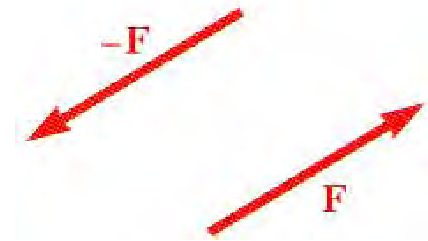
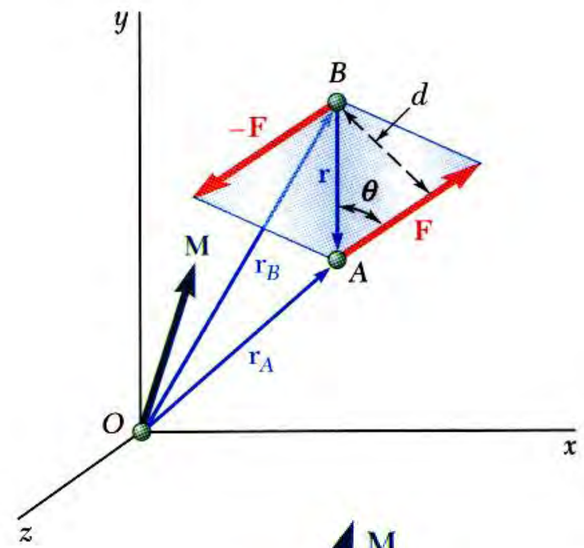
Moment of a Couple

- Two forces ***F*** and ***-F*** having the same magnitude, parallel lines of action, and opposite sense are said to form a *couple*.

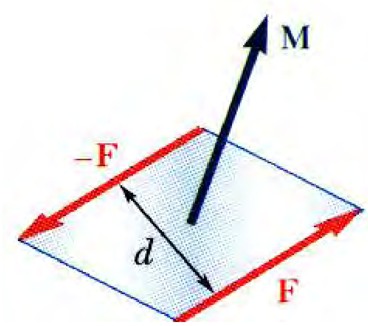


- Moment of the couple,

$$\vec{M} = \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right)$$
$$= \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F}$$
$$= \vec{r} \times \vec{F}$$
$$M = rF\sin\theta = Fd$$



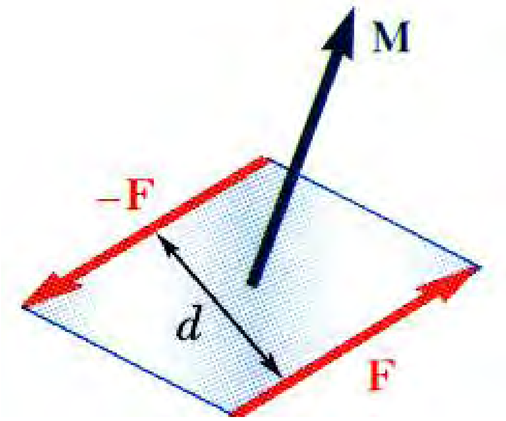
- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.

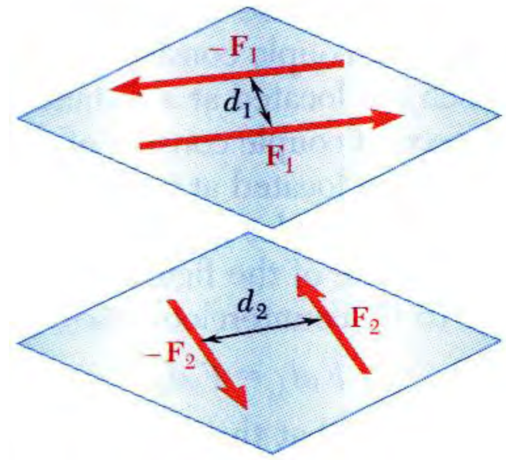


Moment of a Couple

Two couples will have equal moments if

- $F_1 d_1 = F_2 d_2$
- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.



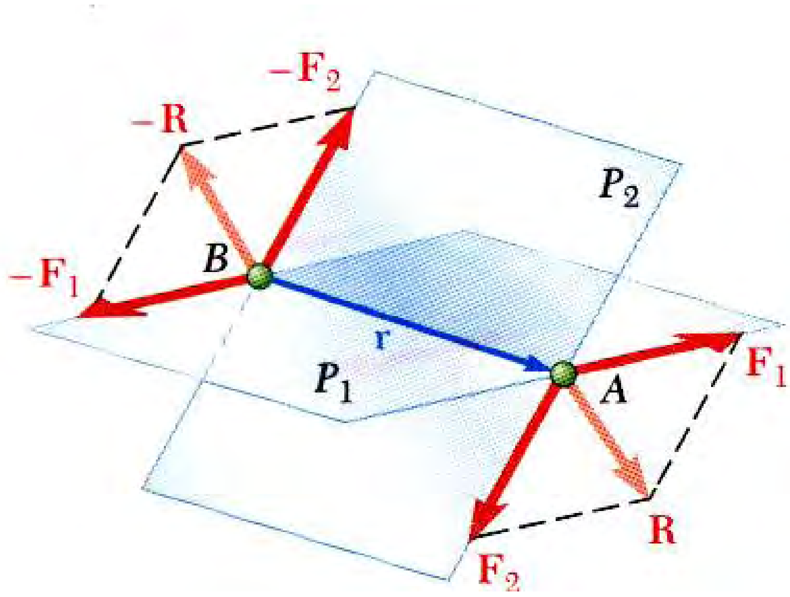


Addition of Couples

- Consider two intersecting planes P_1 and P_2 with each containing a couple

$$\vec{M}_1 = \vec{r} \times \vec{F}_1 \text{ in plane } P_1$$

$$\vec{M}_2 = \vec{r} \times \vec{F}_2 \text{ in plane } P_2$$



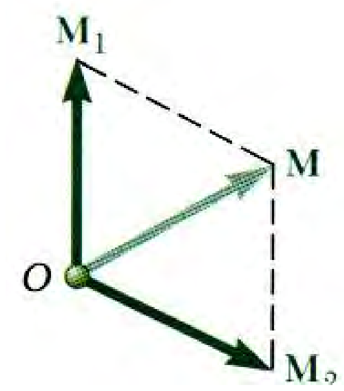
- Resultants of the vectors also form a couple

$$\vec{M} = \vec{r} \times \vec{R} = \vec{r} \times \left(\vec{F}_1 + \vec{F}_2\right)$$

- By Varigon's theorem

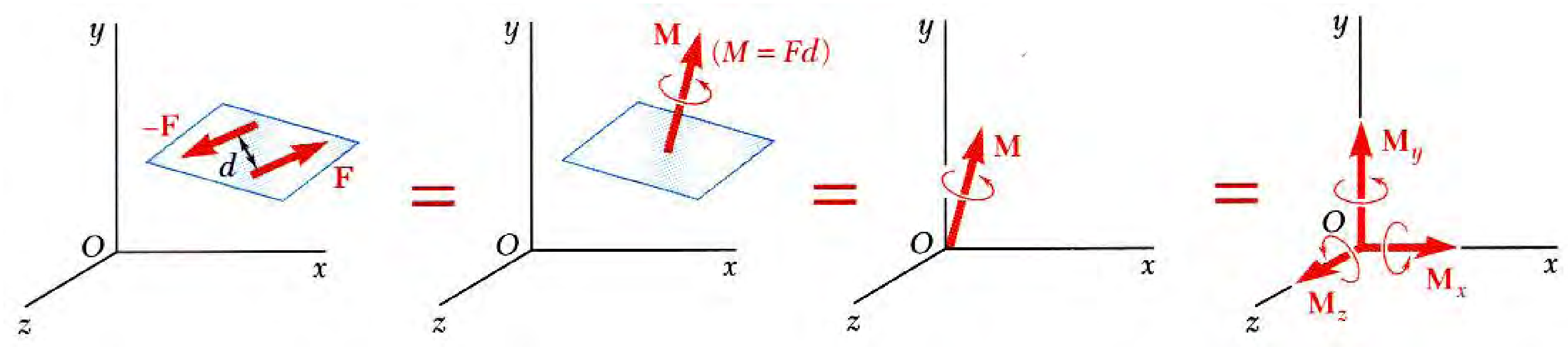
$$\vec{M} = \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2$$

$$= \vec{M}_1 + \vec{M}_2$$



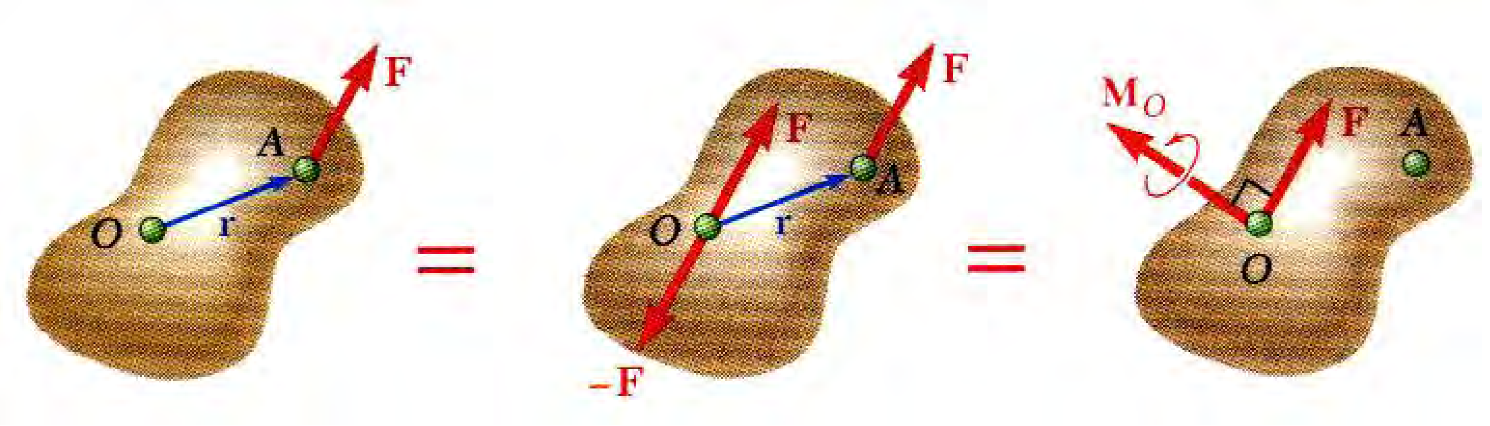
- Sum of two couples is also a couple that is equal to the vector sum of the two couples

Couples Can Be Represented by Vectors



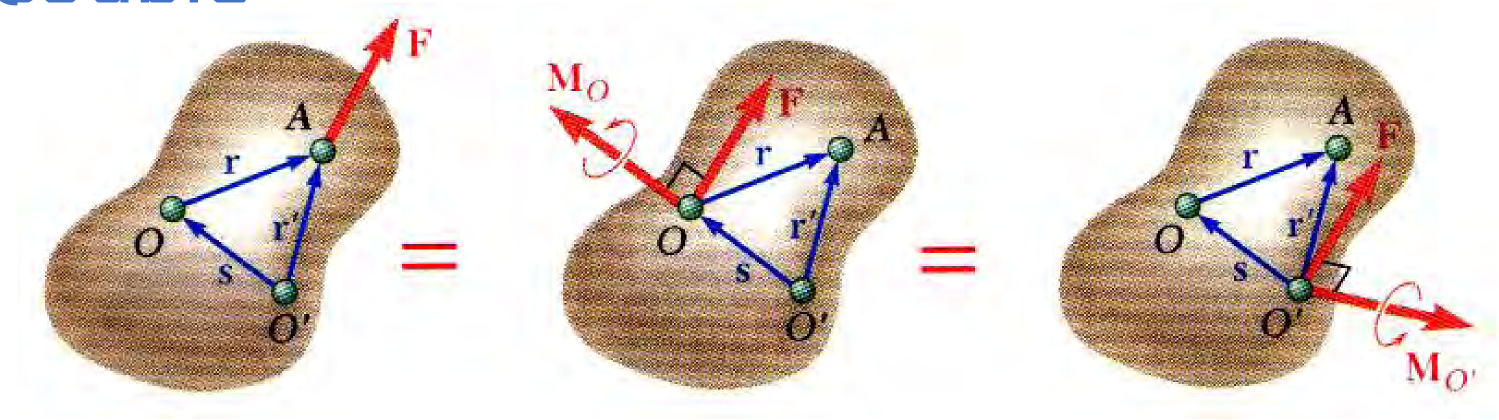
- A couple can be represented by a vector with magnitude and direction equal to the moment of the couple.
- *Couple vectors* obey the law of addition of vectors.
- Couple vectors are free vectors, i.e., the point of application is not significant.
- Couple vectors may be resolved into component vectors.

Resolution of a Force Into a Force at *O* and a Couple



- Force vector ***F*** can not be simply moved to *O* without modifying its action on the body.
- Attaching equal and opposite force vectors at *O* produces no net effect on the body.
- The three forces may be replaced by an equivalent force vector and couple vector, i.e, a *force-couple system*.

