95 \cos(86.51°) = 0.667$ kN

QUIZ

1. If a dot product of two non-zero vectors is 0, then the two vectors must be _______________ to each other.

   A) parallel (pointing in the same direction)

   B) parallel (pointing in the opposite direction)

   C) perpendicular

   D) cannot be determined.

2. If a dot product of two non-zero vectors equals -1, then the vectors must be _________ to each other.

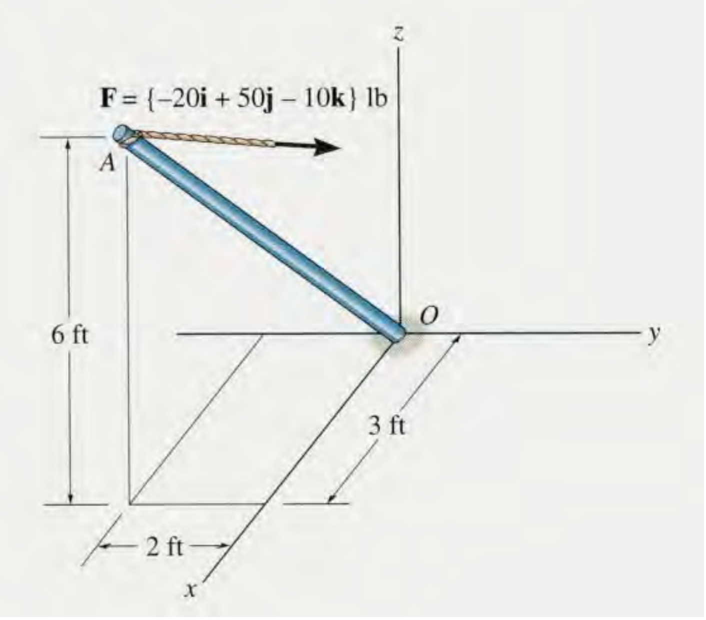
   A) parallel (pointing in the same direction)

   B) parallel (pointing in the opposite direction)

   C) perpendicular

   D) cannot be determined.

EXAMPLE



Given: The force acting on the pole.

Find: The angle between the force vector and the pole, and the magnitude of the projection of the force along the pole AO.

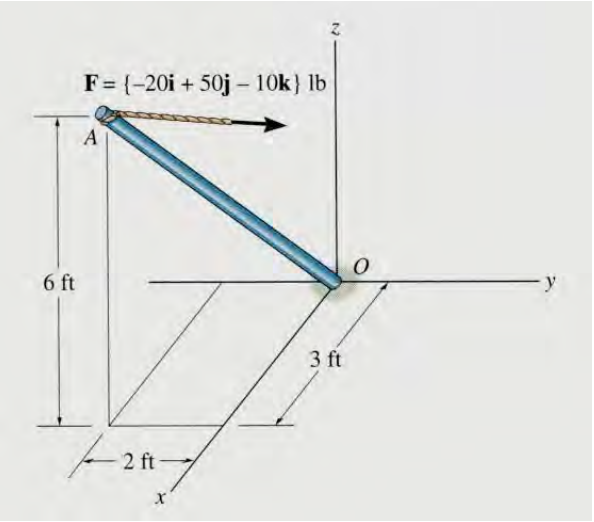
Plan:

1. Get $\boldsymbol{r}_{AO}$

2. $\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r}_{AO})/(F \;\; r_{AO})\}$

3. $F_{OA} = \boldsymbol{F} \bullet \boldsymbol{u}_{AO}$ or $F \cos \theta$

EXAMPLE (continued)



$\boldsymbol{r_{AO}} = \{-3\,\boldsymbol{i} + 2\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ ft.

$r_{AO} = (3^2 + 2^2 + 6^2)^{1/2} = 7$ ft.

$\boldsymbol{F} = \{-20\,\boldsymbol{i} + 50\,\boldsymbol{j} - 10\,\boldsymbol{k}\}$lb

$F = (20^2 + 50^2 + 10^2)^{1/2} = 54.77$ lb

$\boldsymbol{F} \bullet \boldsymbol{r_{AO}} = (-20)(-3) + (50)(2) + (-10)(-6) = 220$ lb·ft

$\theta = \cos^{-1}\{(\boldsymbol{F} \bullet \boldsymbol{r_{AO}})/(F\, r_{AO})\}$

$\theta = \cos^{-1}\{220/(54.77 \times 7)\} = 55.0°$

$\boldsymbol{\lambda_{AO}} = \boldsymbol{r_{AO}}/r_{AO} = \{(-3/7)\,\boldsymbol{i} + (2/7)\,\boldsymbol{j} - (6/7)\,\boldsymbol{k}\}$

$F_{AO} = \boldsymbol{F} \bullet \boldsymbol{\lambda_{AO}} = (-20)(-3/7) + (50)(2/7) + (-10)(-6/7) = 31.4$ lb

Or $F_{AO} = F\cos\theta = 54.77\cos(55.0°) = 31.4$ lb

QUIZ

1. The Dot product can be used to find all of the following except _____ .

   A) sum of two vectors

   B) angle between two vectors

   C) component of a vector parallel to another line

   D) component of a vector perpendicular to another line

2. Find the dot product of the two vectors ***P*** and ***Q*.**

   $\boldsymbol{P} = \{5\,\boldsymbol{i} + 2\,\boldsymbol{j} + 3\,\boldsymbol{k}\}$ m

   $\boldsymbol{Q} = \{-2\,\boldsymbol{i} + 5\,\boldsymbol{j} + 4\,\boldsymbol{k}\}$ m

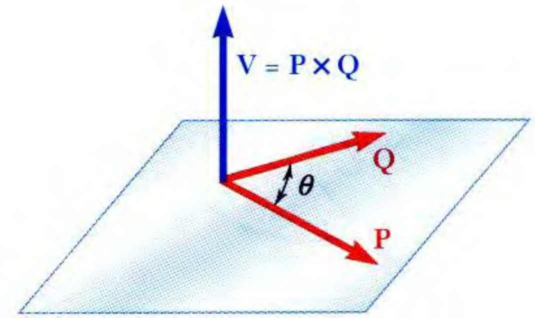
   A) -12 m　　B) 12 m　　C) 12 m^2

   D) -12 m^2　　E) 10 m^2

Vector Product of Two Vectors

- Concept of the moment of a force about a point is more easily understood through applications of the *vector product* or *cross product*.

- Vector product of two vectors $\boldsymbol{P}$ and $\boldsymbol{Q}$ is defined as the vector $\boldsymbol{V}$ which satisfies the following conditions:
  1. Line of action of $\boldsymbol{V}$ is perpendicular to plane containing $\boldsymbol{P}$ and $\boldsymbol{Q}$.
  2. Magnitude of $\boldsymbol{V}$ is $V = PQ\sin\theta$
  3. Direction of $\boldsymbol{V}$ is obtained from the right-hand rule.



(a)

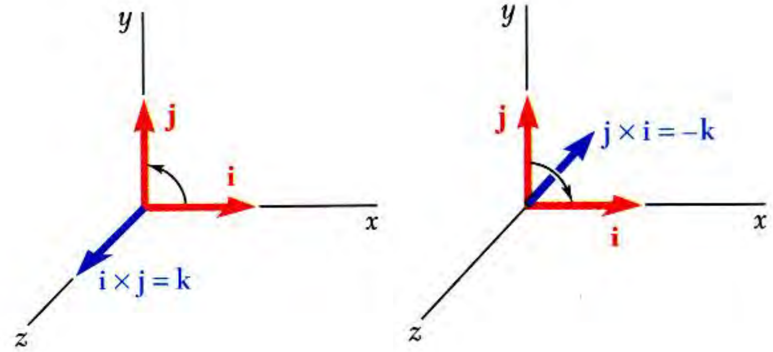
(b)

- Vector products:
  - are not commutative, $\boldsymbol{Q} \times \boldsymbol{P} = -(\boldsymbol{P} \times \boldsymbol{Q})$
  - are distributive, $\boldsymbol{P} \times (\boldsymbol{Q}_1 + \boldsymbol{Q}_2) = \boldsymbol{P} \times \boldsymbol{Q}_1 + \boldsymbol{P} \times \boldsymbol{Q}_2$
  - are not associative, $(\boldsymbol{P} \times \boldsymbol{Q}) \times \boldsymbol{S} \neq \boldsymbol{P} \times (\boldsymbol{Q} \times \boldsymbol{S})$

Vector Products: Rectangular Components

- Vector products of Cartesian unit vectors,

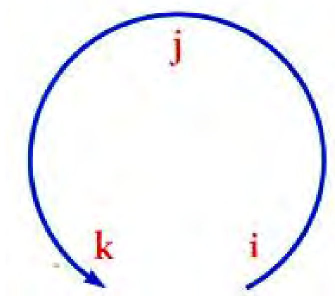
$$\vec{i}\times\vec{i}=0 \qquad \vec{j}\times\vec{i}=-\vec{k} \qquad \vec{k}\times\vec{i}=\vec{j}$$
$$\vec{i}\times\vec{j}=\vec{k} \qquad \vec{j}\times\vec{j}=0 \qquad \vec{k}\times\vec{j}=-\vec{i}$$
$$\vec{i}\times\vec{k}=-\vec{j} \qquad \vec{j}\times\vec{k}=\vec{i} \qquad \vec{k}\times\vec{k}=0$$



- Vector products in terms of rectangular coordinates

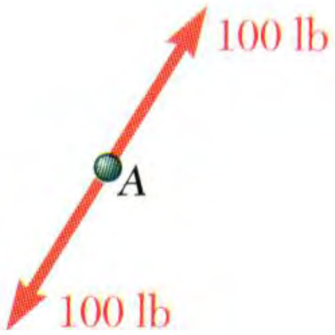
$$\vec{V}=\left(P_x\vec{i}+P_y\vec{j}+P_z\vec{k}\right)\times\left(Q_x\vec{i}+Q_y\vec{j}+Q_z\vec{k}\right)$$
$$=\left(P_yQ_z-P_zQ_y\right)\vec{i}+\left(P_zQ_x-P_xQ_z\right)\vec{j}$$
$$+\left(P_xQ_y-P_yQ_x\right)\vec{k}$$

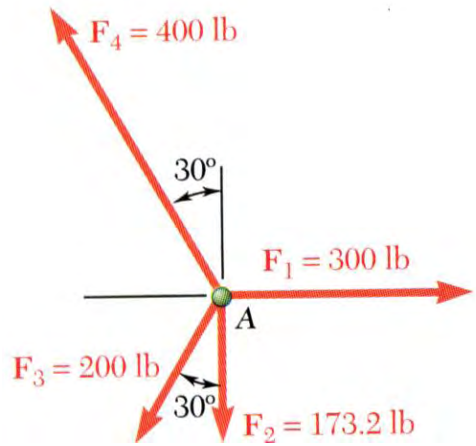
$$=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ P_x & P_y & P_z\\ Q_x & Q_y & Q_z\end{vmatrix}$$

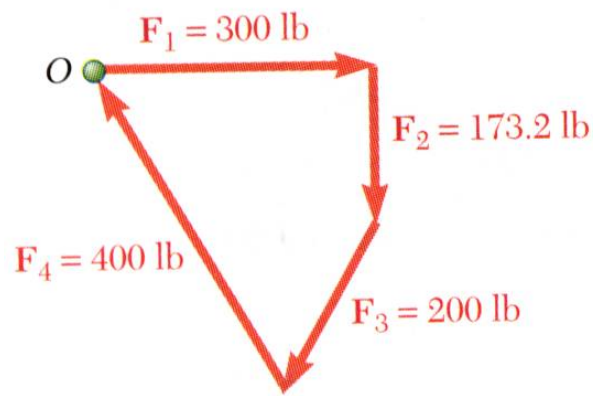


Equilibrium of a Particle

- When the resultant of all forces acting on a particle is zero, the particle is in *equilibrium*.
- *Newton's First Law*: If the resultant force on a particle is zero, the particle will remain at rest or will continue at constant speed in a straight line.







