

Basic Concepts in RF Design

RF Power in dBm

$$P = 10 \cdot \log_{10} \left(\frac{P}{P_0} \right) ; \quad P_0 = \begin{cases} 1\text{W [dBW]} \\ 1\text{mW [dBm]} \end{cases}$$

Power (dBm)	Power (dBW)	Power (watt)	Power (mW)
-30 dBm	-60 dBW	1 μ W	0.001 mW
-20 dBm	-50 dBW	10 μ W	0.01 mW
-10 dBm	-40 dBW	100 μ W	0.1 mW
-1 dBm	-31 dBW	794 μ W	0.794 mW
0 dBm	-30 dBW	1.000 mW	1.000 mW
1 dBm	-29 dBW	1.259 mW	1.259 mW
10 dBm	-20 dBW	10 mW	10 mW
20 dBm	-10 dBW	100 mW	100 mW
30 dBm	0 dBW	1 W	1000 mW

Cont'd

$$P = \frac{1}{2} \times \frac{V^2}{50} = \frac{V^2}{100}$$

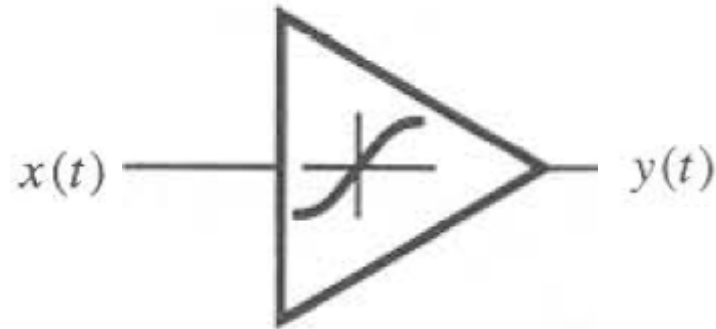
$$P(mW) = P \times 1000 = 10V^2$$

$$P(dBm) = 10 \log(P(mW)) = 10 \log(10V^2) = 10 + 20 \log(V)$$

Amplitude (volte)	Power (dBm)
0.1	-10
1	10
10	30

Effect of Nonlinearity

- Assume $x(t)$ is passing through a nonlinear amplifier



- We have:

$$y(t) \approx \alpha_1 x(t) + \alpha_2 x^2(t) + \alpha_3 x^3(t)$$

Harmonics

if $x(t) = A \cos \omega t$, then

$$\begin{aligned}y(t) &= \alpha_1 A \cos \omega t + \alpha_2 A^2 \cos^2 \omega t + \alpha_3 A^3 \cos^3 \omega t \\&= \alpha_1 A \cos \omega t + \frac{\alpha_2 A^2}{2} (1 + \cos 2\omega t) + \frac{\alpha_3 A^3}{4} (3 \cos \omega t + \cos 3\omega t) \\&= \frac{\alpha_2 A^2}{2} + \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4} \right) \cos \omega t + \frac{\alpha_2 A^2}{2} \cos 2\omega t + \frac{\alpha_3 A^3}{4} \cos 3\omega t\end{aligned}$$

Fractional Harmonic Distortion

- The fractional second-harmonic distortion is a commonly cited metric

$$HD_2 = \frac{\text{ampl of second harmonic}}{\text{ampl of fund}}$$

- If we assume that the square power dominates the second-harmonic

$$HD_2 = \frac{1}{2} \frac{\alpha_2}{\alpha_1} A$$

Third Harmonic Distortion

- The fractional third harmonic distortion is given by

$$HD_3 = \frac{\text{ampl of third harmonic}}{\text{ampl of fund}}$$

- If we assume that the cubic power dominates the third harmonic

$$HD_3 = \frac{1}{4} \frac{\alpha_3}{\alpha_1} A^2$$

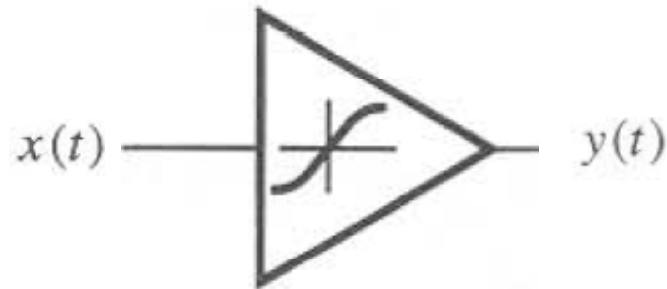
Total Harmonic Distortion

- We define the *Total Harmonic Distortion (THD)* by the following expression

$$THD = \sqrt{HD_2^2 + HD_3^2 + \dots}$$

- Based on the particular application, we specify the maximum tolerable *THD*
- Telephone audio can be pretty distorted ($THD < 10\%$)
- High quality audio is very sensitive ($THD < 1\%$ to $THD < .001\%$)
- Video is also pretty forgiving, $THD < 5\%$ for most applications
- Analog Repeaters $< .001\%$. RF Amplifiers $< 0.1\%$