Resolution of a Force Into a Force at *O* and a Couple



- Moving ***F*** from *A* to a different point *O'* requires the addition of a different couple vector ***M_O'***

$$\vec{M}_{O'} = \vec{r}\,' \times \vec{F}$$

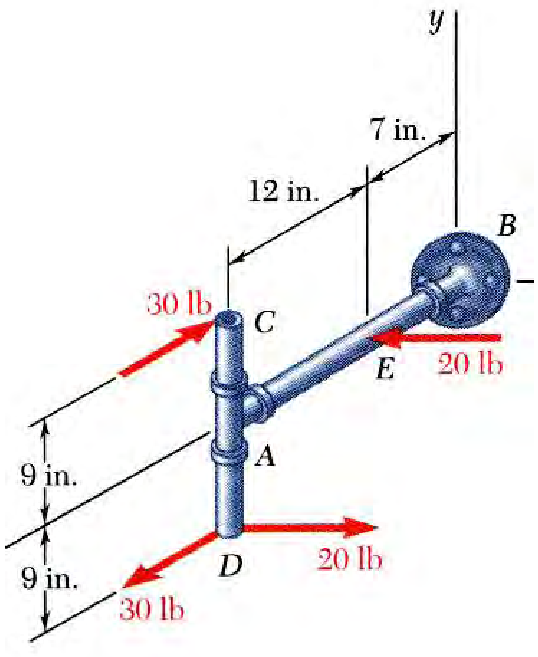
- The moments of ***F*** about O and *O'* are related,

$$\vec{M}_{O'} = \vec{r}\,' \times \vec{F} = (\vec{r} + \vec{s}) \times \vec{F} = \vec{r} \times \vec{F} + \vec{s} \times \vec{F}$$

$$= \vec{M}_O + \vec{s} \times \vec{F}$$

- Moving the force-couple system from *O* to *O'* requires the addition of the moment of the force at *O* about *O'*.

Sample Problem

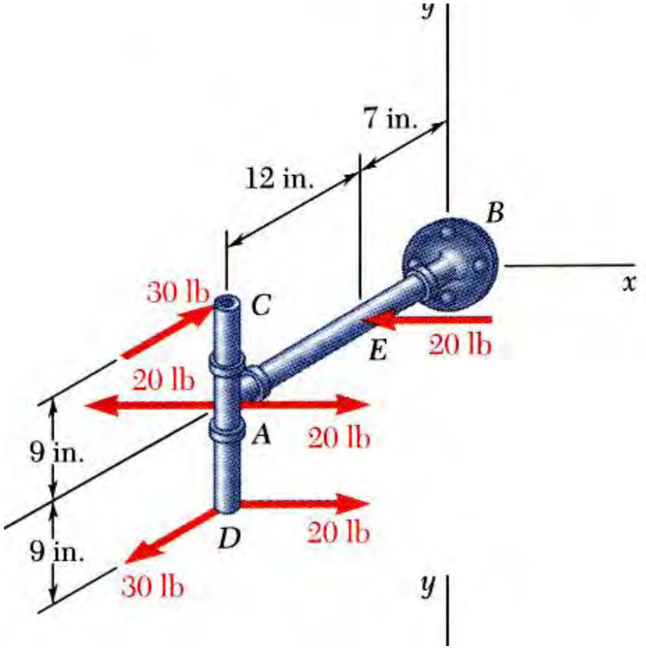


Determine the components of the single couple equivalent to the couples shown.

SOLUTION:

- Attach equal and opposite 20 lb forces in the $\pm x$ direction at *A*, thereby producing 3 couples for which the moment components are easily computed.
- Alternatively, compute the sum of the moments of the four forces about an arbitrary single point. The point *D* is a good choice as only two of the forces will produce non-zero moment contributions..

Sample Problem



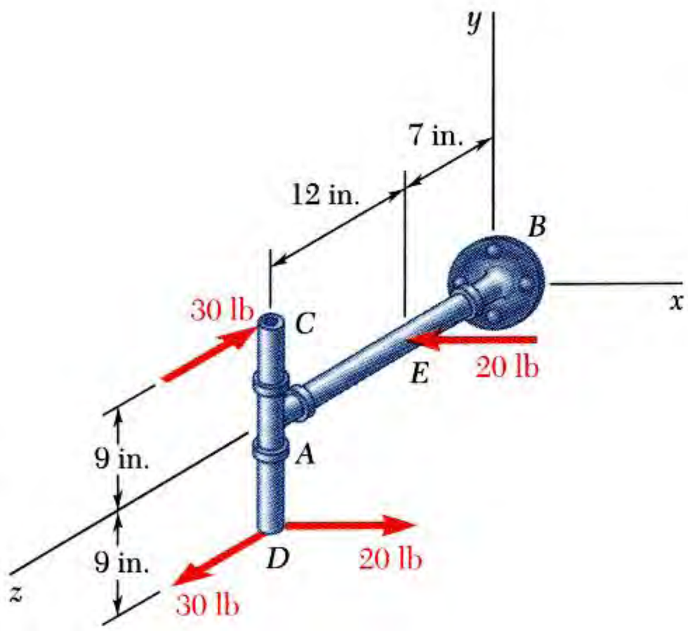
- Attach equal and opposite 20 lb forces in the $\pm x$ direction at *A*

- The three couples may be represented by three couple vectors,

$$M_x = -(30\text{ lb})(18\text{ in.}) = -540\text{ lb}\cdot\text{in.}$$
$$M_y = +(20\text{ lb})(12\text{ in.}) = +240\text{lb}\cdot\text{in.}$$
$$M_z = +(20\text{ lb})(9\text{ in.}) = +180\text{ lb}\cdot\text{in.}$$

$$\vec{M} = -(540\text{ lb}\cdot\text{in.})\vec{i} + (240\text{lb}\cdot\text{in.})\vec{j} + (180\text{ lb}\cdot\text{in.})\vec{k}$$

Sample Problem



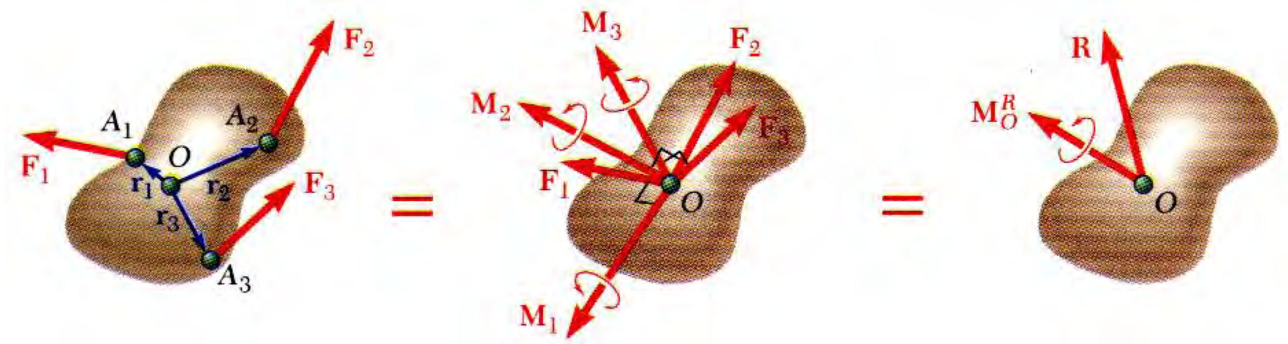
- Alternatively, compute the sum of the moments of the four forces about *D*.

- Only the forces at *C* and *E* contribute to the moment about *D*.

$$\vec{M} = \vec{M}_D = (18\text{ in.})\vec{j} \times (-30\text{ lb})\vec{k} + \left[(9\text{ in.})\vec{j} - (12\text{ in.})\vec{k}\right] \times (-20\text{ lb})\vec{i}$$

$$\vec{M} = -(540\text{ lb}\cdot\text{in.})\vec{i} + (240\text{lb}\cdot\text{in.})\vec{j} + (180\text{ lb}\cdot\text{in.})\vec{k}$$

System of Forces: Reduction to a Force and Couple



- A system of forces may be replaced by a collection of force-couple systems acting a given point *O*

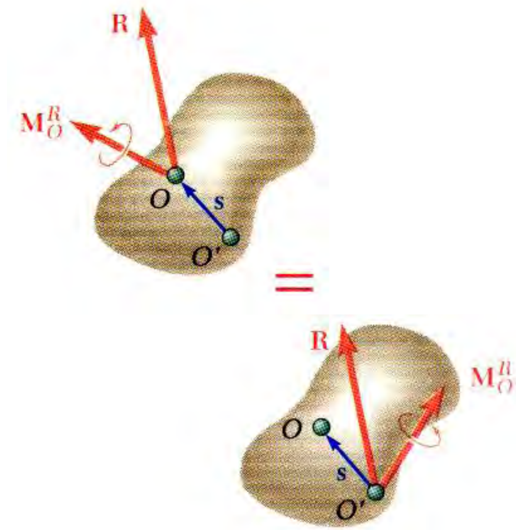
- The force and couple vectors may be combined into a resultant force vector and a resultant couple vector,

$$\vec{R} = \sum \vec{F} \qquad \vec{M}_O^R = \sum \left(\vec{r} \times \vec{F} \right)$$

- The force-couple system at *O* may be moved to *O'* with the addition of the moment of ***R*** about *O'* ,

$$\vec{M}_{O'}^R = \vec{M}_O^R + \vec{s} \times \vec{R}$$

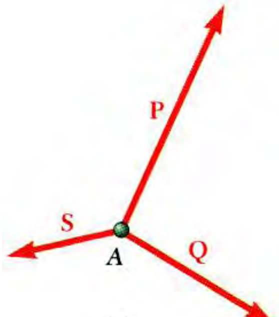
- Two systems of forces are equivalent if they can be reduced to the same force-couple system.



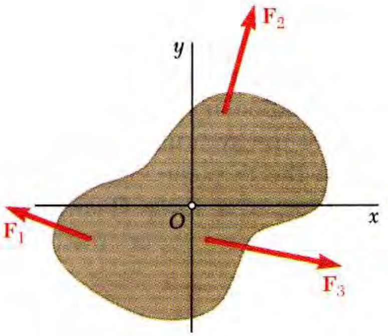
Further Reduction of a System of Forces

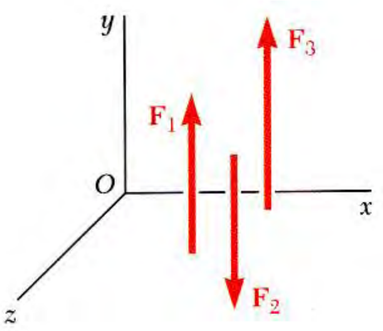
- If the resultant force and couple at *O* are mutually perpendicular, they can be replaced by a single force acting along a new line of action.

- The resultant force-couple system for a system of forces will be mutually perpendicular if:
  1) the forces are concurrent,
  2) the forces are coplanar, or
  3) the forces are parallel.

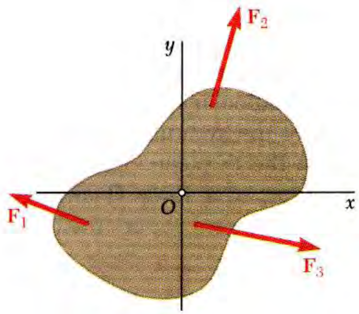


(a)

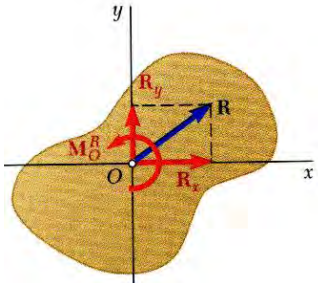


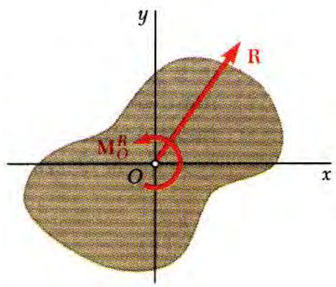


Further Reduction of a System of Forces

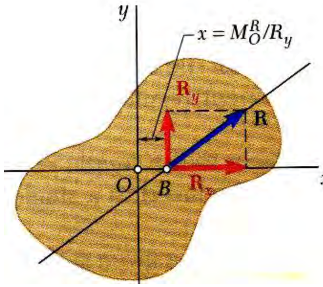


- System of coplanar forces is reduced to a force-couple system $\vec{R}$ and $\vec{M}_O^R$ that is mutually perpendicular.



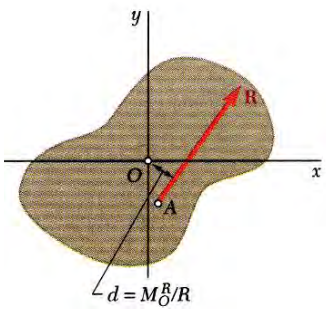


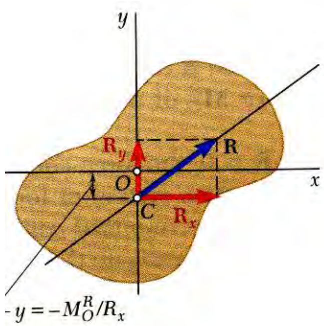
- System can be reduced to a single force by moving the line of action of $\vec{R}$ until its moment about O becomes $\vec{M}_O^R$



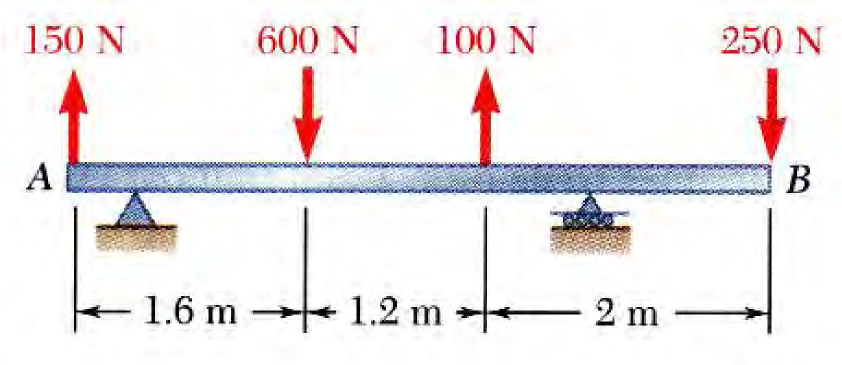
- In terms of rectangular coordinates,

$$xR_y - yR_x = M_O^R$$





Sample Problem



For the beam, reduce the system of forces shown to (a) an equivalent force-couple system at *A*, (b) an equivalent force couple system at *B*, and (c) a single force or resultant.