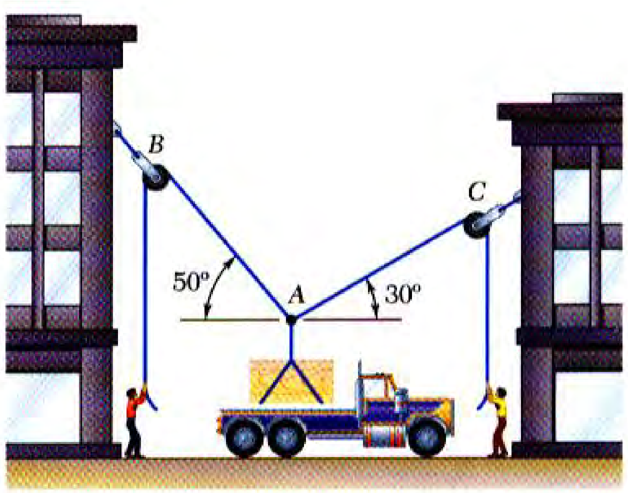
- Particle acted upon by two forces:
  - equal magnitude
  - same line of action
  - opposite sense

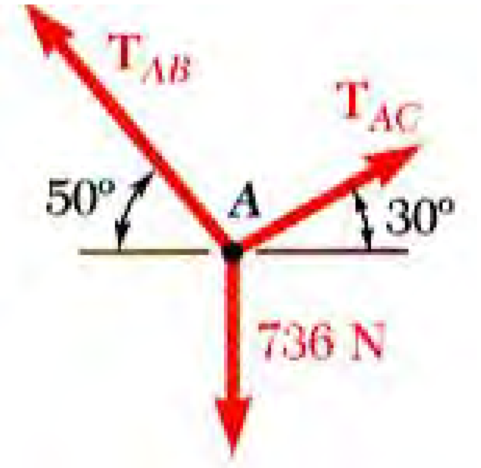
- Particle acted upon by three or more forces:
  - graphical solution yields a closed polygon
  - algebraic solution

$$\vec{R} = \sum \vec{F} = 0$$

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Free-Body Diagrams

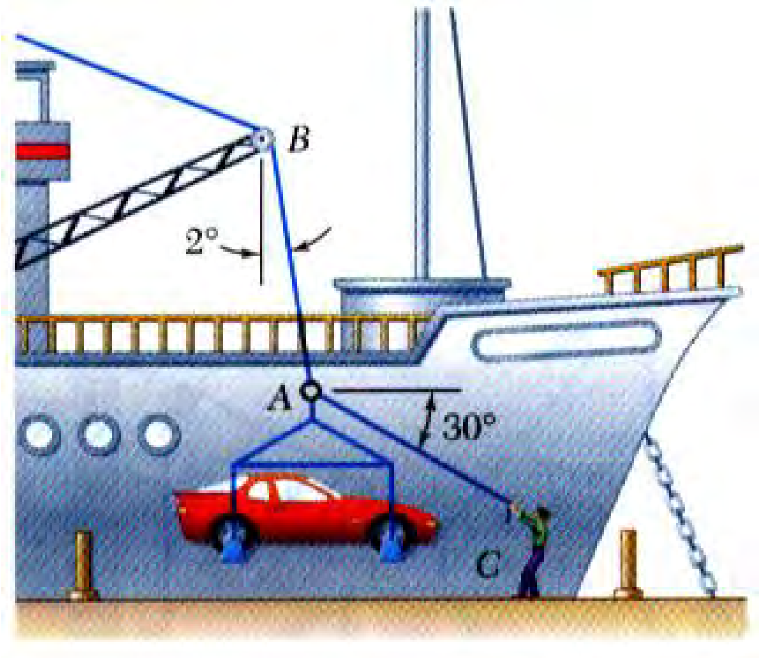




Space Diagram: A sketch showing the physical conditions of the problem.

Free-Body Diagram: A sketch showing only the forces on the selected particle.

Sample Problem

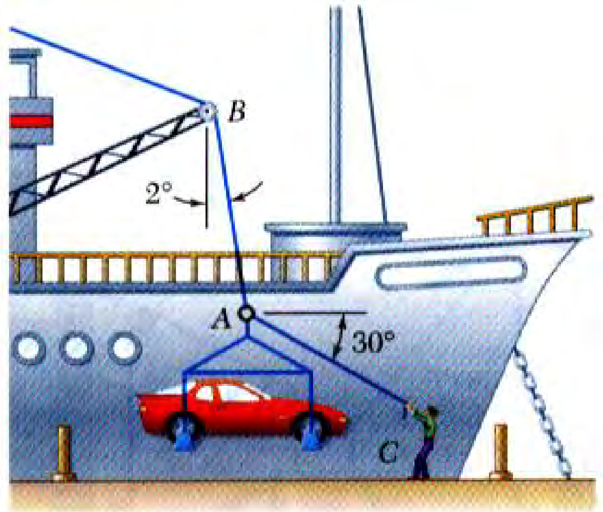


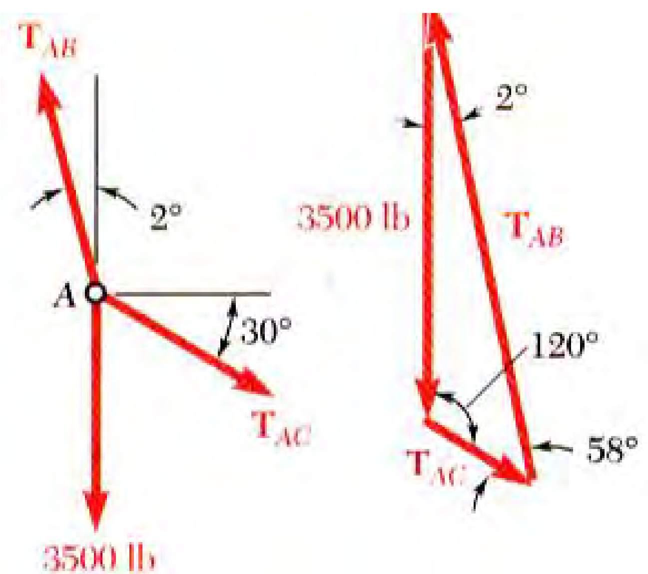
In a ship-unloading operation, a 3500-lb automobile is supported by a cable. A rope is tied to the cable and pulled to center the automobile over its intended position. What is the tension in the rope?

SOLUTION:

- Construct a free-body diagram for the particle at the junction of the rope and cable.
- Apply the conditions for equilibrium by creating a closed polygon from the forces applied to the particle.
- Apply trigonometric relations to determine the unknown force magnitudes.

Sample Problem





SOLUTION:

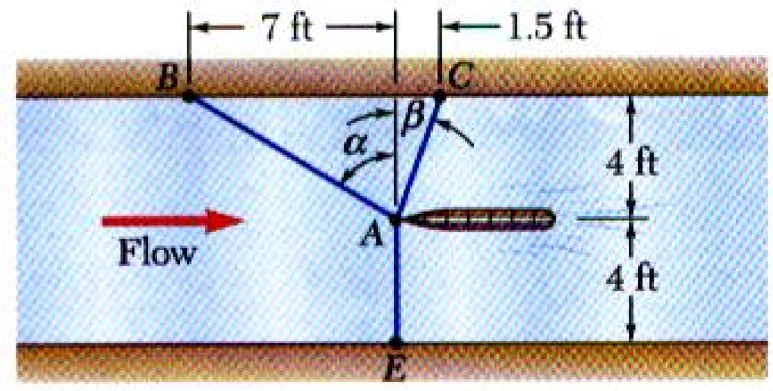
- Construct a free-body diagram for the particle at *A*.
- Apply the conditions for equilibrium.
- Solve for the unknown force magnitudes.

$$\frac{T_{AB}}{\sin 120°} = \frac{T_{AC}}{\sin 2°} = \frac{3500\,\text{lb}}{\sin 58°}$$

$$T_{AB} = 3570\,\text{lb}$$

$$\boxed{T_{AC} = 144\,\text{lb}}$$

Sample Problem



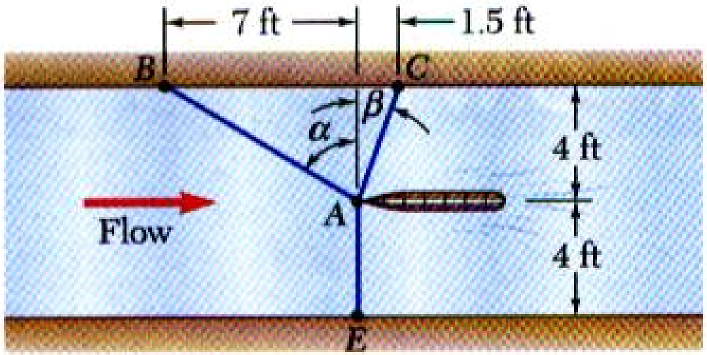
It is desired to determine the drag force at a given speed on a prototype sailboat hull. A model is placed in a test channel and three cables are used to align its bow on the channel centerline. For a given speed, the tension is 40 lb in cable *AB* and 60 lb in cable *AE*.

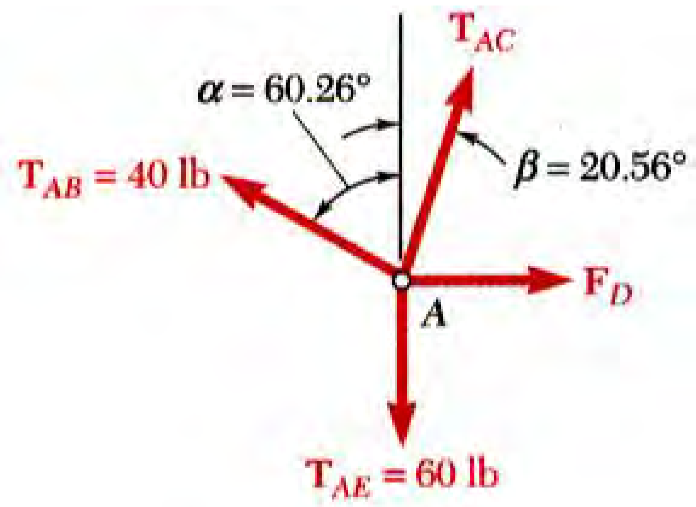
Determine the drag force exerted on the hull and the tension in cable *AC*.

SOLUTION:

- Choosing the hull as the free body, draw a free-body diagram.
- Express the condition for equilibrium for the hull by writing that the sum of all forces must be zero.
- Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.

Sample Problem





SOLUTION:

- Choosing the hull as the free body, draw a free-body diagram.

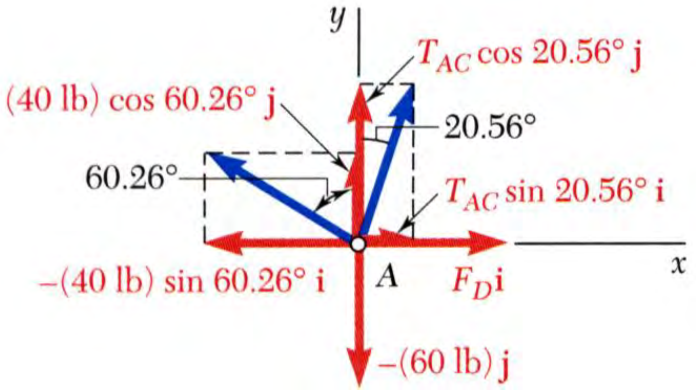
$$\tan\alpha = \frac{7\text{ ft}}{4\text{ ft}} = 1.75 \qquad \tan\beta = \frac{1.5\text{ ft}}{4\text{ ft}} = 0.375$$

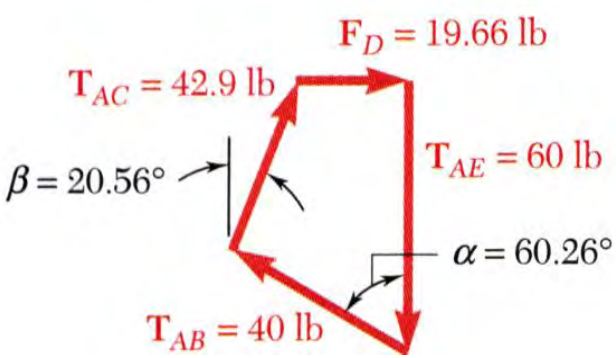
$$\alpha = 60.25° \qquad \beta = 20.56°$$

- Express the condition for equilibrium for the hull by writing that the sum of all forces must be zero.

$$\vec{R} = \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AE} + \vec{F}_D = 0$$

Sample Problem





- Resolve the vector equilibrium equation into two component equations. Solve for the two unknown cable tensions.

$$\vec{T}_{AB} = -(40\,\mathrm{lb})\sin 60.26^\circ\,\vec{i} + (40\,\mathrm{lb})\cos 60.26^\circ\,\vec{j}$$
$$= -(34.73\,\mathrm{lb})\vec{i} + (19.84\,\mathrm{lb})\vec{j}$$