Note: Since the support reactions are not included, the given system will not maintain the beam in equilibrium.

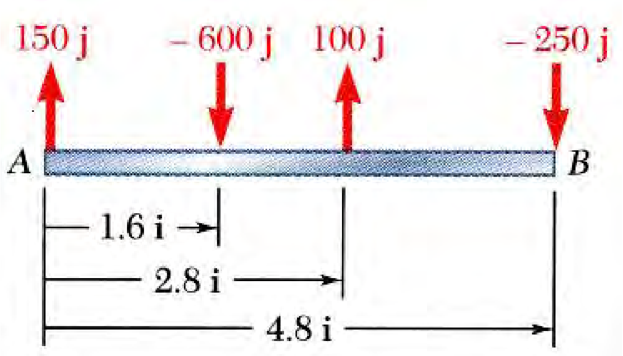
<u>SOLUTION:</u>

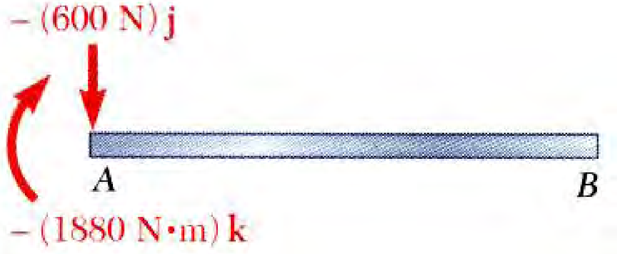
a) Compute the resultant force for the forces shown and the resultant couple for the moments of the forces about *A*.

b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

c) Determine the point of application for the resultant force such that its moment about *A* is equal to the resultant couple at *A*.

Sample Problem





<u>SOLUTION:</u>

a) Compute the resultant force and the resultant couple at *A*.

$$\vec{R} = \sum \vec{F}$$

$$= (150\text{ N})\vec{j} - (600\text{ N})\vec{j} + (100\text{ N})\vec{j} - (250\text{ N})\vec{j}$$

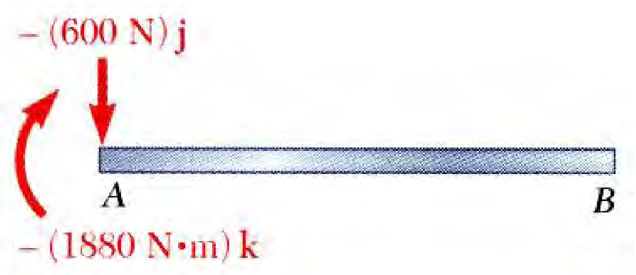
$$\boxed{\vec{R} = -(600\text{ N})\vec{j}}$$

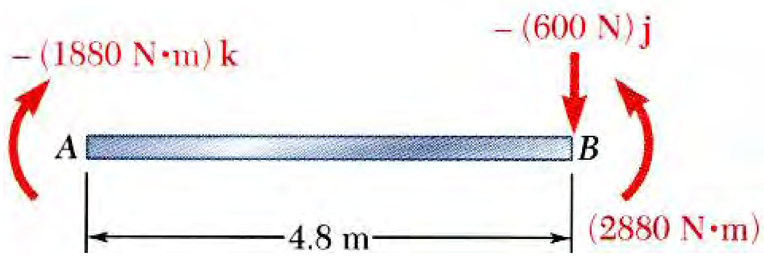
$$\vec{M}_A^R = \sum(\vec{r} \times \vec{F})$$

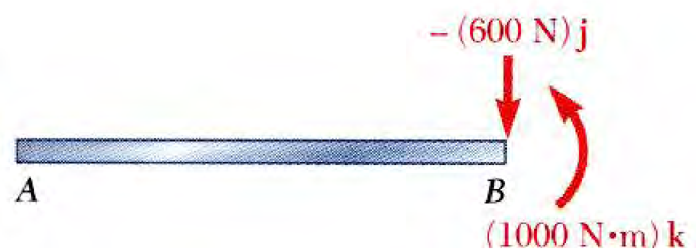
$$= (1.6\vec{i}) \times (-600\vec{j}) + (2.8\vec{i}) \times (100\vec{j}) + (4.8\vec{i}) \times (-250\vec{j})$$

$$\boxed{\vec{M}_A^R = -(1880\text{ N}\cdot\text{m})\vec{k}}$$

Sample Problem







b) Find an equivalent force-couple system at *B* based on the force-couple system at *A*.

The force is unchanged by the movement of the force-couple system from *A* to *B*.

$$\vec{R} = -(600\ \text{N})\vec{j}$$

The couple at *B* is equal to the moment about *B* of the force-couple system found at *A*.

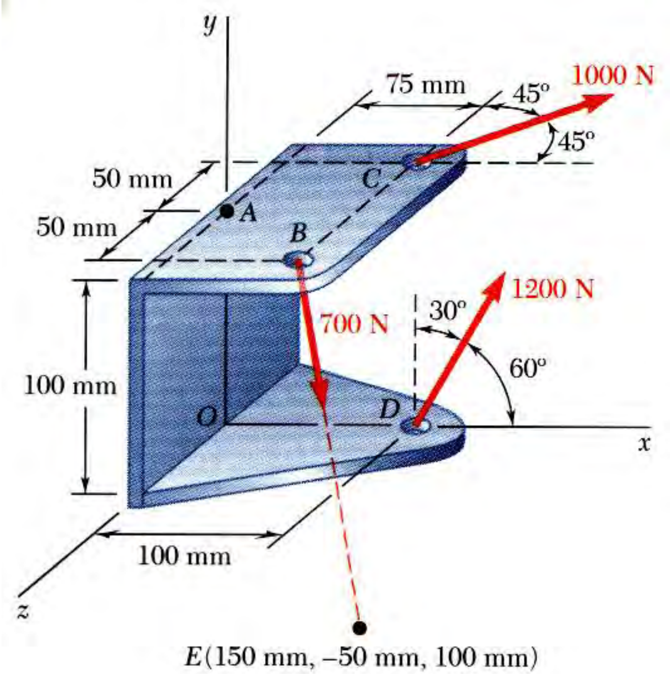
$$\vec{M}_B^R = \vec{M}_A^R + \vec{r}_{B/A} \times \vec{R}$$

$$= -(1880\ \text{N}\cdot\text{m})\vec{k} + (-4.8\ \text{m})\vec{i} \times (-600\ \text{N})\vec{j}$$

$$= -(1880\ \text{N}\cdot\text{m})\vec{k} + (2880\ \text{N}\cdot\text{m})\vec{k}$$

$$\vec{M}_B^R = +(1000\ \text{N}\cdot\text{m})\vec{k}$$

Sample Problem



Three cables are attached to the bracket as shown. Replace the forces with an equivalent force-couple system at *A*.

<u>SOLUTION:</u>

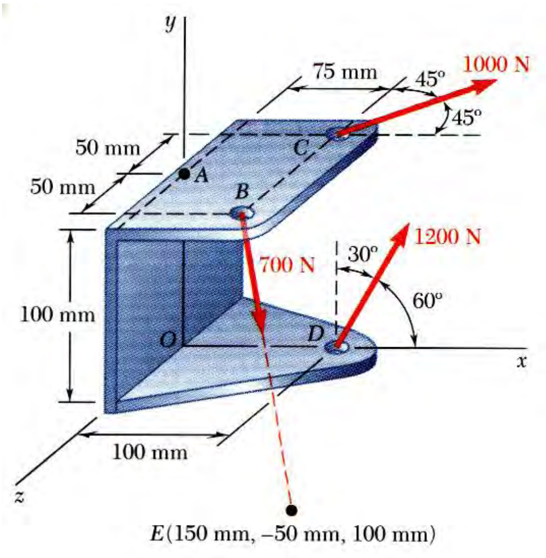
- Determine the relative position vectors for the points of application of the cable forces with respect to *A*.
- Resolve the forces into rectangular components.
- Compute the equivalent force,

$$\vec{R} = \sum \vec{F}$$

- Compute the equivalent couple,

$$\vec{M}_A^R = \sum \left(\vec{r} \times \vec{F} \right)$$

Sample Problem



SOLUTION:

- Determine the relative position vectors with respect to *A*.

$$\vec{r}_{B/A} = 0.075\vec{i} + 0.050\vec{k}\ (\text{m})$$

$$\vec{r}_{C/A} = 0.075\vec{i} - 0.050\vec{k}\ (\text{m})$$

$$\vec{r}_{D/A} = 0.100\vec{i} - 0.100\vec{j}\ (\text{m})$$

- Resolve the forces into rectangular components.

$$\vec{F}_B = (700\ \text{N})\vec{\lambda}$$

$$\vec{\lambda} = \frac{\vec{r}_{E/B}}{r_{E/B}} = \frac{75\vec{i} - 150\vec{j} + 50\vec{k}}{175}$$

$$= 0.429\vec{i} - 0.857\vec{j} + 0.289\vec{k}$$

$$\vec{F}_B = 300\vec{i} - 600\vec{j} + 200\vec{k}\ \ (\text{N})$$

$$\vec{F}_C = (1000\ \text{N})(\cos 45\vec{i} - \cos 45\vec{j})$$

$$= 707\vec{i} - 707\vec{j}\ \ (\text{N})$$

$$\vec{F}_D = (1200\ \text{N})(\cos 60\vec{i} + \cos 30\vec{j})$$

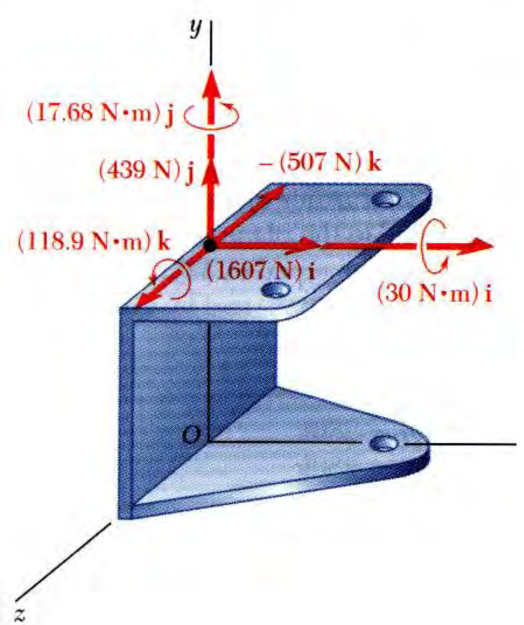
$$= 600\vec{i} + 1039\vec{j}\ \ (\text{N})$$

Sample Problem

- Compute the equivalent force,

$$\vec{R} = \sum \vec{F}$$
$$= (300 + 707 + 600)\vec{i}$$
$$+ (-600 + 1039)\vec{j}$$
$$+ (200 - 707)\vec{k}$$

$$\vec{R} = 1607\vec{i} + 439\vec{j} - 507\vec{k} \quad (\text{N})$$



- Compute the equivalent couple,

$$\vec{M}_A^R = \sum (\vec{r} \times \vec{F})$$

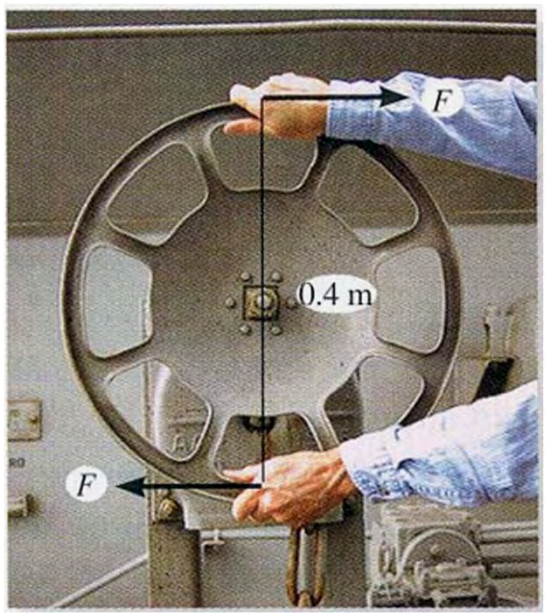
$$\vec{r}_{B/A} \times \vec{F}_B = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.075 & 0 & 0.050 \\ 300 & -600 & 200 \end{vmatrix} = 30\vec{i} - 45\vec{k}$$

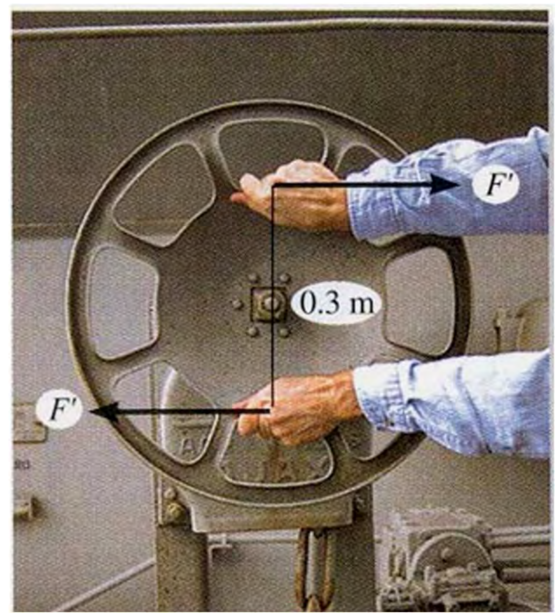
$$\vec{r}_{C/A} \times \vec{F}_c = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.075 & 0 & -0.050 \\ 707 & 0 & -707 \end{vmatrix} = 17.68\vec{j}$$

$$\vec{r}_{D/A} \times \vec{F}_D = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.100 & -0.100 & 0 \\ 600 & 1039 & 0 \end{vmatrix} = 163.9\vec{k}$$

$$\vec{M}_A^R = 30\vec{i} + 17.68\vec{j} + 118.9\vec{k}$$

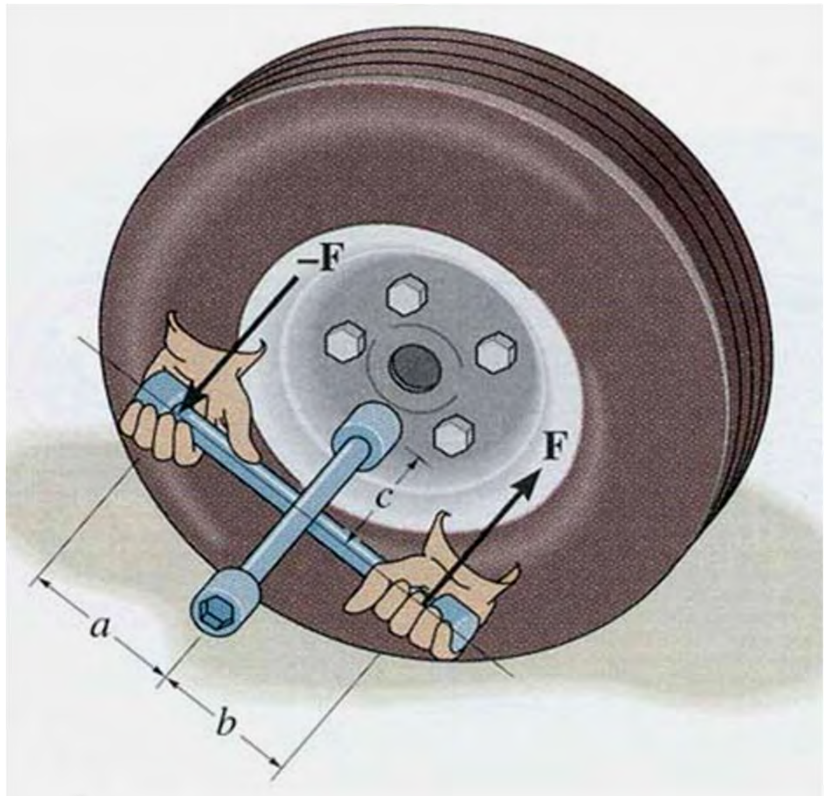
APPLICATIONS





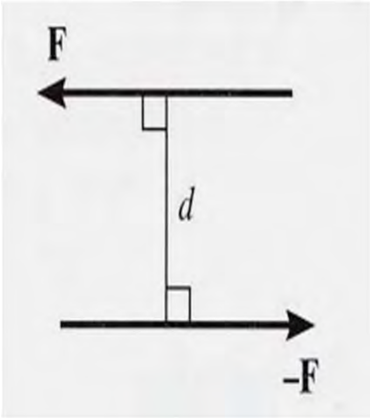
A torque or moment of 12 N · m is required to rotate the wheel. Which one of the two grips of the wheel above will require less force to rotate the wheel?

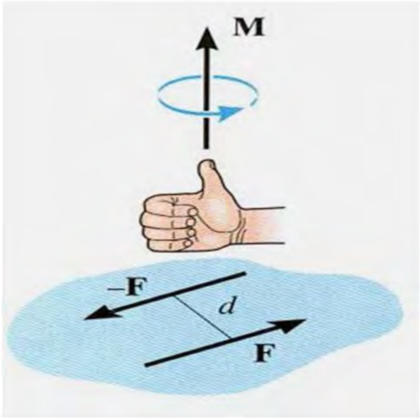
APPLICATIONS (continued)



The crossbar lug wrench is being used to loosen a lug net. What is the effect of changing dimensions a, b, or c on the force that must be applied?

MOMENT OF A COUPLE (Section 4.6)





A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a perpendicular distance d.

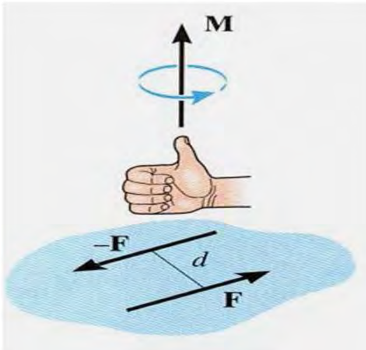
The moment of a couple is defined as

M_O = F d (using a scalar analysis) or as

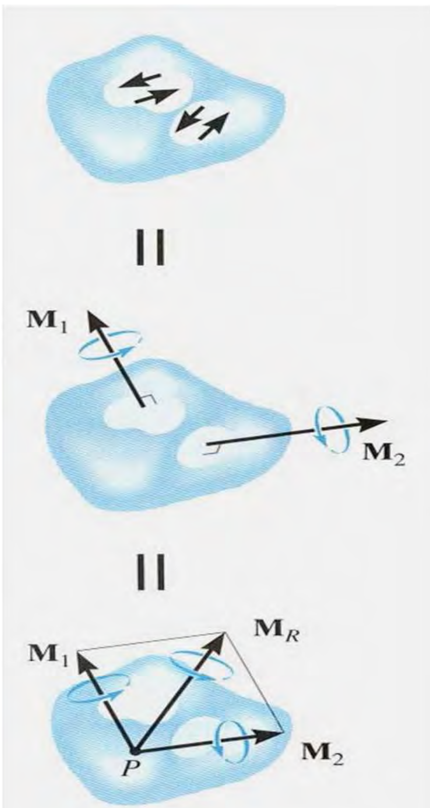
$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis).

Here ***r*** is any position vector from the line of action of ***–F*** to the line of action of ***F***.

MOMENT OF A COUPLE (continued)



The net external effect of a couple is that the net force equals zero and the magnitude of the net moment equals F d

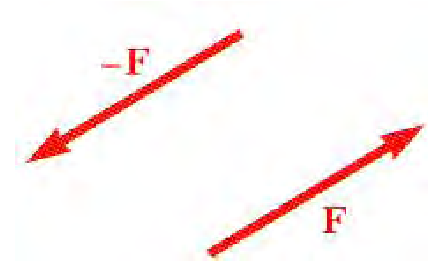


Since the moment of a couple depends only on the distance between the forces, the moment of a couple is a **free vector**. It can be moved anywhere on the body and have the same external effect on the body.

Moments due to couples can be added using the same rules as adding any vectors.

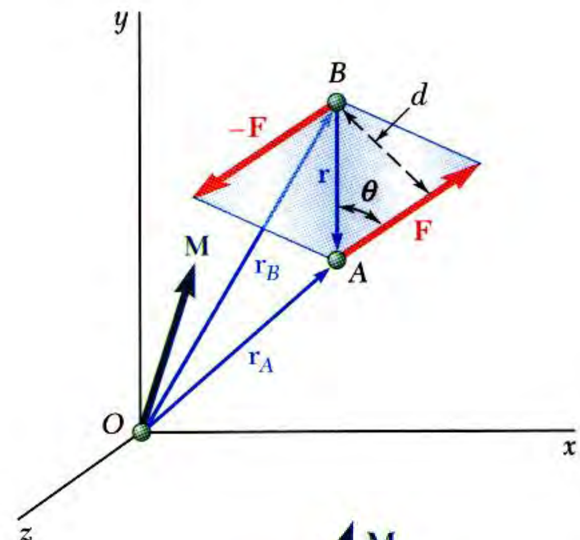
Moment of a Couple

- Two forces ***F*** and ***-F*** having the same magnitude, parallel lines of action, and opposite sense are said to form a *couple*.

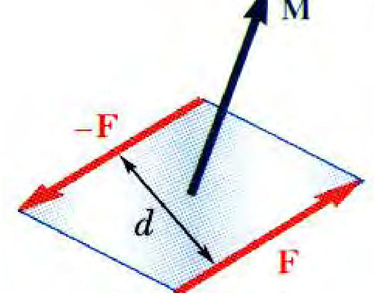


- Moment of the couple,

$$\begin{aligned} \vec{M} &= \vec{r}_A \times \vec{F} + \vec{r}_B \times \left(-\vec{F}\right) \\ &= \left(\vec{r}_A - \vec{r}_B\right) \times \vec{F} \\ &= \vec{r} \times \vec{F} \\ M &= rF \sin\theta = Fd \end{aligned}$$



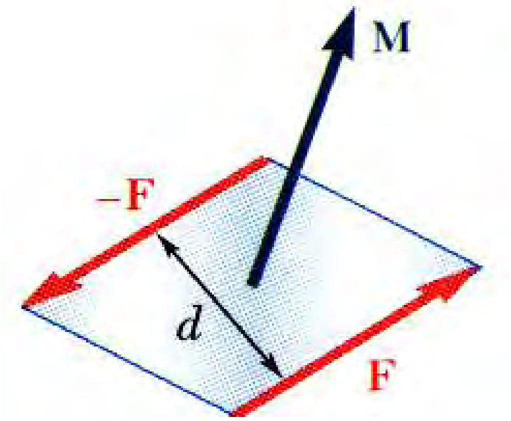
- The moment vector of the couple is independent of the choice of the origin of the coordinate axes, i.e., it is a *free vector* that can be applied at any point with the same effect.

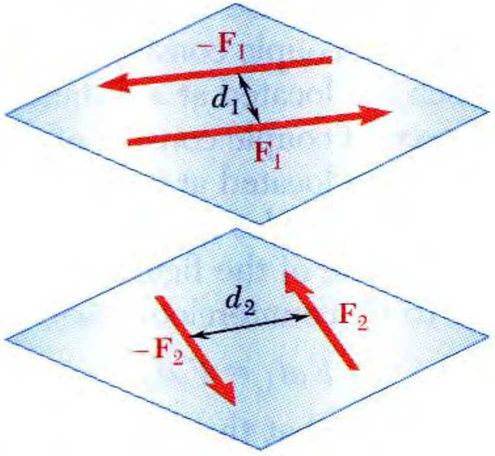


Moment of a Couple

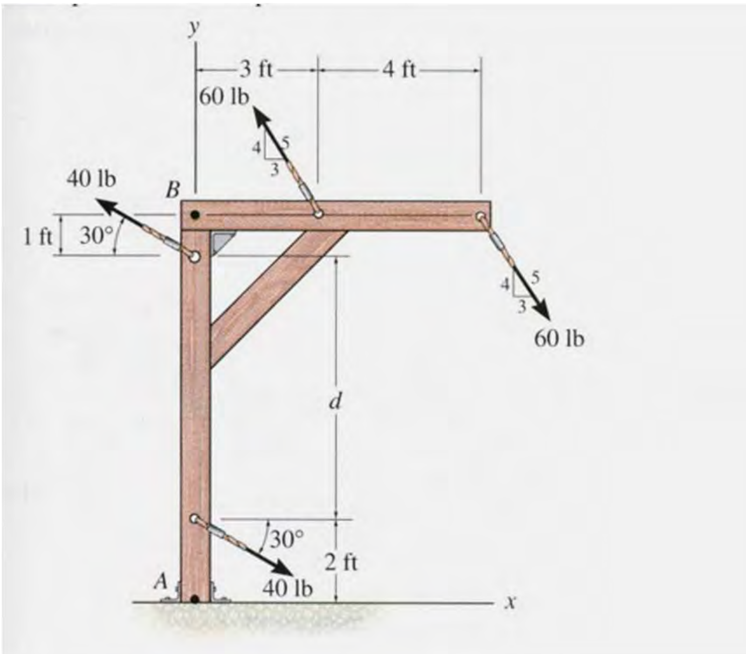
Two couples will have equal moments if

- $F_1 d_1 = F_2 d_2$
- the two couples lie in parallel planes, and
- the two couples have the same sense or the tendency to cause rotation in the same direction.





EXAMPLE - SCALAR APPROACH



Given: Two couples act on the beam and d equals 8 ft.