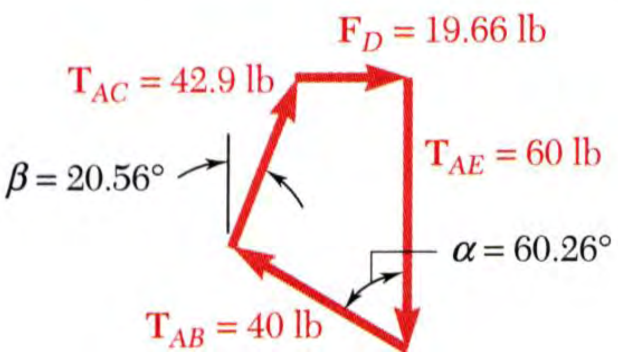
$$\vec{T}_{AC} = T_{AC}\sin 20.56^\circ\,\vec{i} + T_{AC}\cos 20.56^\circ\,\vec{j}$$
$$= 0.3512\,T_{AC}\,\vec{i} + 0.9363\,T_{AC}\,\vec{j}$$

$$\vec{T} = -(60\,\mathrm{lb})\vec{i}$$

$$\vec{F}_D = F_D\,\vec{i}$$

$$\vec{R} = 0$$
$$= (-34.73 + 0.3512\,T_{AC} + F_D)\vec{i}$$
$$+ (19.84 + 0.9363\,T_{AC} - 60)\,\vec{j}$$

Sample Problem



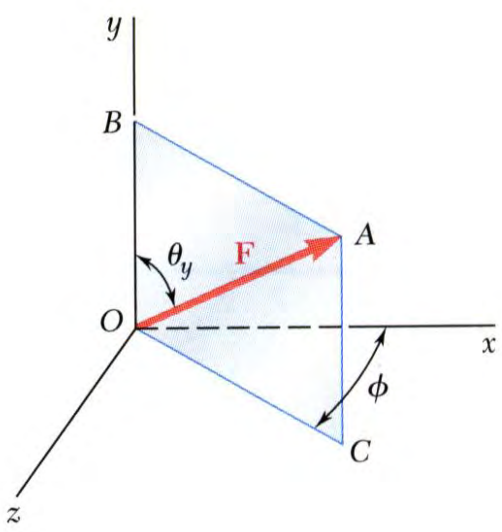
$$\vec{R} = 0$$
$$= (-34.73 + 0.3512T_{AC} + F_D)\vec{i}$$
$$+ (19.84 + 0.9363T_{AC} - 60)\vec{j}$$

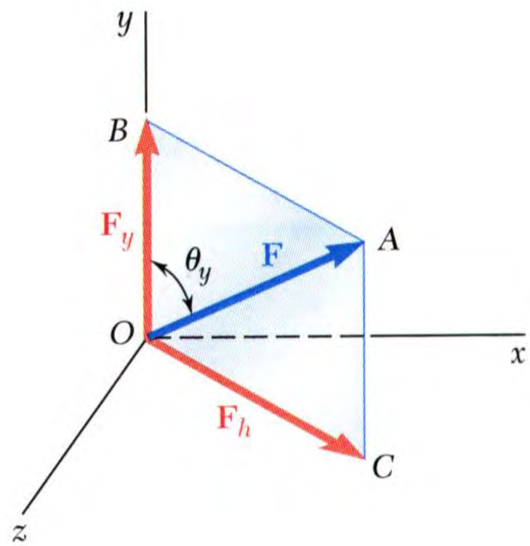
This equation is satisfied only if each component of the resultant is equal to zero

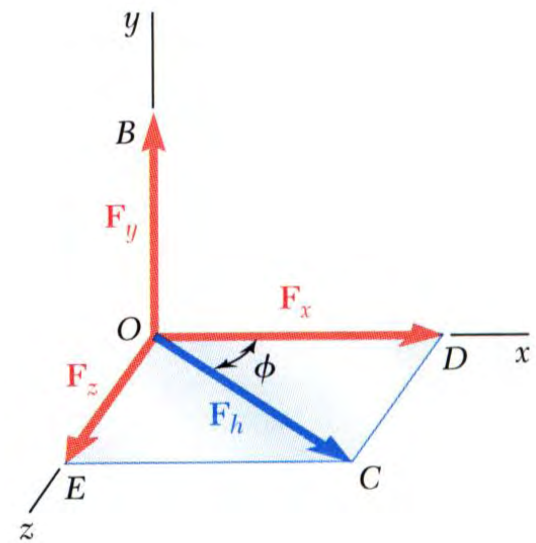
$$\left(\sum F_x = 0\right) \quad 0 = -34.73 + 0.3512T_{AC} + F_D$$
$$\left(\sum F_y = 0\right) \quad 0 = 19.84 + 0.9363T_{AC} - 60$$

$$\boxed{\begin{aligned} T_{AC} &= +42.9\text{ lb} \\ F_D &= +19.66\text{ lb} \end{aligned}}$$

Rectangular Components in Space







- The vector $\vec{F}$ is contained in the plane *OBAC*.

- Resolve $\vec{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

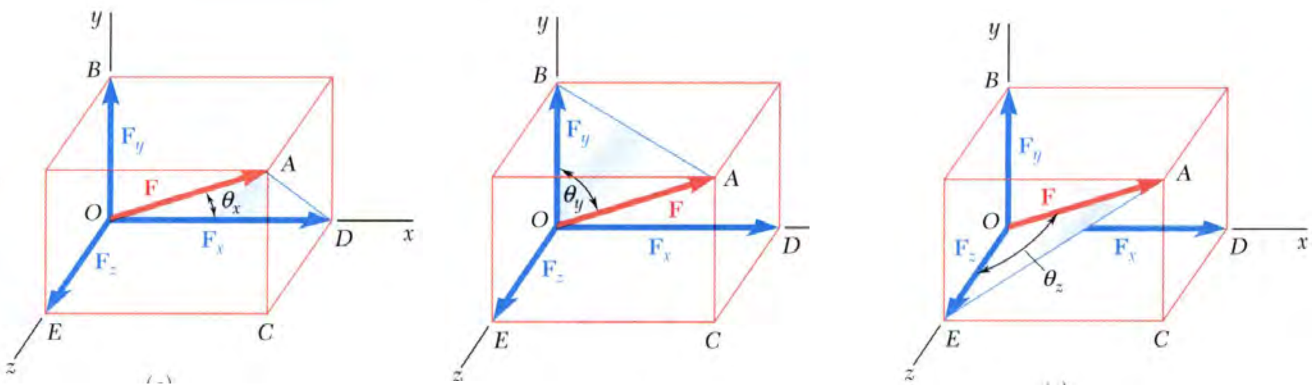
$$F_h = F\sin\theta_y$$

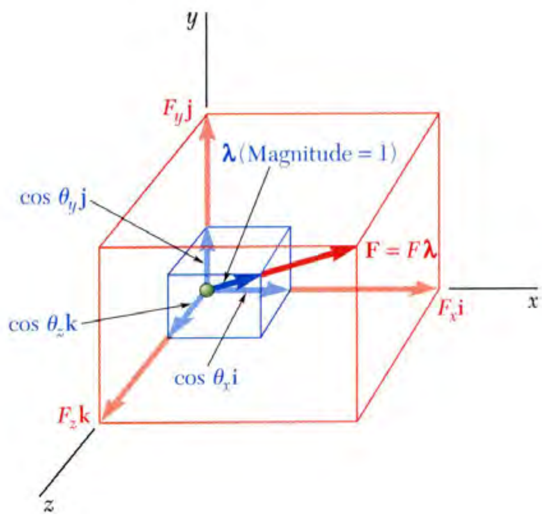
- Resolve F_h into rectangular components

$$F_x = F_h\cos\phi$$
$$= F\sin\theta_y\cos\phi$$

$$F_y = F_h\sin\phi$$
$$= F\sin\theta_y\sin\phi$$

Rectangular Components in Space





- With the angles between $\vec{F}$ and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

$$= F\vec{\lambda}$$

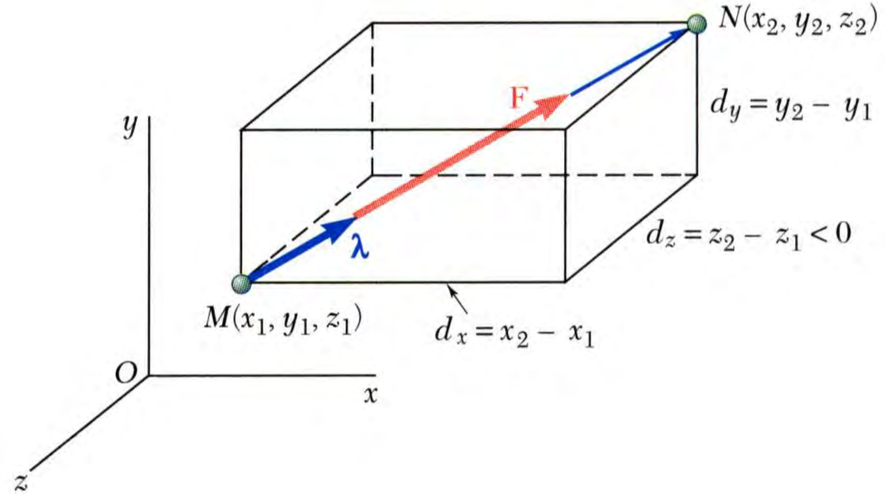
$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the direction cosines for $\vec{F}$

Rectangular Components in Space

Direction of the force is defined by the location of two points,

$$M(x_1, y_1, z_1) \text{ and } N(x_2, y_2, z_2)$$



$$\vec{d} = \text{vector joining } M \text{ and } N$$

$$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$$

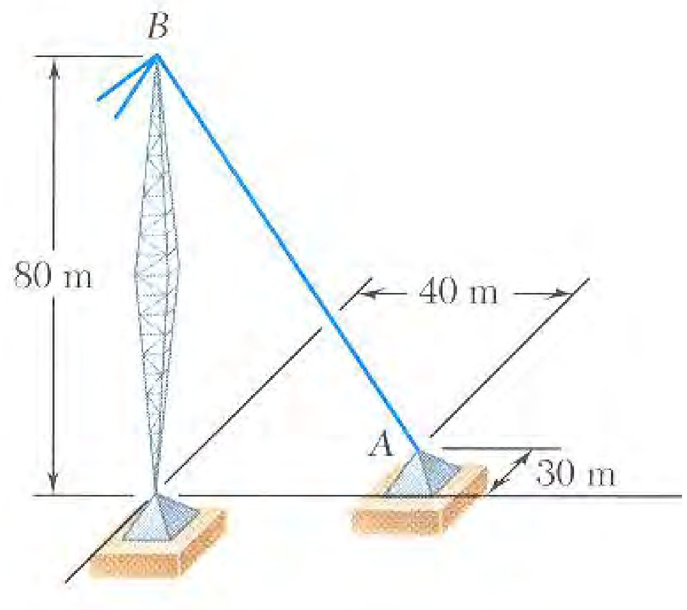
$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$

$$\vec{F} = F\vec{\lambda}$$

$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$

$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$

Sample Problem



The tension in the guy wire is 2500 N. Determine:

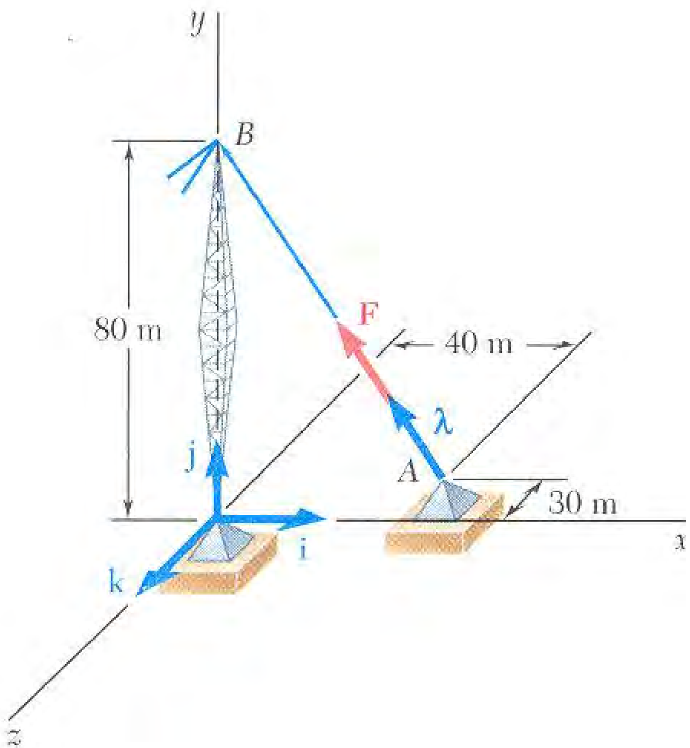
a) components F_x, F_y, F_z of the force acting on the bolt at *A*,

b) the angles θ_x, θ_y, θ_z defining the direction of the force

SOLUTION:

- Based on the relative locations of the points *A* and *B*, determine the unit vector pointing from *A* towards *B*.
- Apply the unit vector to determine the components of the force acting on *A*.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

Sample Problem



SOLUTION:

- Determine the unit vector pointing from *A* towards *B*.

$$\overrightarrow{AB} = (-40\,\mathrm{m})\vec{i} + (80\,\mathrm{m})\vec{j} + (30\,\mathrm{m})\vec{k}$$

$$AB = \sqrt{(-40\,\mathrm{m})^2 + (80\,\mathrm{m})^2 + (30\,\mathrm{m})^2}$$

$$= 94.3\,\mathrm{m}$$

$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k}$$

$$= -0.424\,\vec{i} + 0.848\,\vec{j} + 0.318\vec{k}$$

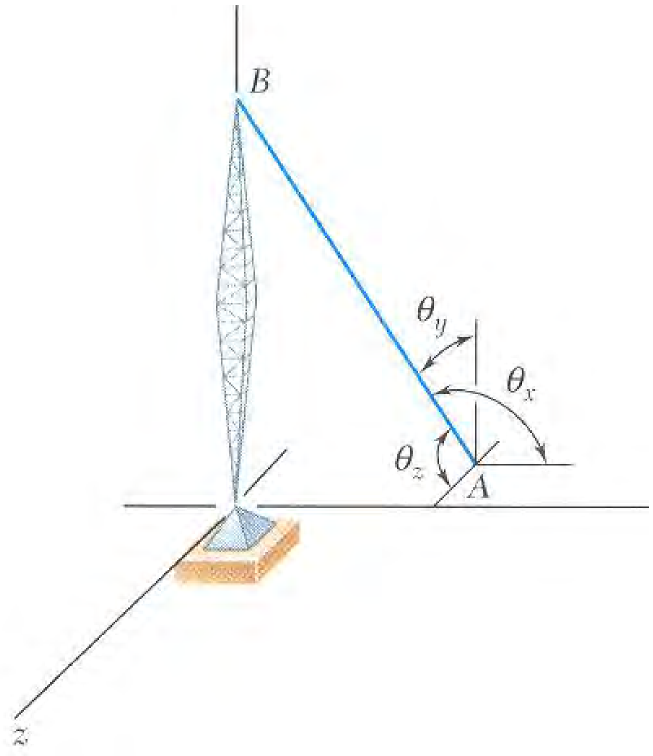
- Determine the components of the force.

$$\vec{F} = F\vec{\lambda}$$

$$= (2500\,\mathrm{N})\left(-0.424\,\vec{i} + 0.848\,\vec{j} + 0.318\vec{k}\right)$$

$$= (-1060\,\mathrm{N})\vec{i} + (2120\,\mathrm{N})\vec{j} + (795\,\mathrm{N})\vec{k}$$

Sample Problem



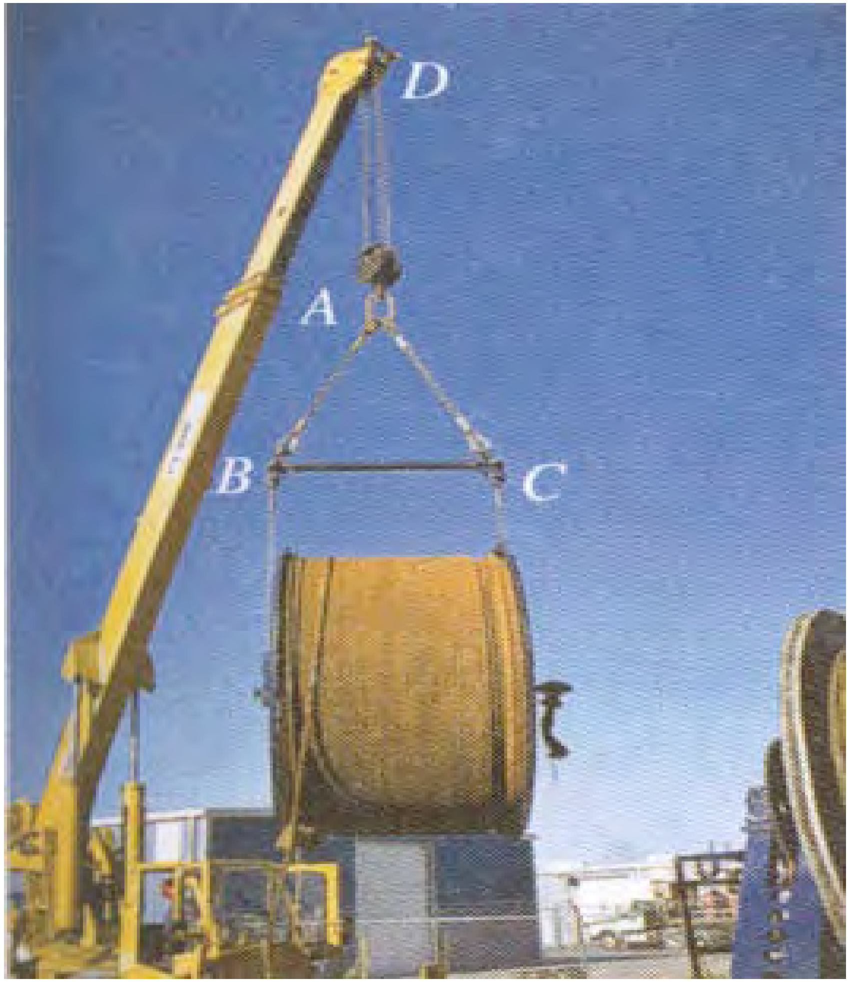
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\vec{\lambda} = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$

$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\boxed{\begin{aligned} \theta_x &= 115.1^\circ \\ \theta_y &= 32.0^\circ \\ \theta_z &= 71.5^\circ \end{aligned}}$$

APPLICATIONS

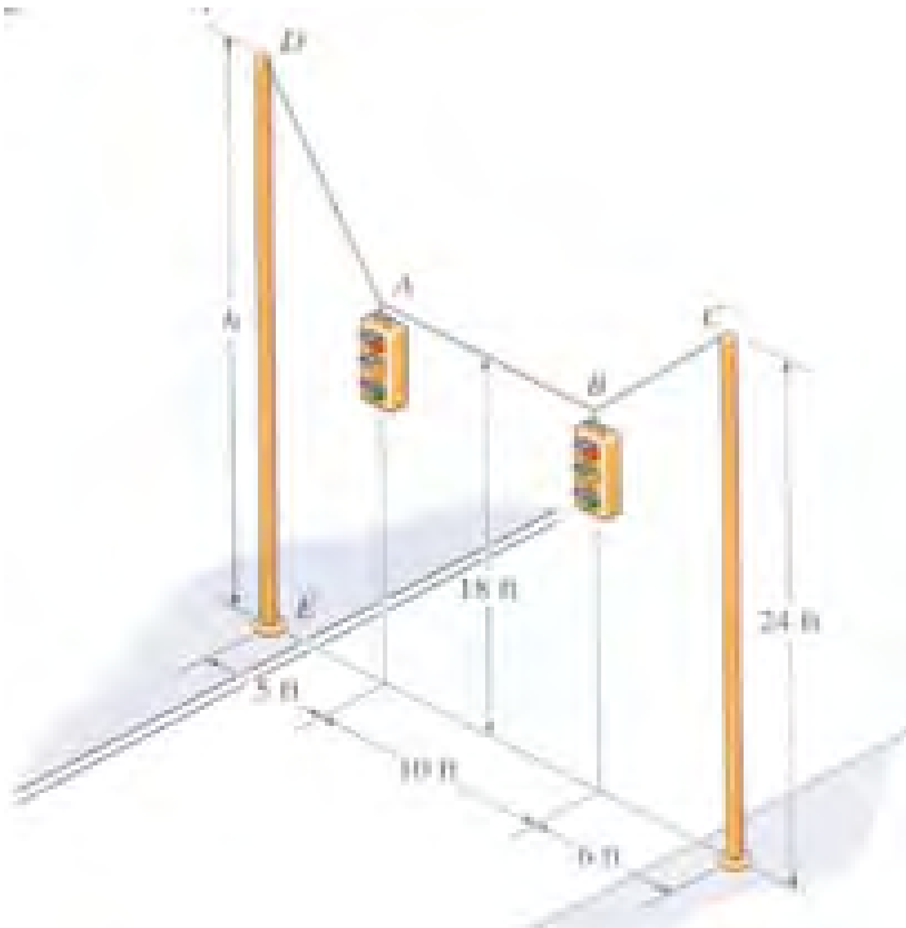


For a spool of given weight, what are the forces in cables AB and AC ?

APPLICATIONS (continued)

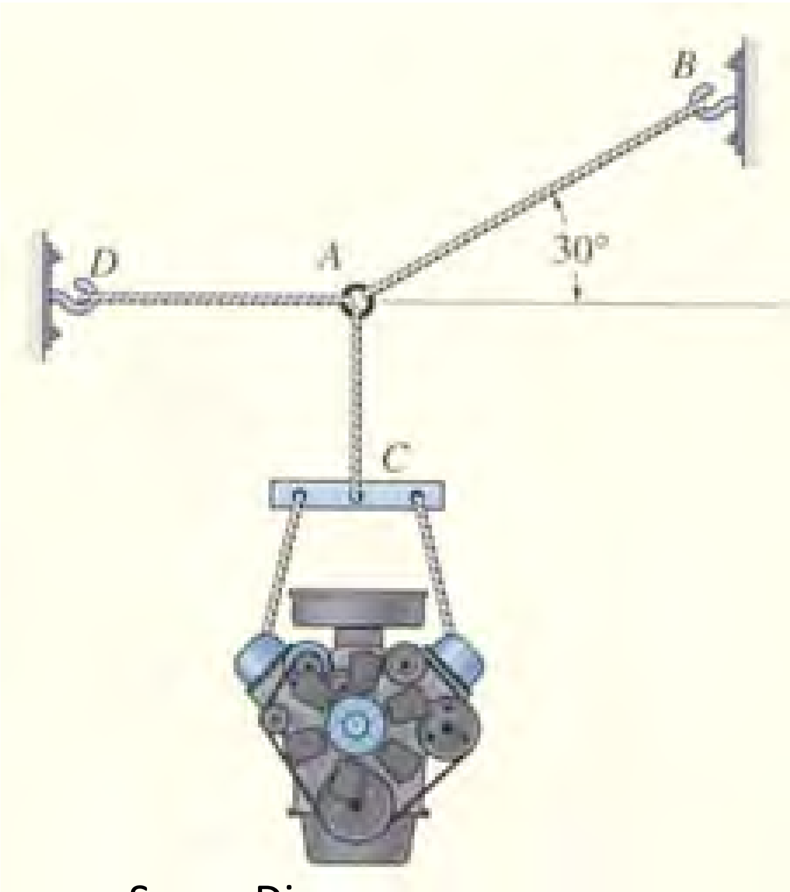
For a given cable strength, what is the maximum weight that can be lifted ?

APPLICATIONS (continued)



For a given weight of the lights, what are the forces in the cables? What size of cable must you use ?