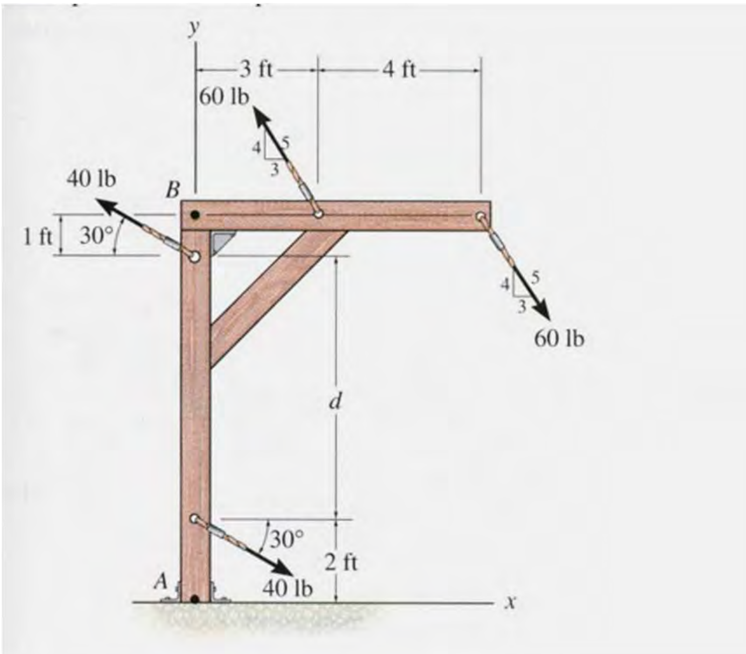
Find: The resultant couple

Plan:

1) Resolve the forces in x and y directions so they can be treated as couples.

2) Determine the net moment due to the two couples.

Solution:



The x and y components of the top 60 lb force are:

(4/5)(60 lb) = 48 lb vertically up

(3/5)(60 lb) = 36 lb to the left

Similarly for the top 40 lb force:

(40 lb) (sin 30°) up

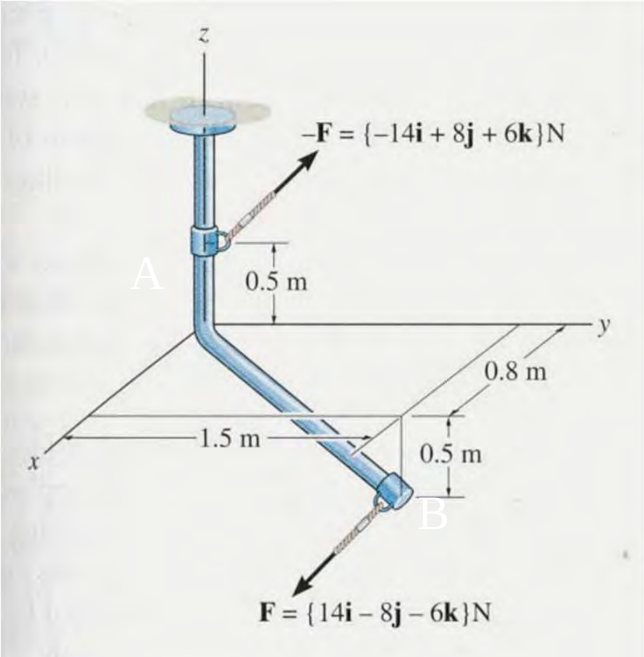
(40 lb) (cos 30°) to the left

The net moment equals to

$$+\circlearrowright \Sigma M = -(48\text{ lb})(4\text{ ft}) + (40\text{ lb})(\cos 30^\circ)(8\text{ft})$$

$$= -192.0 + 277.1 = 85.1\text{ ft}\cdot\text{lb}$$

EXAMPLE - VECTOR



Given: A force couple acting on the rod.

Find: The couple moment acting on the rod in Cartesian vector notation.

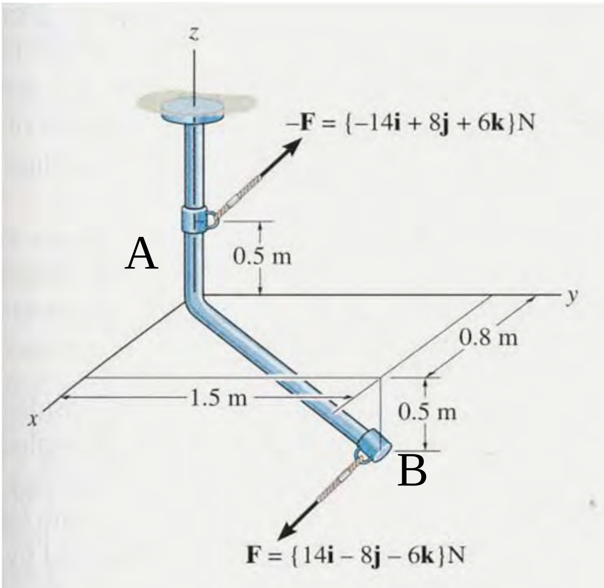
Plan:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{14\,\boldsymbol{i} - 8\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

Solution:



$\boldsymbol{r}_{AB} = \{0.8\,\boldsymbol{i} + 1.5\,\boldsymbol{j} - 1\,\boldsymbol{k}\}$ m

$\boldsymbol{F} = \{14\,\boldsymbol{i} - 8\,\boldsymbol{j} - 6\,\boldsymbol{k}\}$ N

$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$

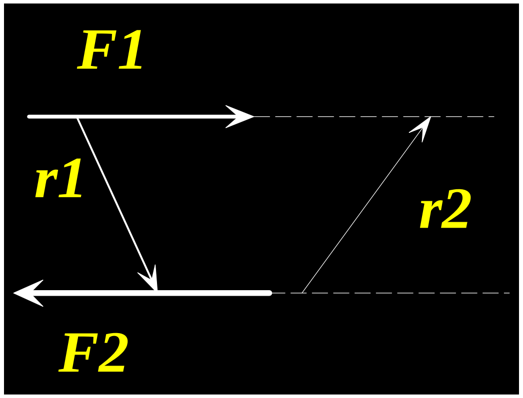
$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.8 & 1.5 & -1 \\ 14 & -8 & -6 \end{vmatrix} \text{ N}\cdot\text{m}$$

$= \{\boldsymbol{i}\,(-9 - (-8)) - \boldsymbol{j}\,(-\,4.8 - (-14)) + \boldsymbol{k}\,(-4.8 - -14(1.5))\}$ N·m

$= \{-17\,\boldsymbol{i} - 9.2\,\boldsymbol{j} - 21\,\boldsymbol{k}\}$ N·m

CONCEPT QUIZ

1. $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ form a couple. The moment of the couple is given by ____ .



A) $\boldsymbol{r_1} \times \boldsymbol{F_1}$ B) $\boldsymbol{r_2} \times \boldsymbol{F_1}$

C) $\boldsymbol{F_2} \times \boldsymbol{r_1}$ D) $\boldsymbol{r_2} \times \boldsymbol{F_2}$

2. If three couples act on a body, the overall result is that

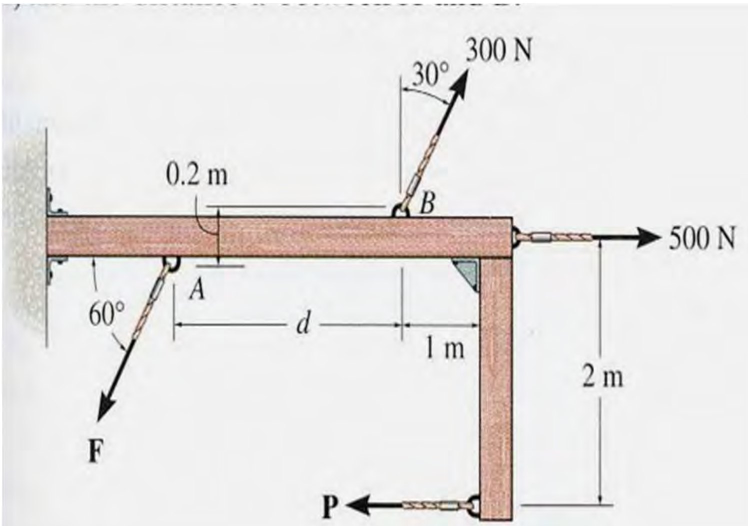
A) the net force is not equal to 0.

B) the net force and net moment are equal to 0.

C) the net moment equals 0 but the net force is not necessarily equal to 0.

D) the net force equals 0 but the net moment is not necessarily equal to 0 .

GROUP PROBLEM SOLVING - SCALAR



Given: Two couples act on the beam. The resultant couple is zero.

Find: The magnitudes of the forces P and F and the distance d.

PLAN:

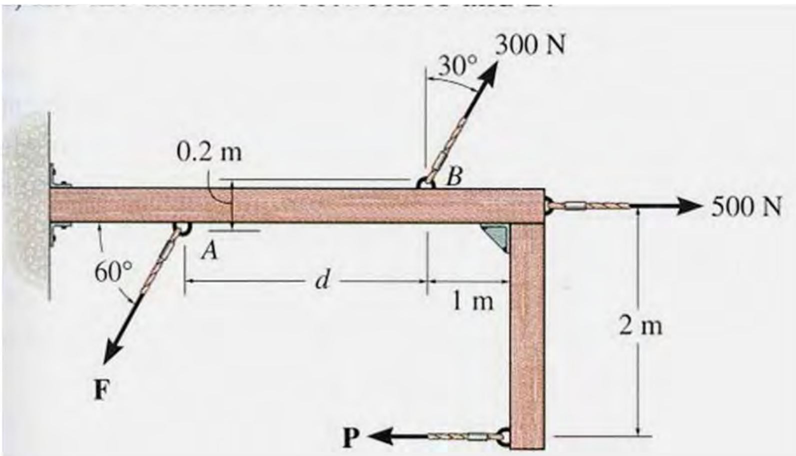
1) Use definition of a couple to find P and F.

2) Resolve the 300 N force in x and y directions.

3) Determine the net moment.

4) Equate the net moment to zero to find d.

Solution:



From the definition of a couple

P = 500 N and

F = 300 N.

Resolve the 300 N force into vertical and horizontal components. The vertical component is ($300 \cos 30^\circ$) N and the horizontal component is ($300 \sin 30^\circ$) N.

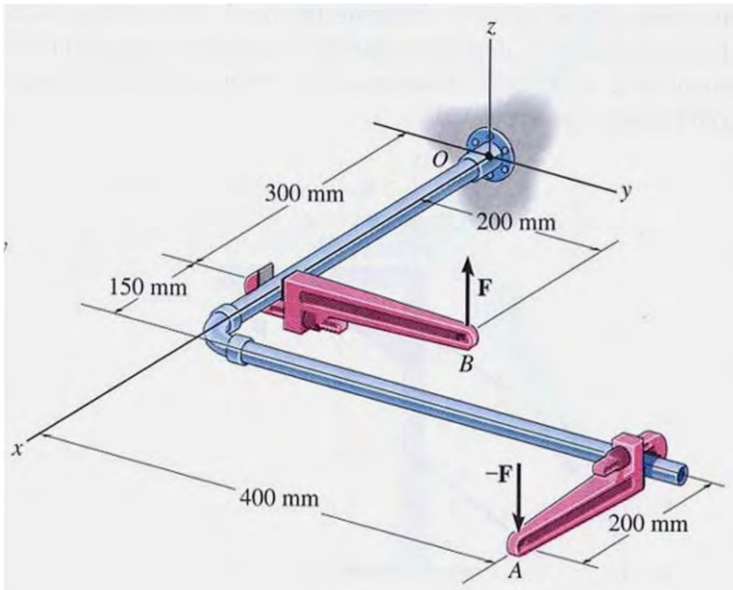
It was given that the net moment equals zero. So

$$+\circlearrowright \Sigma M = -(500)(2) + (300 \cos 30^\circ)(d) + (300 \sin 30^\circ)(0.2) = 0$$

Now solve this equation for d.

$$d = (1000 - 60 \sin 30^\circ) / (300 \cos 30^\circ) = 3.96 \text{ m}$$

GROUP PROBLEM SOLVING - VECTOR



Given: $\boldsymbol{F} = \{25\ \boldsymbol{k}\}$ N and

$-\boldsymbol{F} = \{-\ 25\ \boldsymbol{k}\}$ N

Find: The couple moment acting on the pipe assembly using Cartesian vector notation.

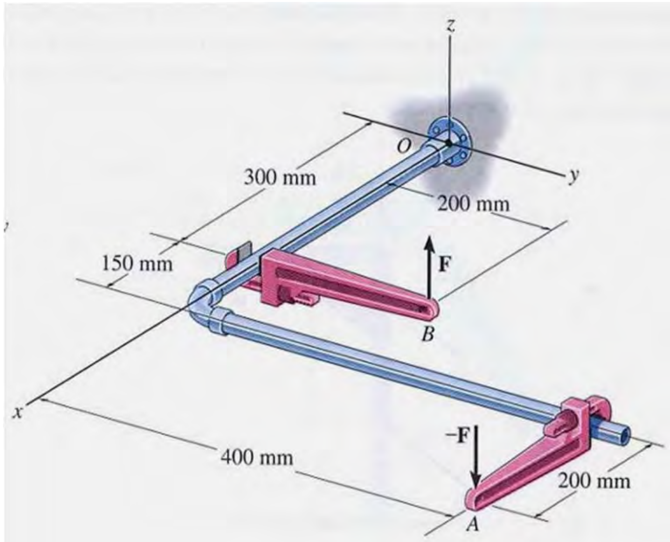
PLAN:

1) Use $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$ to find the couple moment.

2) Set $\boldsymbol{r} = \boldsymbol{r}_{AB}$ and $\boldsymbol{F} = \{25\ \boldsymbol{k}\}$ N .

3) Calculate the cross product to find $\boldsymbol{M}$.

SOLUTION



$$\boldsymbol{r}_{AB} = \{ -\ 350\ \boldsymbol{i} - 200\ \boldsymbol{j} \}\ \text{mm}$$

$$= \{ -\ 0.35\ \boldsymbol{i} - 0.2\ \boldsymbol{j} \}\ \text{m}$$

$$\boldsymbol{F} = \{25\ \boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{M} = \boldsymbol{r}_{AB} \times \boldsymbol{F}$$

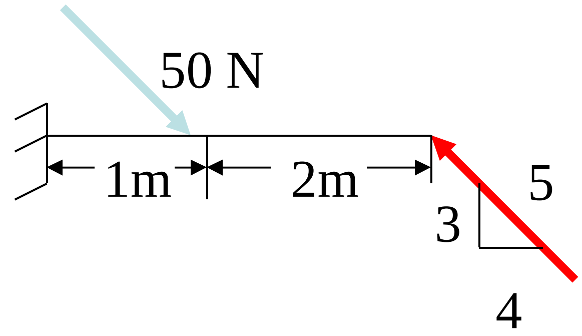
$$= \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix}\ \text{N} \cdot \text{m}$$

$$= \{ \boldsymbol{i}\ (-\ 5 - 0\ ) - \boldsymbol{j}\ (-\ 8.75 - 0) + \boldsymbol{k}\ (0) \}\ \text{N} \cdot \text{m}$$

$$= \{ -5\ \boldsymbol{i} + 8.75\ \boldsymbol{j} \}\ \text{N} \cdot \text{m}$$

ATTENTION QUIZ

1. A couple is applied to the beam as shown. Its moment equals _____ N·m.

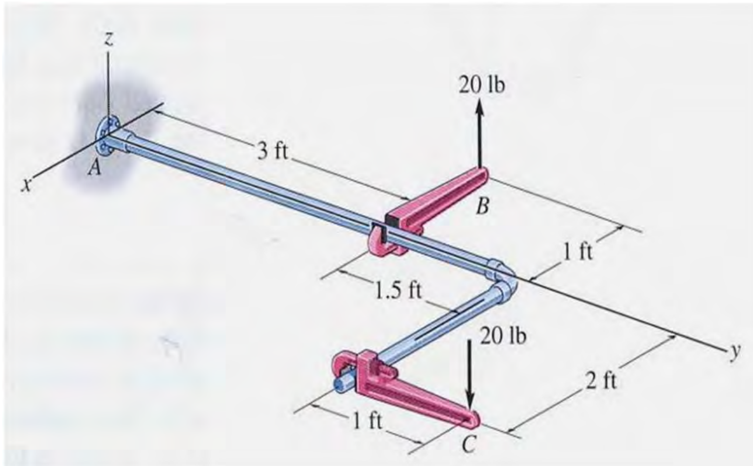


A) 50 B) 60

C) 80 D) 100

2. You can determine the couple moment as $\boldsymbol{M} = \boldsymbol{r} \times \boldsymbol{F}$

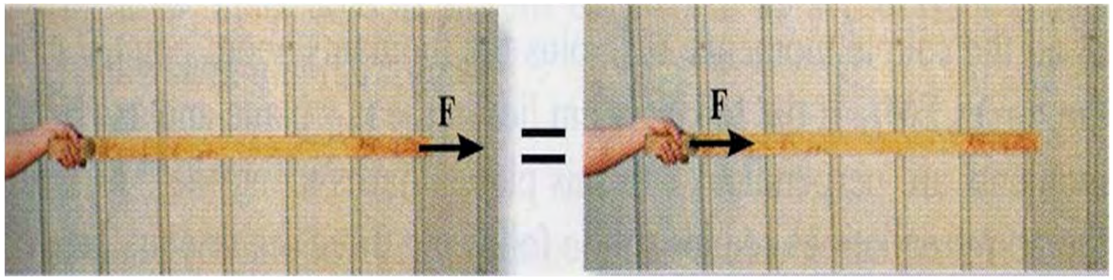
If $\boldsymbol{F}$ = { -20 $\boldsymbol{k}$} lb, then $\boldsymbol{r}$ is

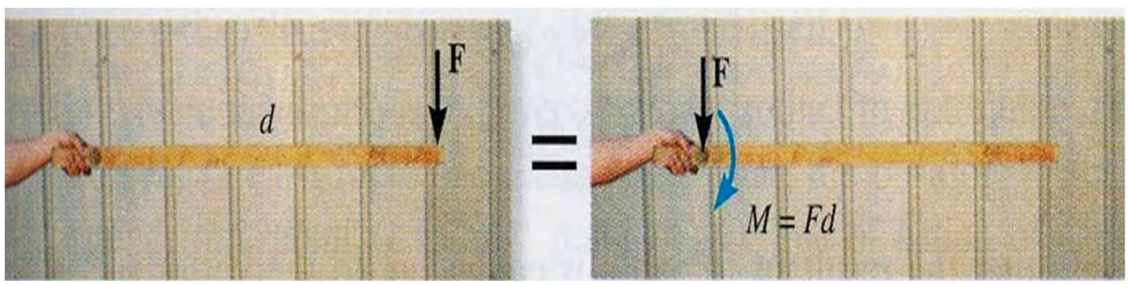


A) $\boldsymbol{r}_{BC}$ B) $\boldsymbol{r}_{AB}$

C) $\boldsymbol{r}_{CB}$ D) $\boldsymbol{r}_{AC}$

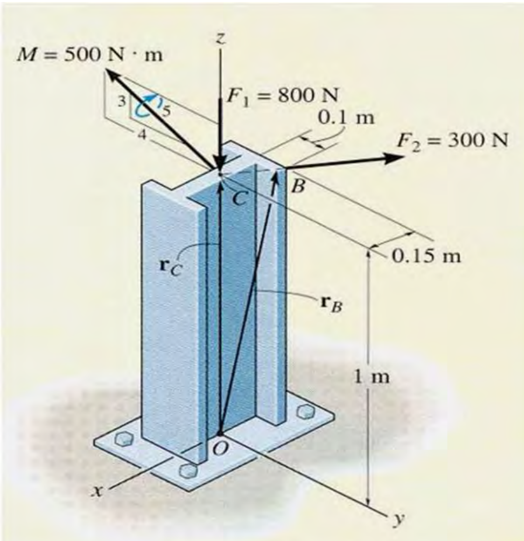
APPLICATIONS





What is the resultant effect on the person's hand when the force is applied in four different ways ?

APPLICATIONS (continued)



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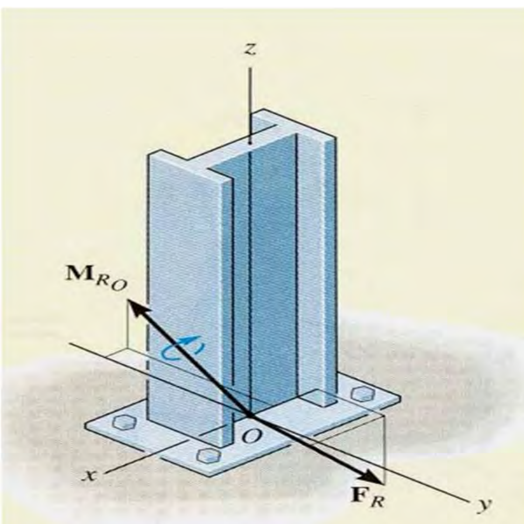
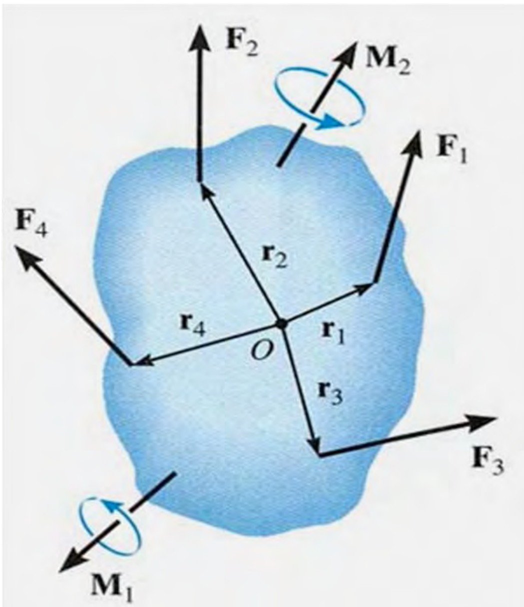


Several forces and a couple moment are acting on this vertical section of an I-beam.

Can you replace them with just one force and one couple moment at point O that will have the same external effect? If yes, how will you do that?

AN EQUIVALENT SYSTEM (Section 4.7)



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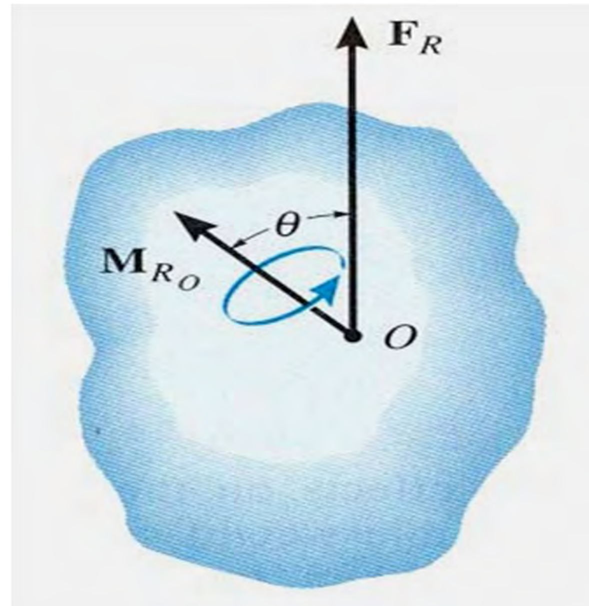
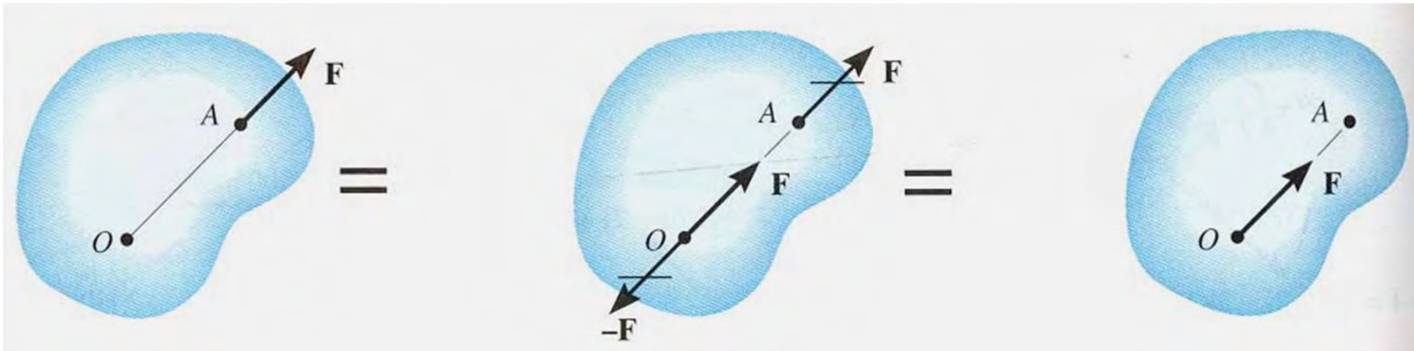


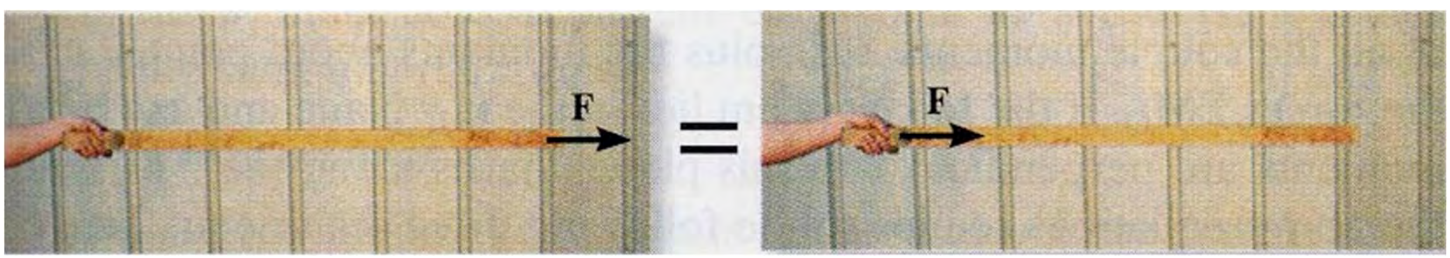
When a number of forces and couple moments are acting on a body, it is easier to understand their overall effect on the body if they are combined into a single force and couple moment having the same external effect

The two force and couple systems are called equivalent systems since they have the same external effect on the body.