COPLANAR FORCE SYSTEMS



Space Diagram

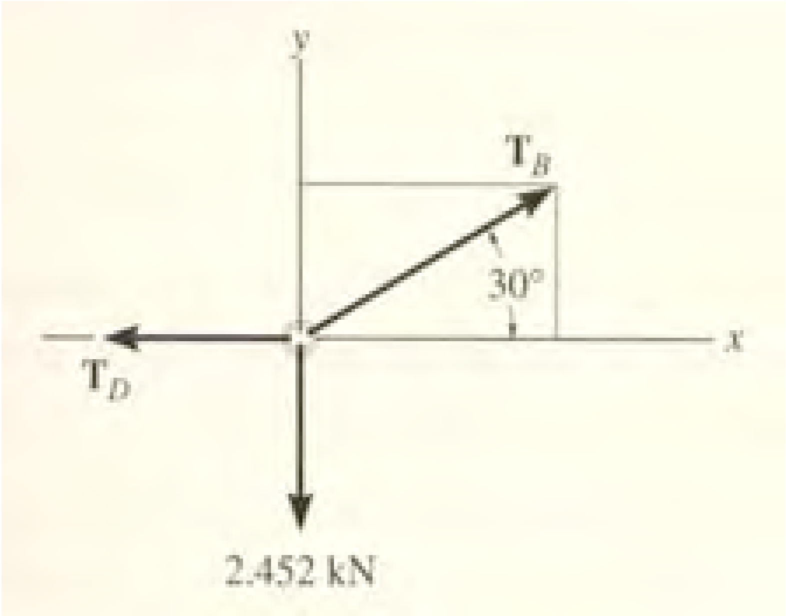
This is an example of a 2-D or coplanar force system. If the whole assembly is in equilibrium, then particle A is also in equilibrium.

To determine the tensions in the cables for a given weight of the engine, we need to learn how to draw a free body diagram and apply equations of equilibrium.

THE WHAT, WHY AND HOW OF A FREE BODY DIAGRAM (FBD)

Free Body Diagrams are one of the most important things for you to know how to draw and use.

What ? - It is a drawing that shows all external forces acting on the particle.



Why ? - It helps you write the equations of equilibrium used to solve for the unknowns (usually forces or angles).

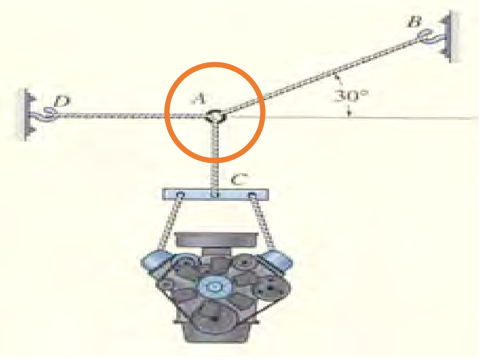
How ?

1. Imagine the particle to be isolated or cut free from its surroundings.

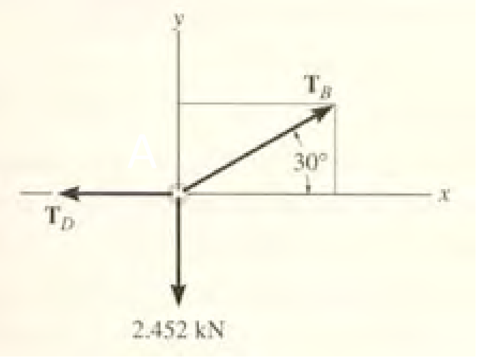
2. Show all the forces that act on the particle.

Active forces: They want to move the particle. Reactive forces: They tend to resist the motion.

3. Identify each force and show all known magnitudes and directions. Show all unknown magnitudes and / or directions as variables. Include a reference frame.

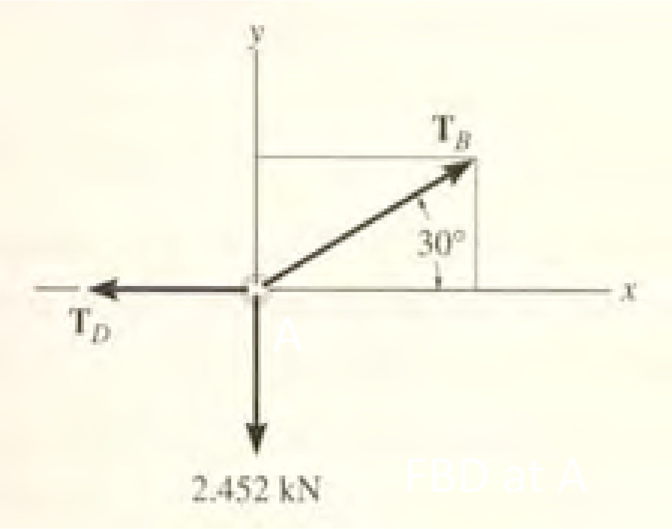


Note : Engine mass = 250 Kg



FBD at A

EQUATIONS OF 2-D EQUILIBRIUM



Since particle A is in equilibrium, the net force at A is zero.

So $\boldsymbol{F}_{AB}$ + $\boldsymbol{F}_{AC}$ + $\boldsymbol{F}_{AD}$ = 0

or Σ $\boldsymbol{F}$ = 0

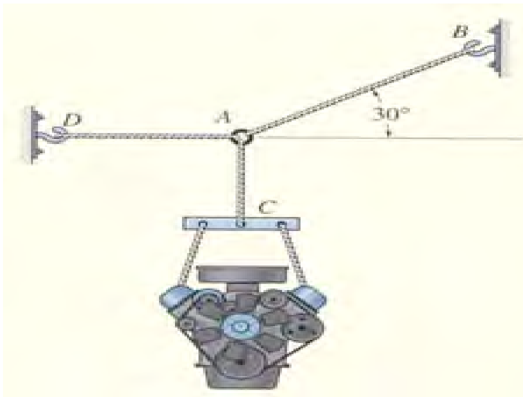
In general, for a particle in equilibrium, Σ $\boldsymbol{F}$ = 0 or
$\Sigma F_x \, \boldsymbol{i} + \Sigma F_y \, \boldsymbol{j} = 0 = 0 \, \boldsymbol{i} + 0 \, \boldsymbol{j}$ (A vector equation)

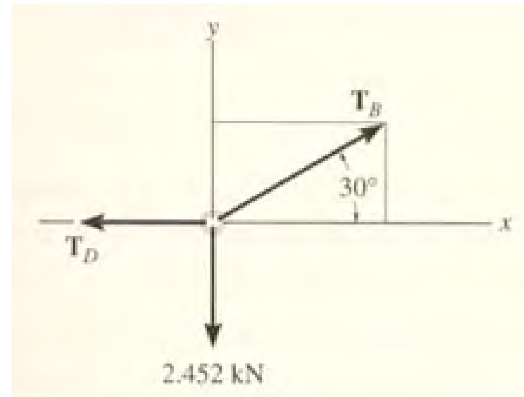
Or, written in a scalar form,

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

These are two scalar equations of equilibrium. They can be used to solve for up to two unknowns.

EXAMPLE





Note : Engine mass = 250 Kg

FBD at A

Write the scalar EofE:

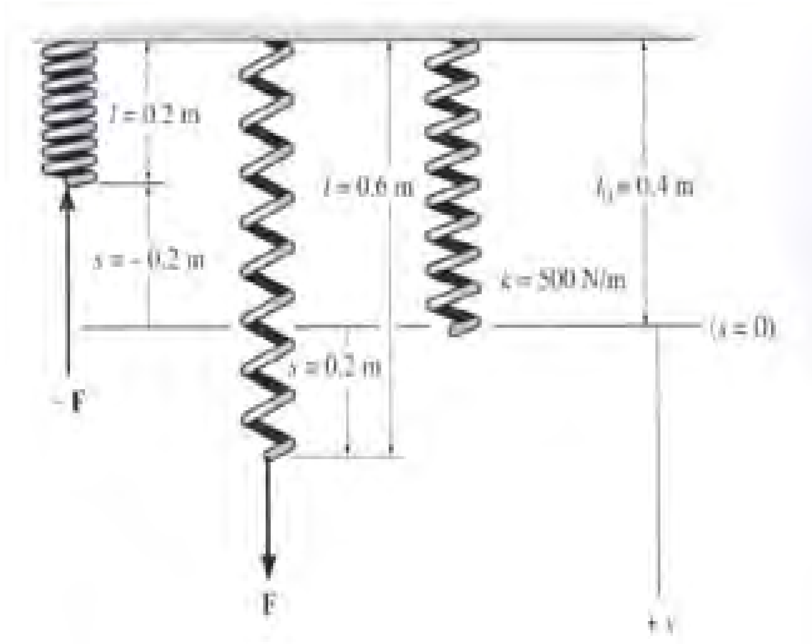
$$+\rightarrow \Sigma F_x = T_B \cos 30^\circ - T_D = 0$$

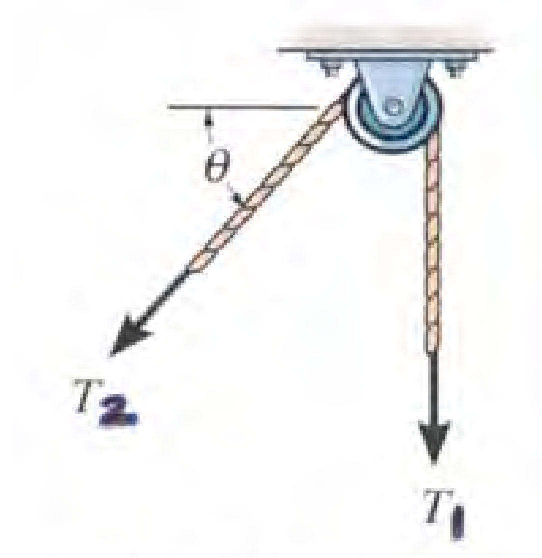
$$+\uparrow \Sigma F_y = T_B \sin 30^\circ - 2.452 \text{ kN} = 0$$

Solving the second equation gives: $T_B = 4.90$ kN

From the first equation, we get: $T_D = 4.25$ kN

SPRINGS, CABLES, AND PULLEYS





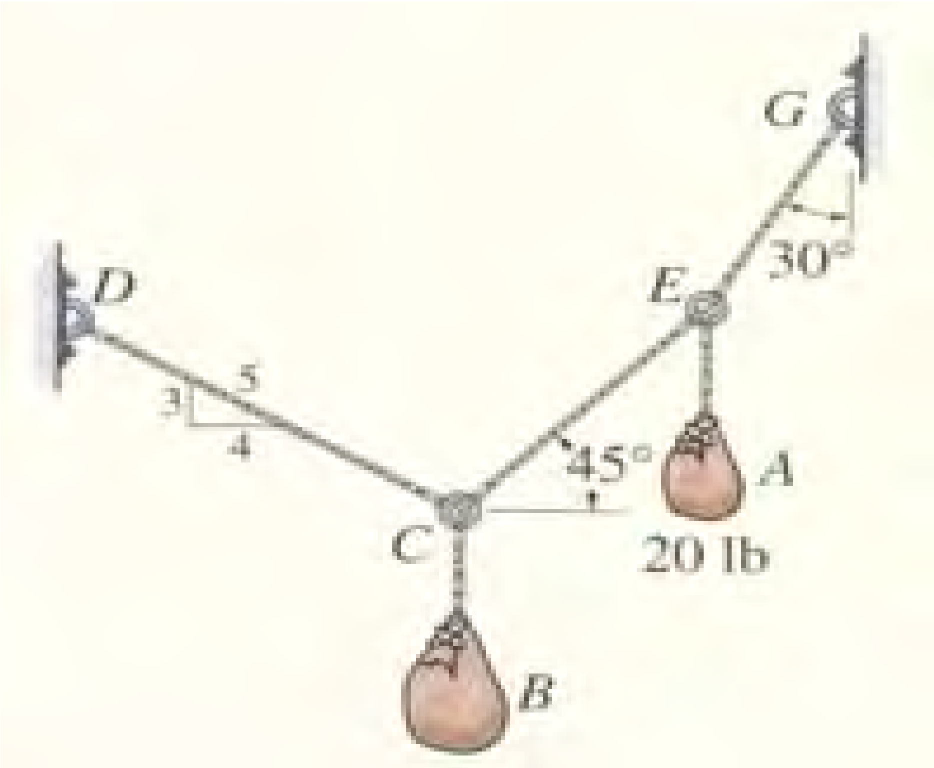
Cable is in tension

Spring Force = spring constant * deformation, or

$$F = k * S$$

With a frictionless pulley, $T_1 = T_2$.

EXAMPLE



Given: Sack A weighs 20 lb. and geometry is as shown.

Find: Forces in the cables and weight of sack B.