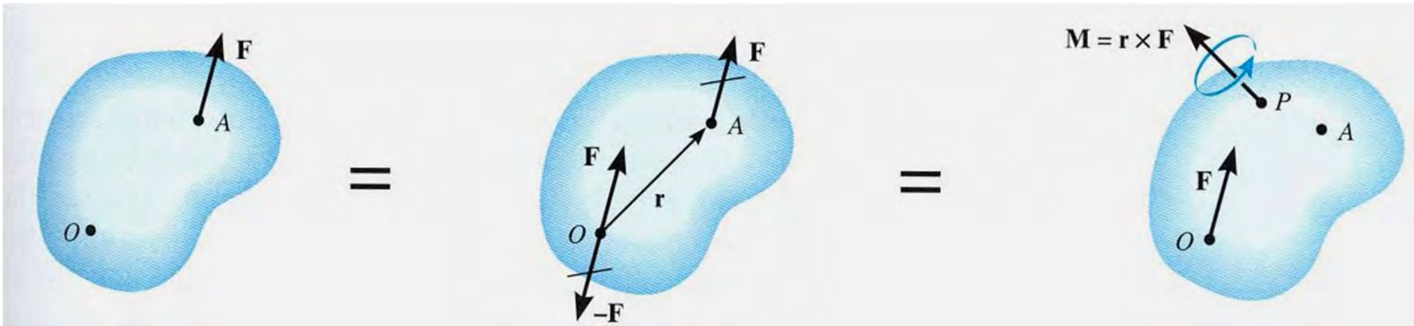
MOVING A FORCE ON ITS LINE OF ACTION



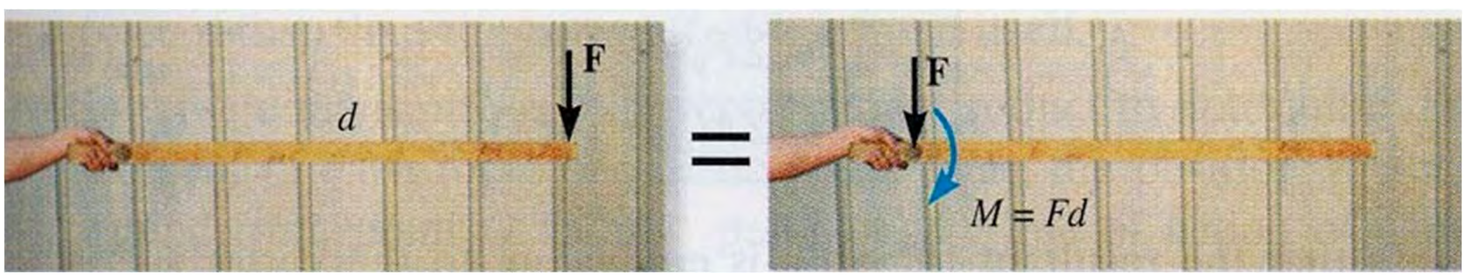
Moving a force from A to O, when both points are on the vectors' line of action, does not change the external effect. Hence, a force vector is called a sliding vector. (But the internal effect of the force on the body does depend on where the force is applied).

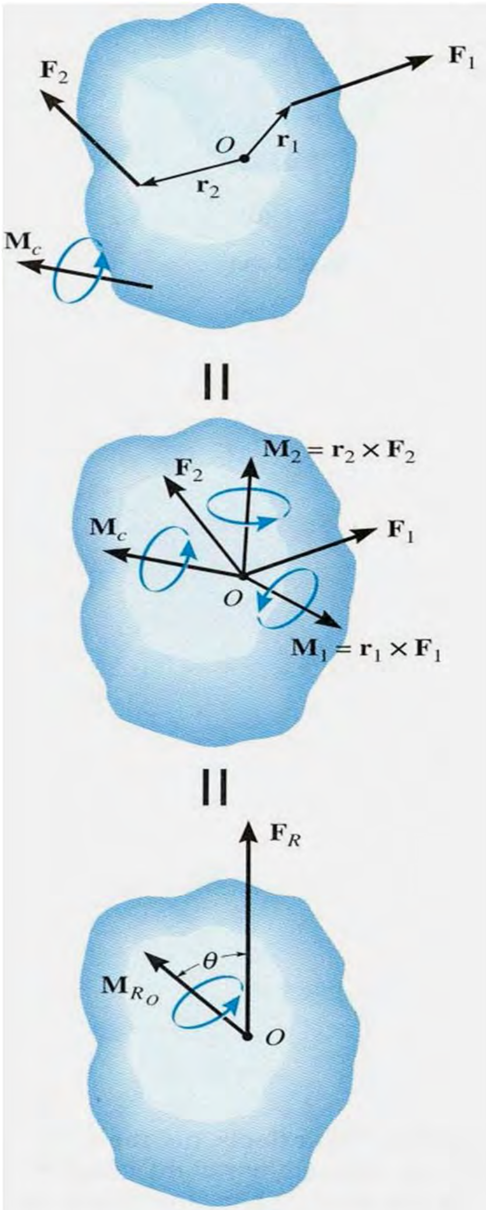


MOVING A FORCE OFF OF ITS LINE OF ACTION



Moving a force from point A to O (as shown above) requires creating an additional couple moment. Since this new couple moment is a "free" vector, it can be applied at any point P on the body.





FINDING THE RESULTANT OF A FORCE AND COUPLE SYSTEM (Section 4.8)

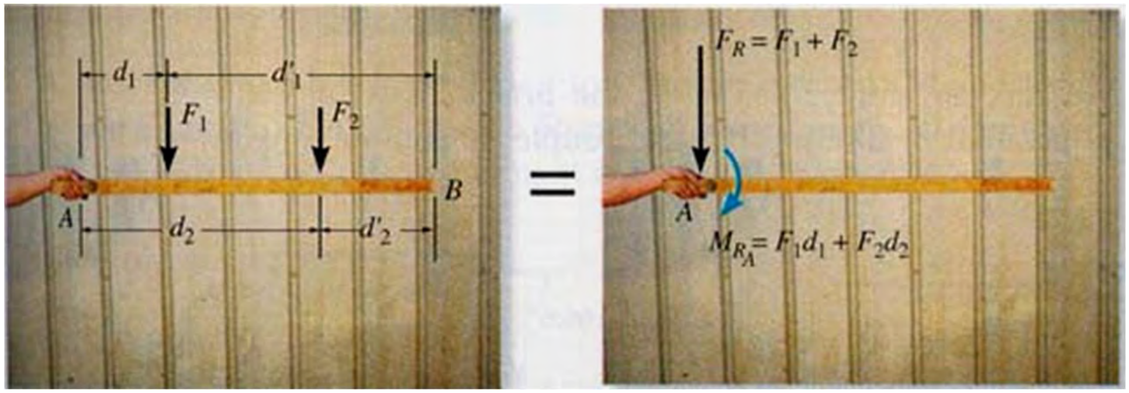
When several forces and couple moments act on a body, you can move each force and its associated couple moment to a common point O.

Now you can add all the forces and couple moments together and find one resultant force-couple moment pair.

$$\mathbf{F}_R = \Sigma \mathbf{F}$$

$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

RESULTANT OF A FORCE AND COUPLE SYSTEM (continued)



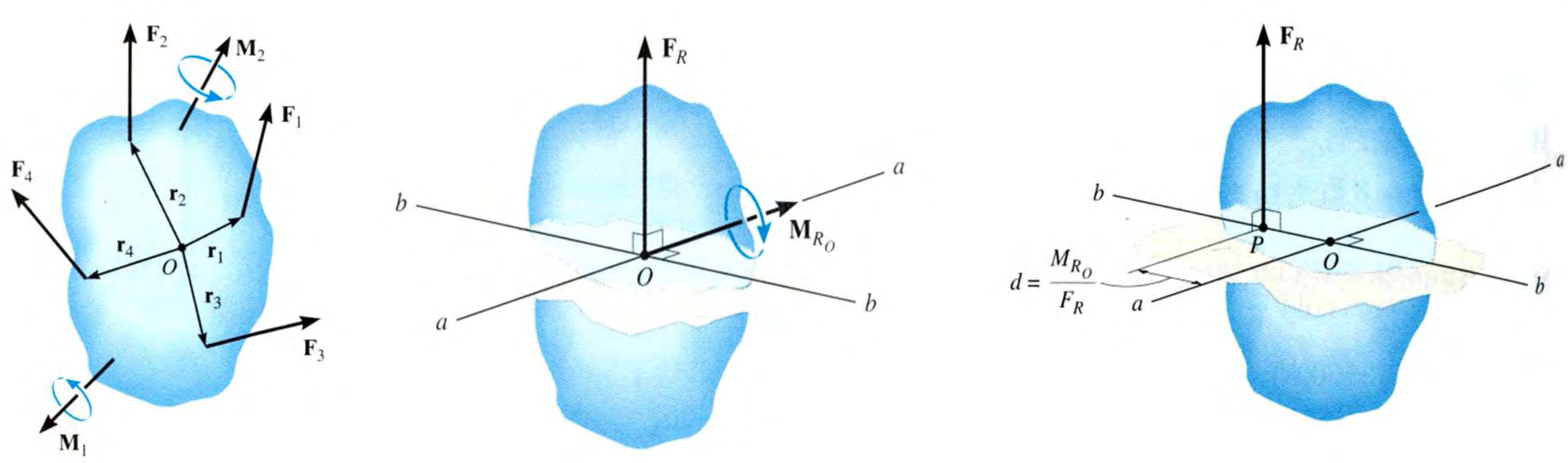
If the force system lies in the x-y plane (the 2-D case), then the reduced equivalent system can be obtained using the following three scalar equations.

$$F_{R_x} = \Sigma F_x$$

$$F_{R_y} = \Sigma F_y$$

$$M_{R_O} = \Sigma M_c + \Sigma M_O$$

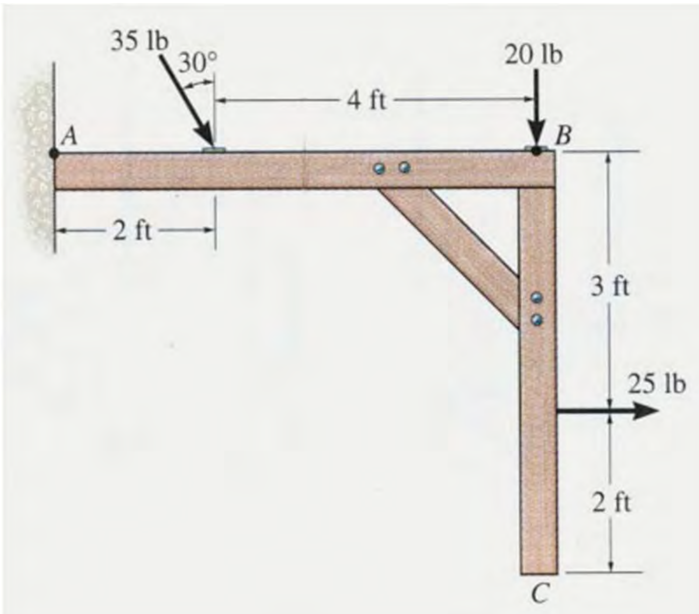
REDUCING A FORCE-MOMENT TO A SINGLE FORCE (Section 4.9)



If $\boldsymbol{F_R}$ and $\boldsymbol{M_{RO}}$ are perpendicular to each other, then the system can be further reduced to a single force, $\boldsymbol{F_R}$, by simply moving $\boldsymbol{F_R}$ from O to P.

In three special cases, concurrent, coplanar, and parallel systems of forces, the system can always be reduced to a single force.

EXAMPLE 1



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A and then the equivalent single force location along the beam AB.

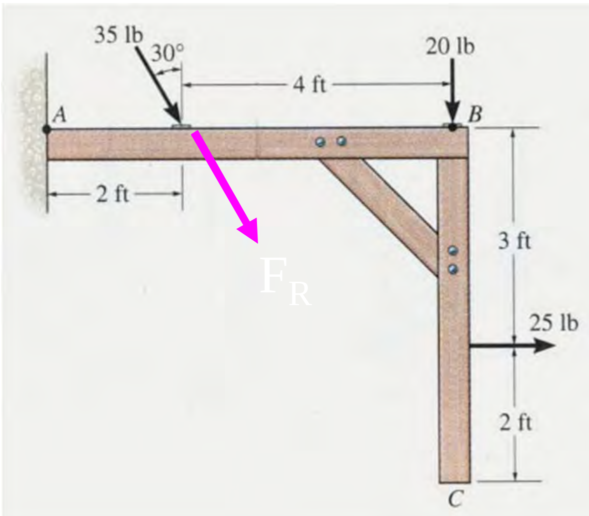
Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A.