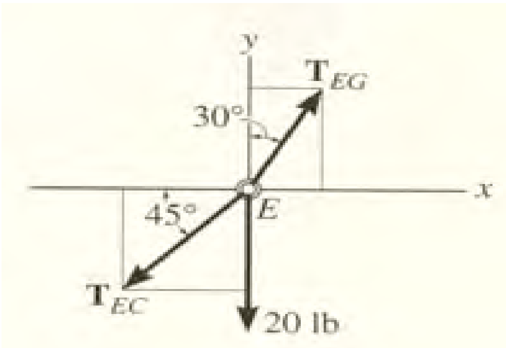
<u>**Plan**</u>**:**

1. Draw a FBD for Point E.

2. Apply EofE at Point E to solve for the unknowns (T_{EG} & T_{EC}).

3. Repeat this process at C.

EXAMPLE (continued)



A FBD at E should look like the one to the left. Note the assumed directions for the two cable tensions.

The scalar E-of-E are:

$+\rightarrow \quad \Sigma F_x = T_{EG} \sin 30º - T_{EC} \cos 45º = 0$

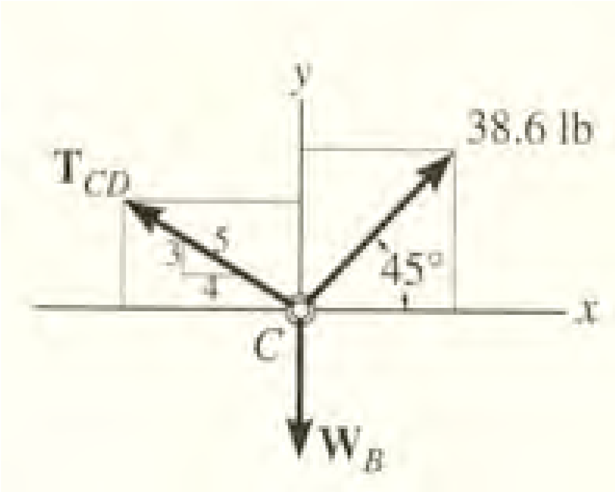
$+\uparrow \quad \Sigma F_y = T_{EG} \cos 30º - T_{EC} \sin 45º - 20 \text{ lbs} = 0$

Solving these two simultaneous equations for the two unknowns yields:

T_{EC} = 38.6 lb

T_{EG} = 54.6 lb

EXAMPLE (continued)



Now move on to ring C. A FBD for C should look like the one to the left.

The scalar E-of-E are:

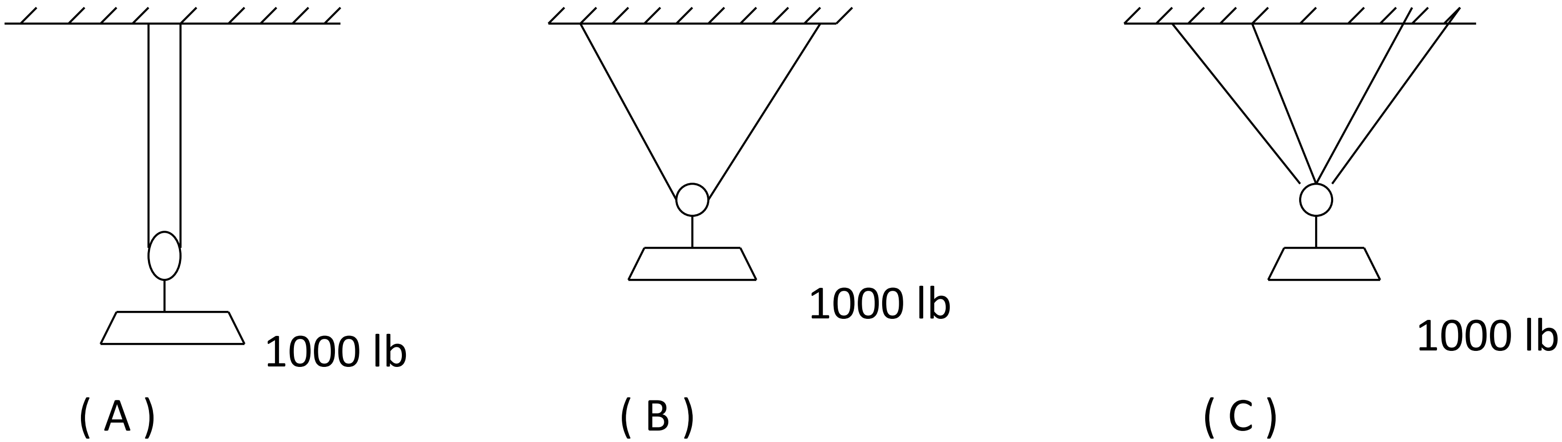
$+\rightarrow \Sigma F_x = 38.64 \cos 45° - (4/5)\, T_{CD} = 0$

$+\uparrow \Sigma F_y = (3/5)\, T_{CD} + 38.64 \sin 45° - W_B = 0$

Solving the first equation and then the second yields

$T_{CD} = 34.2$ lb and $W_B = 47.8$ lb .

CONCEPT QUESTIONS



1) Assuming you know the geometry of the ropes, you cannot determine the forces in the cables in which system above?

2) Why?

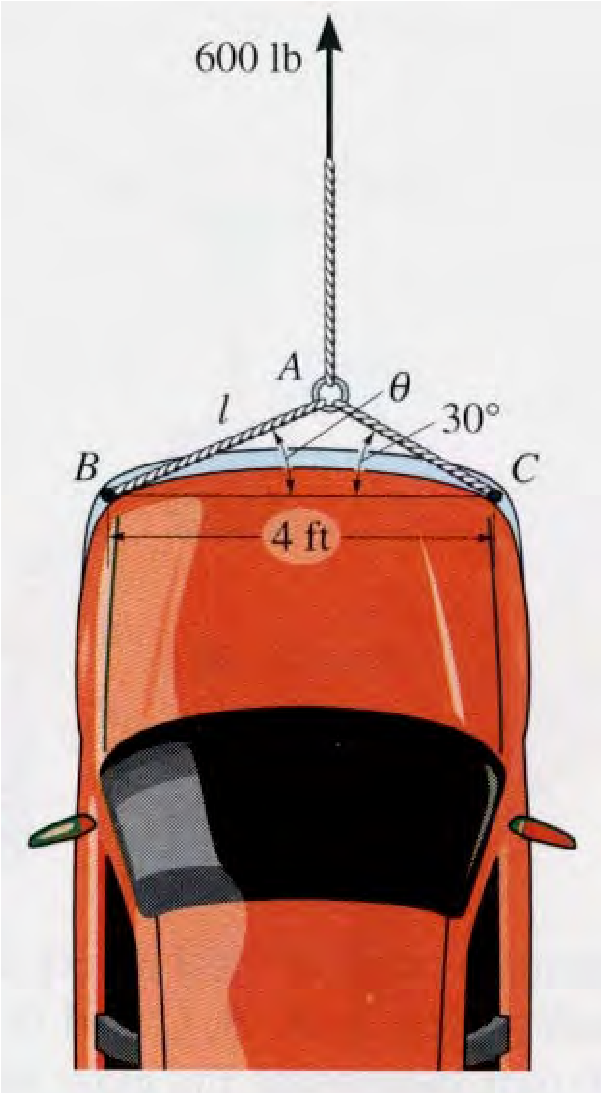
A) The weight is too heavy.

B) The cables are too thin.

C) There are more unknowns than equations.

D) There are too few cables for a 1000 lb weight.

GROUP PROBLEM SOLVING



Given: The car is towed at constant speed by the 600 lb force and the angle θ is 25°.

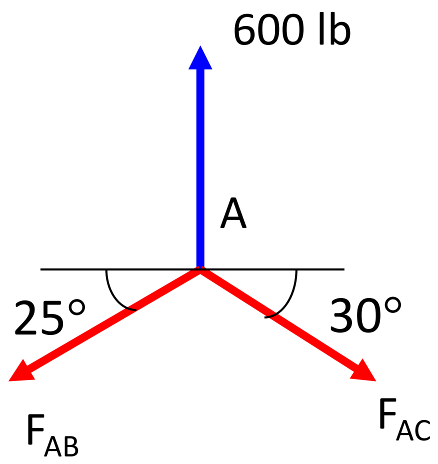
Find: The forces in the ropes AB and AC.

<u>Plan:</u>

1. Draw a FBD for point A.

2. Apply the E-of-E to solve for the forces in ropes AB and AC.

GROUP PROBLEM SOLVING (continued)



FBD at point A

Applying the scalar E-of-E at A, we get;

$+ \rightarrow \sum F_x = F_{AC} \cos 30° - F_{AB} \cos 25° = 0$

$+ \rightarrow \sum F_y = -F_{AC} \sin 30° - F_{AB} \sin 25° + 600 = 0$

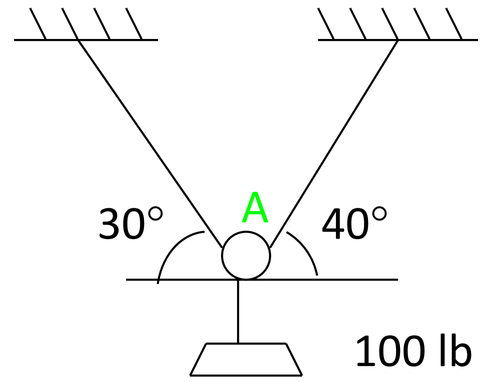
Solving the above equations, we get;

$F_{AB} = 634$ lb

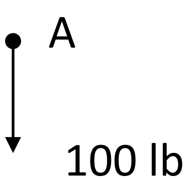
$F_{AC} = 664$ lb

ATTENTION QUIZ

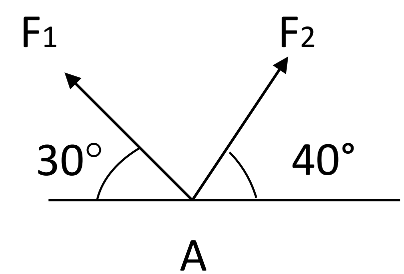
1. Select the correct FBD of particle A.



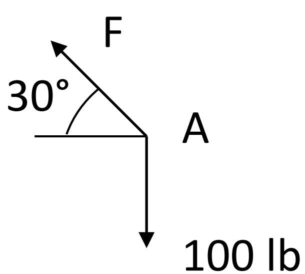
A)



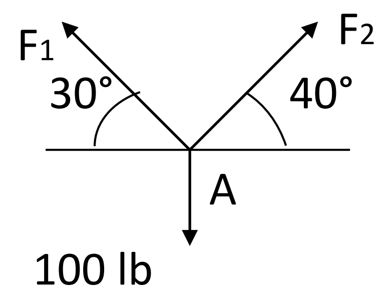
B)



C)

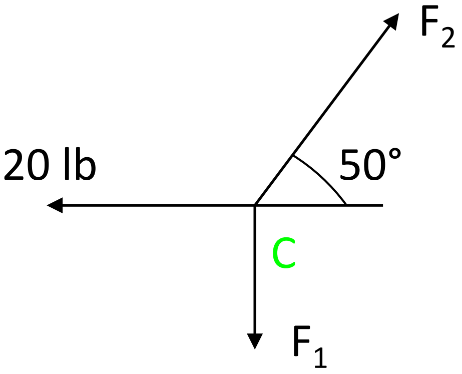


D)



ATTENTION QUIZ

2. Using this FBD of Point C, the sum of forces in the x-direction (ΣF_X) is ___ . Use a sign convention of + $\rightarrow$.



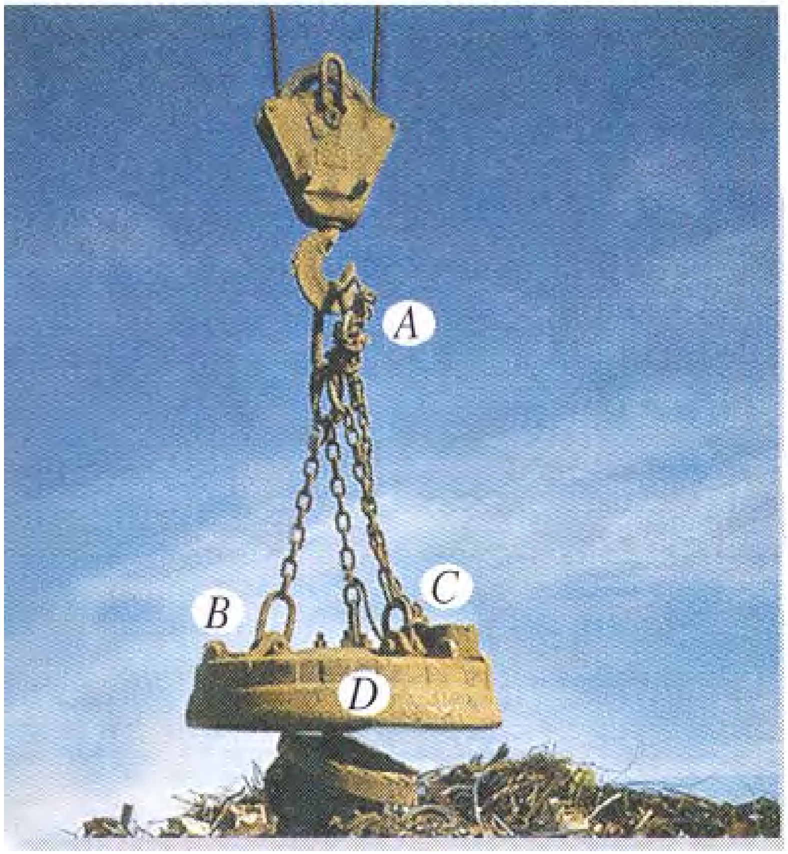
A) $F_2 \sin 50° - 20 = 0$

B) $F_2 \cos 50° - 20 = 0$

C) $F_2 \sin 50° - F_1 = 0$

D) $F_2 \cos 50° + 20 = 0$

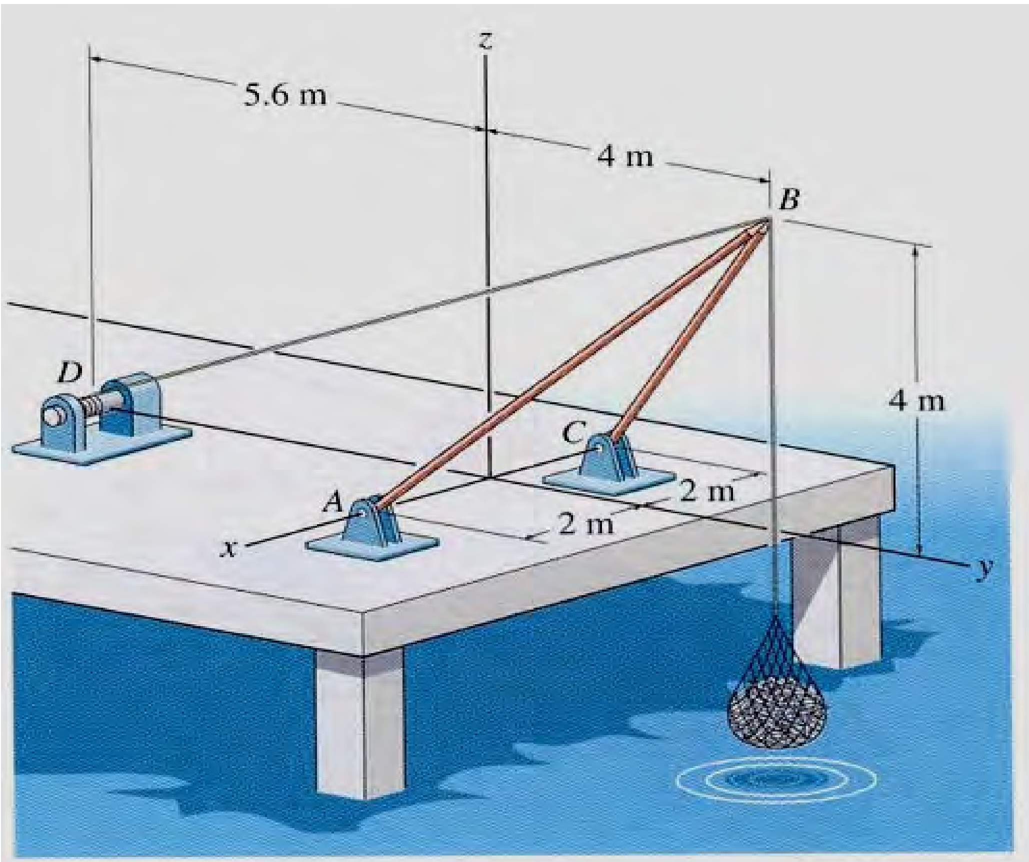
APPLICATIONS



The weights of the electromagnet and the loads are given.

Can you determine the forces in the chains?

APPLICATIONS (continued)

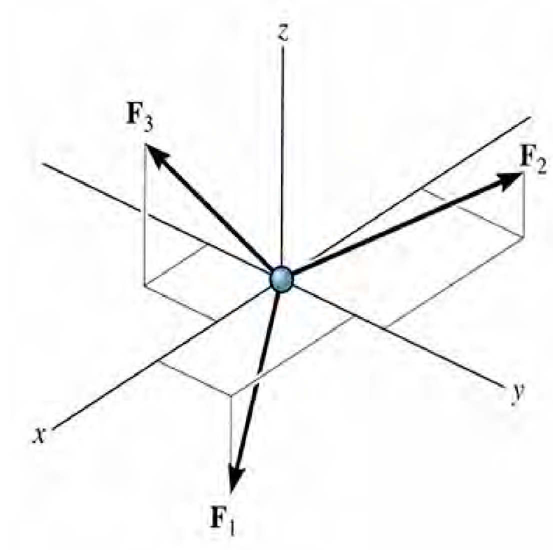


The shear leg derrick is to be designed to lift a maximum of 500 kg of fish.

What is the effect of different offset distances on the forces in the cable and derrick legs?

THE EQUATIONS OF 3-D EQUILIBRIUM

When a particle is in equilibrium, the vector sum of all the forces acting on it must be zero ($\Sigma \boldsymbol{F} = 0$) .



This equation can be written in terms of its x, y and z components. This form is written as follows.

$$(\Sigma F_x)\,\boldsymbol{i} + (\Sigma F_y)\,\boldsymbol{j} + (\Sigma F_z)\,\boldsymbol{k} = 0$$

This vector equation will be satisfied only when

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

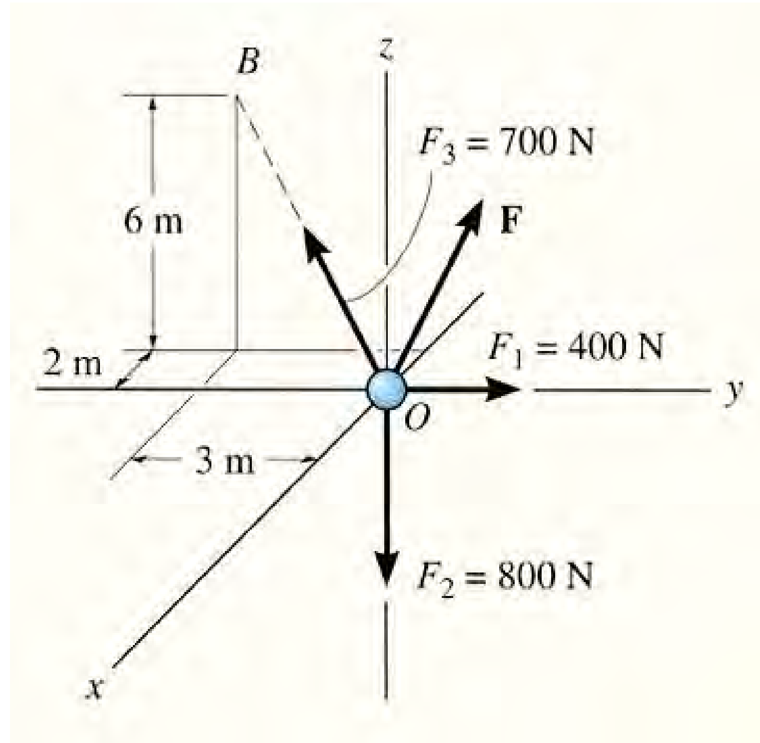
$$\Sigma F_z = 0$$

These equations are the three scalar equations of equilibrium. They are valid at any point in equilibrium and allow you to solve for up to three unknowns.

EXAMPLE #1

Given: $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$.

Find: The force $\boldsymbol{F}$ required to keep particle O in equilibrium.



Plan:

1) Draw a FBD of particle O.

2) Write the unknown force as

$$\boldsymbol{F} = \{F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j} + F_z\,\boldsymbol{k}\}\text{ N}$$

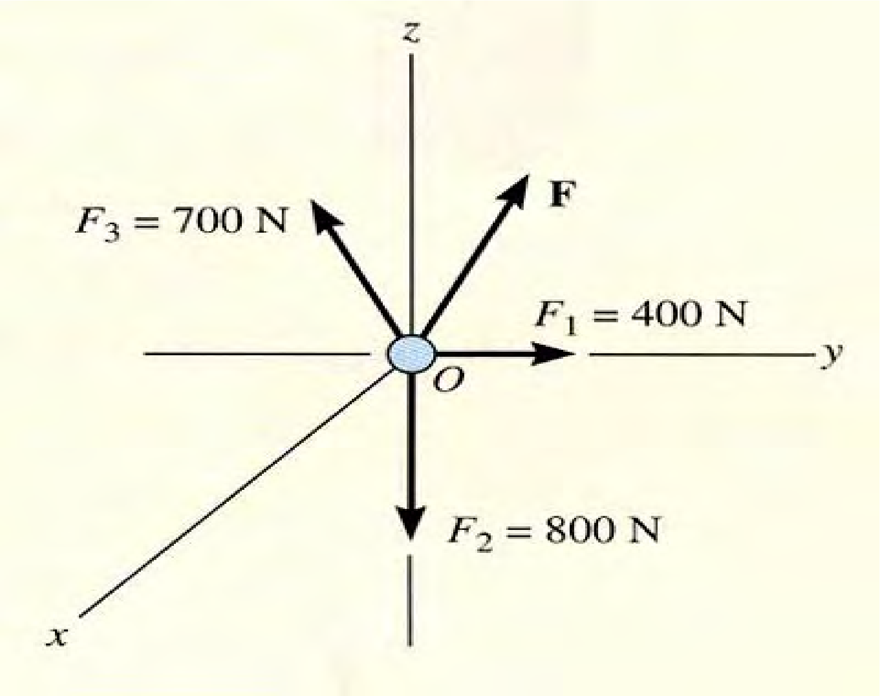
3) Write $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ in Cartesian vector form.

4) Apply the three equilibrium equations to solve for the three unknowns F_x, F_y, and F_z.

EXAMPLE #1 (continued)

$\boldsymbol{F_1} = \{400\ \boldsymbol{j}\}$N

$\boldsymbol{F_2} = \{-800\ \boldsymbol{k}\}$N



$\boldsymbol{F_3} = F_3\,(\boldsymbol{r_B}/\,r_B)$

$= 700\text{ N}\,[(-2\,\boldsymbol{i} - 3\,\boldsymbol{j} + 6\boldsymbol{k})/(2^2 + 3^2 + 6^2)^{½}]$

$= \{-200\,\boldsymbol{i} - 300\,\boldsymbol{j} + 600\,\boldsymbol{k}\}$ N

EXAMPLE #1 (continued)

Equating the respective ***i***, ***j***, ***k*** components to zero, we have

$\Sigma F_x = -200 + F_X = 0$; solving gives $F_x = 200$ N

$\Sigma F_y = 400 - 300 + F_y = 0$; solving gives $F_y = -100$ N

$\Sigma F_z = -800 + 600 + F_z = 0$; solving gives $F_z = 200$ N

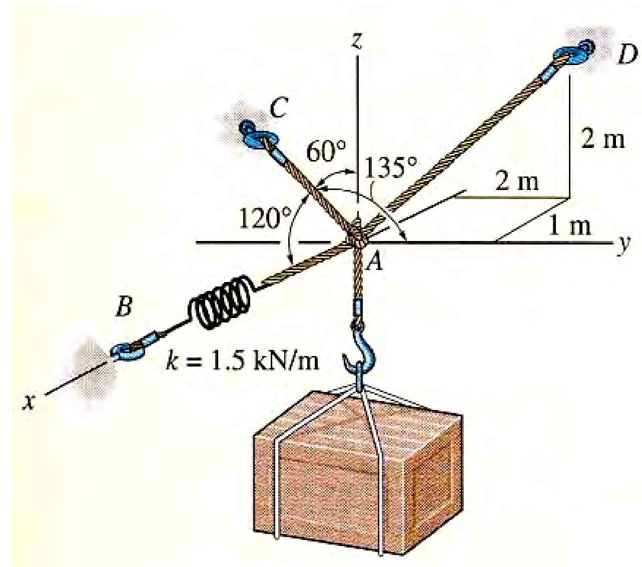
Thus, $\boldsymbol{F} = \{200\,\boldsymbol{i} - 100\,\boldsymbol{j} + 200\,\boldsymbol{k}\}$ N

Using this force vector, you can determine the force's magnitude and coordinate direction angles as needed.