3) Shift the F_{RA} to a distance d such that $d = M_{RA}/F_{Ry}$

EXAMPLE (continued)



$$+\rightarrow \Sigma F_{Rx} = 25 + 35 \sin 30^\circ = 42.5 \text{ lb}$$

$$+\downarrow \Sigma F_{Ry} = 20 + 35 \cos 30^\circ = 50.31 \text{ lb}$$

$$+\circlearrowleft M_{RA} = 35 \cos 30^\circ (2) + 20(6) - 25(3)$$

$$= 105.6 \text{ lb}\cdot\text{ft}$$

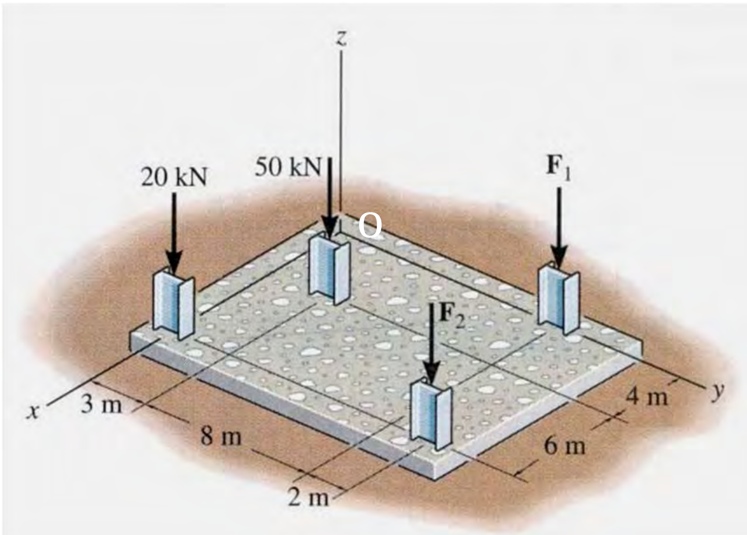
$$F_R = (42.5^2 + 50.31^2)^{1/2} = 65.9 \text{ lb}$$

$$\theta = \tan^{-1}(50.31/42.5) = 49.8^\circ$$

The equivalent single force F_R can be located on the beam AB at a distance d measured from A.

$$d = M_{RA}/F_{Ry} = 105.6/50.31 = 2.10 \text{ ft.}$$

EXAMPLE 2



Given: The building slab has four columns. F_1 and $F_2 = 0$.

Find: The equivalent resultant force and couple moment at the origin O. Also find the location (x,y) of the single equivalent resultant force.

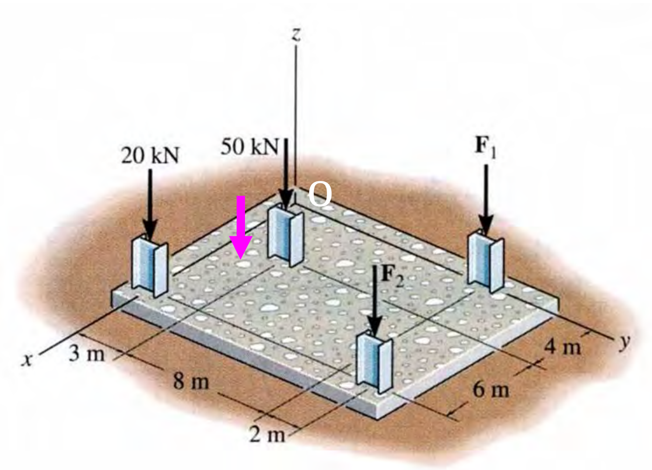
Plan:

1) Find $\boldsymbol{F_{RO}} = \sum \boldsymbol{F_i} = F_{Rzo}\,\boldsymbol{k}$

2) Find $\boldsymbol{M_{RO}} = \sum (\boldsymbol{r_i} \times \boldsymbol{F_i}) = M_{RxO}\,\boldsymbol{i} + M_{RyO}\,\boldsymbol{j}$

3) The location of the single equivalent resultant force is given as $x = -M_{RyO}/F_{RzO}$ and $y = M_{RxO}/F_{RzO}$

EXAMPLE 2 (continued)



$\boldsymbol{F_{RO}} = \{-50\ \boldsymbol{k} - 20\ \boldsymbol{k}\} = \{-70\ \boldsymbol{k}\}$ kN

$\boldsymbol{M_{RO}} = (10\ \boldsymbol{i}) \times (-20\ \boldsymbol{k}) + (4\ \boldsymbol{i} + 3\ \boldsymbol{j}) x (-50\ \boldsymbol{k})$

$= \{200\ \boldsymbol{j} + 200\ \boldsymbol{j} - 150\ \boldsymbol{i}\}$ kN·m

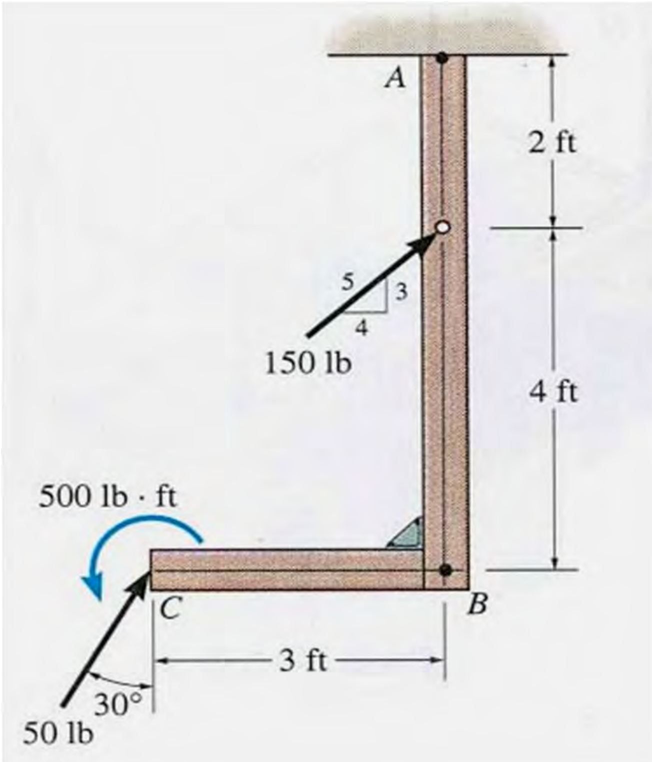
$= \{-150\ \boldsymbol{i} + 400\ \boldsymbol{j}\}$ kN·m

The location of the single equivalent resultant force is given as,

$x = -M_{Ryo}/F_{Rzo} = -400/(-70) = 5.71$ m

$y = M_{Rxo}/F_{Rzo} = (-150)/(-70) = 2.14$ m

GROUP PROBLEM SOLVING



Given: A 2-D force and couple system as shown.

Find: The equivalent resultant force and couple moment acting at A.

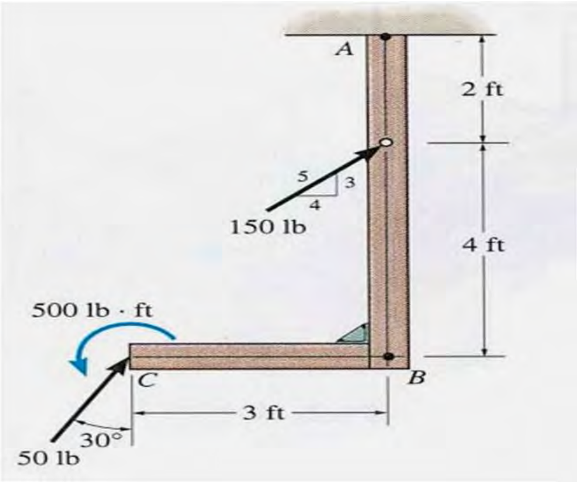
Plan:

1) Sum all the x and y components of the forces to find F_{RA}.

2) Find and sum all the moments resulting from moving each force to A and add them to the 500 lb - ft free moment to find the resultant M_{RA} .

GROUP PROBLEM SOLVING (continued)

Summing the force components:



$+\rightarrow \Sigma F_x = (4/5)\ 150\ \text{lb} + 50\ \text{lb} \sin 30° = 145\ \text{lb}$

$+\uparrow \Sigma F_y = (3/5)\ 150\ \text{lb} + 50\ \text{lb} \cos 30° = 133.3\ \text{lb}$

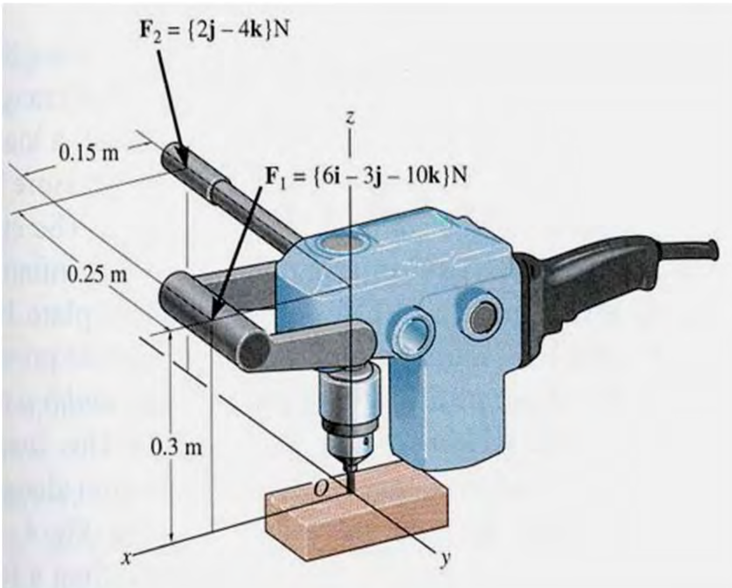
Now find the magnitude and direction of the resultant.

$F_{RA} = (145^2 + 133.3^2)^{1/2} = 197\ \text{lb}$ and $\theta = \tan^{-1}(133.3/145) = 42.6°$

$+\circlearrowleft M_{RA} = \{ (4/5)(150)(2) - 50 \cos 30° (3) + 50 \sin 30° (6) + 500 \}$

$= 760\ \text{lb}\cdot\text{ft}$

GROUP PROBLEM SOLVING (continued)



Given: Handle forces F_1 and F_2 are applied to the electric drill.

Find: An equivalent resultant force and couple moment at point O.

Plan:

a) Find $\boldsymbol{F_{RO}} = \Sigma\, \boldsymbol{F_i}$

b) Find $\boldsymbol{M_{RO}} = \Sigma\, \boldsymbol{M_C} + \Sigma\, (\, \boldsymbol{r_i} \times \boldsymbol{F_i}\,)$

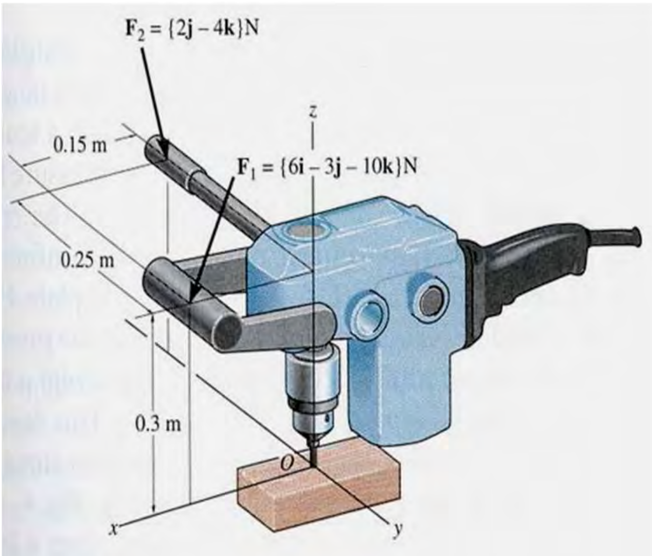
Where,

$\boldsymbol{F_i}$ are the individual forces in Cartesian vector notation (CVN).

$\boldsymbol{M_C}$ are any free couple moments in CVN (none in this example).

$\boldsymbol{R_i}$ are the position vectors from the point O to any point on the line of action of $\boldsymbol{F_i}$.

SOLUTION



$$\boldsymbol{F_1} = \{6\,\boldsymbol{i} - 3\,\boldsymbol{j} - 10\,\boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F_2} = \{0\,\boldsymbol{i} + 2\,\boldsymbol{j} - 4\,\boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{F_{RO}} = \{6\,\boldsymbol{i} - 1\,\boldsymbol{j} - 14\,\boldsymbol{k}\}\ \text{N}$$

$$\boldsymbol{r_1} = \{0.15\,\boldsymbol{i} + 0.3\,\boldsymbol{k}\}\ \text{m}$$

$$\boldsymbol{r_2} = \{-0.25\,\boldsymbol{j} + 0.3\,\boldsymbol{k}\}\ \text{m}$$

$$\boldsymbol{M_{RO}} = \boldsymbol{r_1} \times \boldsymbol{F_1} + \boldsymbol{r_2} \times \boldsymbol{F_2}$$

$$\boldsymbol{M_{RO}} = \left\{ \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0.15 & 0 & 0.3 \\ 6 & -3 & -10 \end{vmatrix} + \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ 0 & -0.25 & 0.3 \\ 0 & 2 & -4 \end{vmatrix} \right\}\ \text{N·m}$$

$$= \{0.9\,\boldsymbol{i} + 3.3\,\boldsymbol{j} - 0.45\,\boldsymbol{k} + 0.4\,\boldsymbol{i} + 0\,\boldsymbol{j} + 0\,\boldsymbol{k}\}\ \text{N·m}$$

$$= \{1.3\,\boldsymbol{i} + 3.3\,\boldsymbol{j} - 0.45\,\boldsymbol{k}\}\ \text{N·m}$$

ATTENTION QUIZ

1. For this force system, the equivalent system at P is ___________ .

A) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +60 ft ·lb

B) F_{RP} = 0 lb and M_{RP} = +30 ft · lb

C) F_{RP} = 30 lb (along +y-dir.) and M_{RP} = -30 ft ·lb

D) F_{RP} = 40 lb (along +x-dir.) and M_{RP} = +30 ft ·lb