EXAMPLE #2

Given: A 100 Kg crate, as shown, is supported by three cords. One cord has a spring in it.

Find: Tension in cords AC and AD and the stretch of the spring.



<u>Plan</u>:

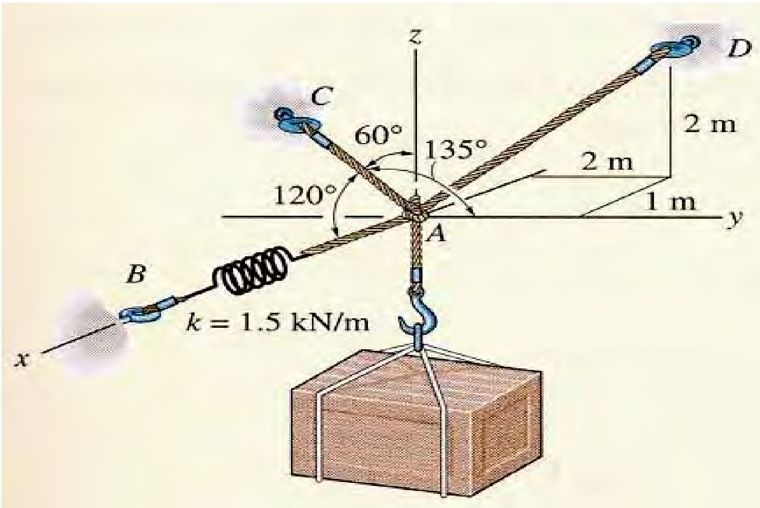
1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

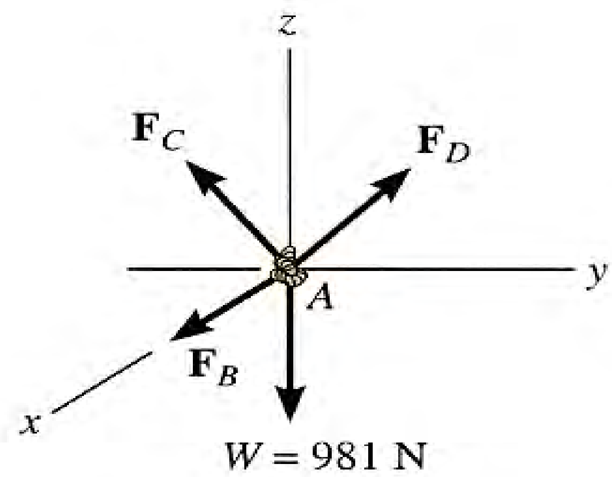
2) Represent each force in the Cartesian vector form.

3) Apply equilibrium equations to solve for the three unknowns.

4) Find the spring stretch using $F_B = K * S$.

EXAMPLE #2 (continued)





FBD at A

$\boldsymbol{F_B} = F_B$ N $\boldsymbol{i}$

$\boldsymbol{F_C} = F_C$ N $(\cos 120° \, \boldsymbol{i} + \cos 135° \, \boldsymbol{j} + \cos 60° \, \boldsymbol{k})$

$= \{- 0.5 \, F_C \, \boldsymbol{i} - 0.707 \, F_C \, \boldsymbol{j} + 0.5 \, F_C \, \boldsymbol{k}\}$ N

$\boldsymbol{F_D} = F_D(\boldsymbol{r_{AD}}/r_{AD})$

$= F_D \, N[(-1 \, \boldsymbol{i} + 2 \, \boldsymbol{j} + 2 \, \boldsymbol{k})/(1^2 + 2^2 + 2^2)^{½}]$

$= \{- 0.3333 \, F_D \, \boldsymbol{i} + 0.667 \, F_D \, \boldsymbol{j} + 0.667 \, F_D \, \boldsymbol{k}\}$N

EXAMPLE #2 (continued)

The weight is $\boldsymbol{W} = (-\ mg)\ \boldsymbol{k} = (-100\ kg * 9.81\ m/sec^2)\ \boldsymbol{k} = \{-\ 981\ \boldsymbol{k}\}\ N$

Now equate the respective ***i*** , ***j*** , ***k*** components to zero.

$$\sum F_x = F_B - 0.5F_C - 0.333F_D = 0$$

$$\sum F_y = -\ 0.707\ F_C + 0.667\ F_D = 0$$

$$\sum F_z = 0.5\ F_C + 0.667\ F_D - 981\ N = 0$$

Solving the three simultaneous equations yields

$F_C = 813\ N$

$F_D = 862\ N$

$F_B = 693.7\ N$

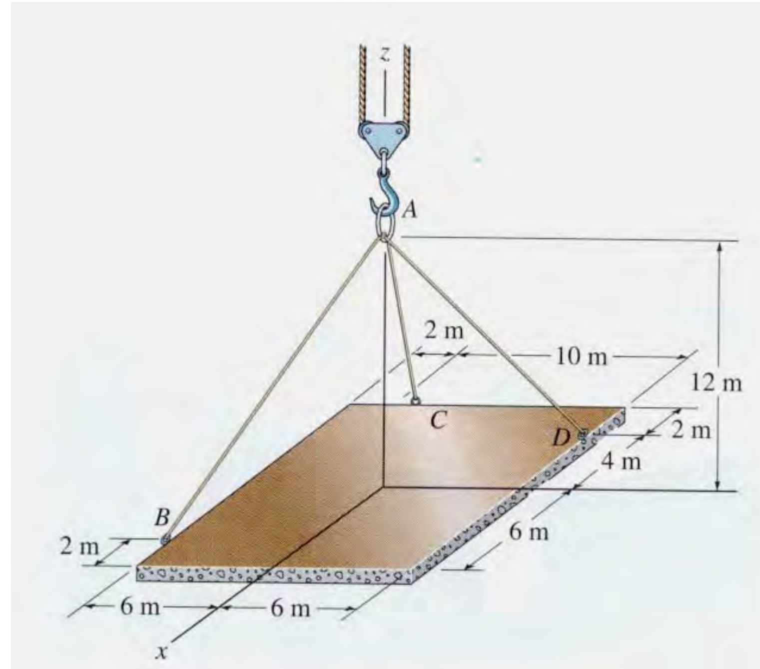
The spring stretch is (from F = k * s)

$$s = F_B / k = 693.7\ N / 1500\ N/m = 0.462\ m$$

GROUP PROBLEM SOLVING

Given: A 150 Kg plate, as shown, is supported by three cables and is in equilibrium.

Find: Tension in each of the cables.



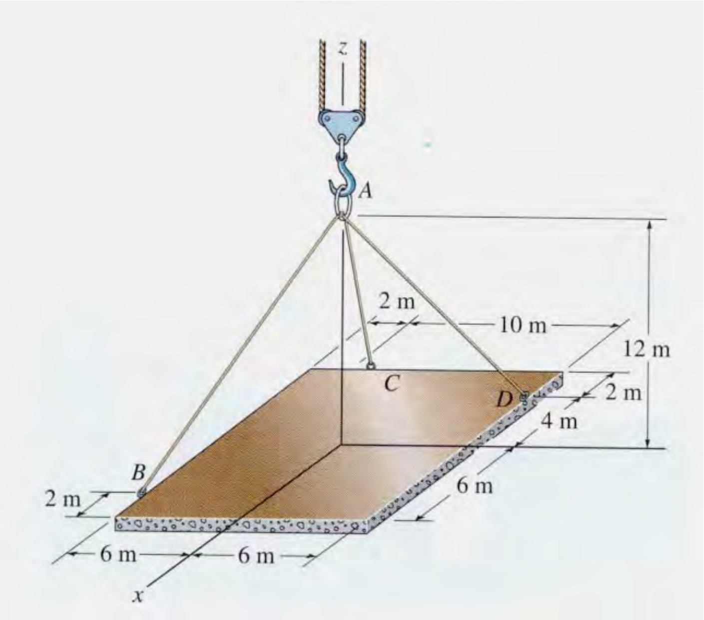
<u>Plan</u>:

1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_B, F_C, F_D .

2) Represent each force in the Cartesian vector form.

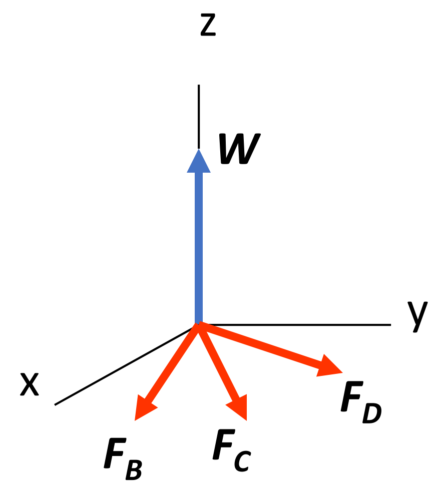
3) Apply equilibrium equations to solve for the three unknowns.

GROUP PROBLEM SOLVING (continued)



FBD of Point A:

Also include some geometry to help with 3D perspective



$\boldsymbol{W}$ = load or weight of plate = (mass)(gravity)
= 150 (9.81) $\boldsymbol{k}$ = 1472 $\boldsymbol{k}$ N

$\boldsymbol{F_B} = F_B(\boldsymbol{r_{AB}}/r_{AB}) = F_B\,\text{N}\,(4\,\boldsymbol{i} - 6\,\boldsymbol{j} - 12\,\boldsymbol{k})\text{m}/(14\text{ m})$

$\boldsymbol{F_C} = F_C(\boldsymbol{r_{AC}}/r_{AC}) = F_C(-6\,\boldsymbol{i} - 4\,\boldsymbol{j} - 12\,\boldsymbol{k})\text{m}/(14\text{ m})$

$\boldsymbol{F_D} = F_D(\boldsymbol{r_{AD}}/r_{AD}) = F_D(-4\,\boldsymbol{i} + 6\,\boldsymbol{j} - 12\,\boldsymbol{k})\text{m}/(14\text{ m})$

GROUP PROBLEM SOLVING (continued)

The particle A is in equilibrium, hence

$\boldsymbol{F_B} + \boldsymbol{F_C} + \boldsymbol{F_D} + \boldsymbol{W} = 0$

Now equate the respective ***i***, ***j***, ***k*** components to zero (i.e., apply the three scalar equations of equilibrium).

$\sum F_x = (4/14)F_B - (6/14)F_C - (4/14)F_D = 0$

$\sum F_y = (-6/14)F_B - (4/14)F_C + (6/14)F_D = 0$

$\sum F_z = (-12/14)F_B - (12/14)F_C - (12/14)F_D + 1472 = 0$

Solving the three simultaneous equations gives

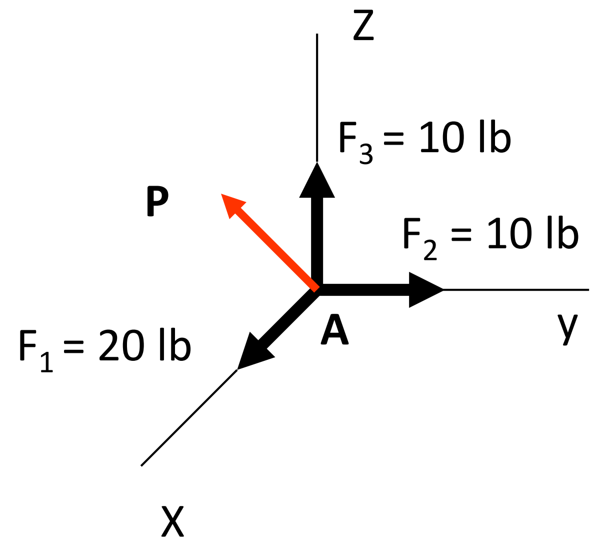
$F_B = 858$ N

$F_C = 0$ N

$F_D = 858$ N

QUIZ

1. Four forces act at point A and point A is in equilibrium. Select the correct force vector ***P.***



A) {-20 ***i*** + 10 ***j*** – 10 ***k***}lb

B) {-10 ***i*** – 20 ***j*** – 10 ***k***} lb

C) {+ 20 ***i*** – 10 ***j*** – 10 ***k***}lb

D) None of the above.

2. In 3-D, when you don't know the direction or the magnitude of a force, how many unknowns do you have corresponding to that force?

A) One B) Two C) Three D) Four