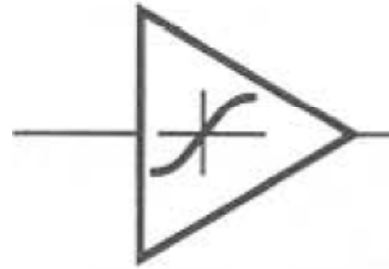
Gain Compression (I)



$$\text{if } x(t) = A \cos \omega t, \text{ then } y(t) \approx \left(\alpha_1 A + \frac{3\alpha_3 A^3}{4} \right) \cos \omega t$$

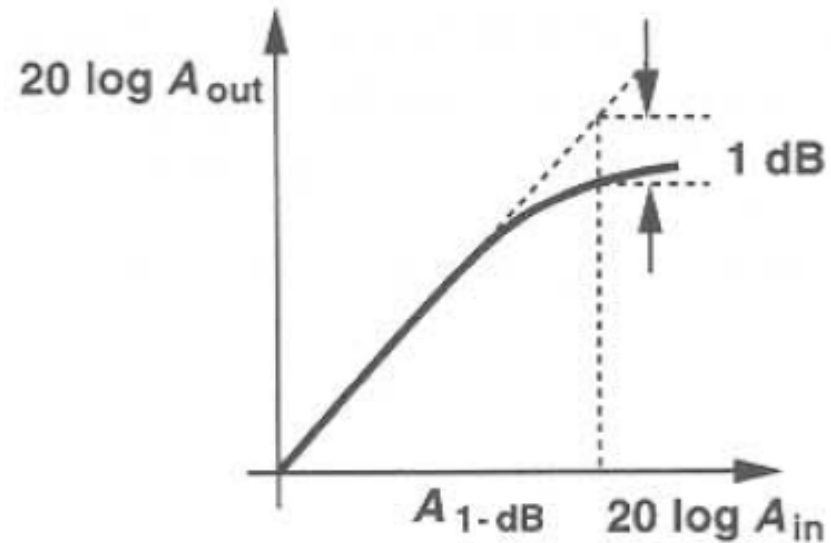
- Gain: $\alpha_1 + 3\alpha_3 A^2/4$.
- In practice we have: $\alpha_3 < 0$
- Since α_3 is negative, so we have gain compression.
- The “1-dB compression point” defined as the input signal level that causes the small-signal gain drop by 1 dB.

Gain Compression (II)

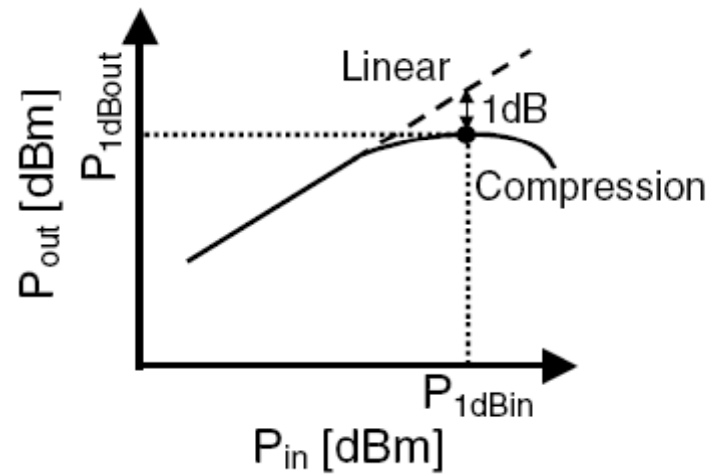
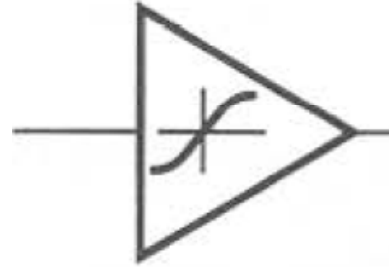


$$20 \log |\alpha_1 A + \frac{3}{4} \alpha_3 A^3| = 20 \log |\alpha_1 A| - 1 \text{ dB}$$

$$A_{1\text{-dB}} = \sqrt{0.145 \left| \frac{\alpha_1}{\alpha_3} \right|}$$



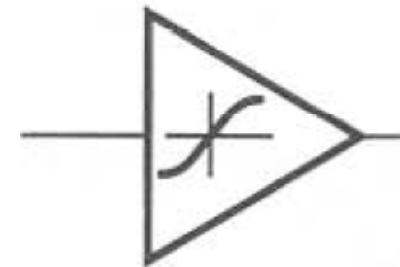
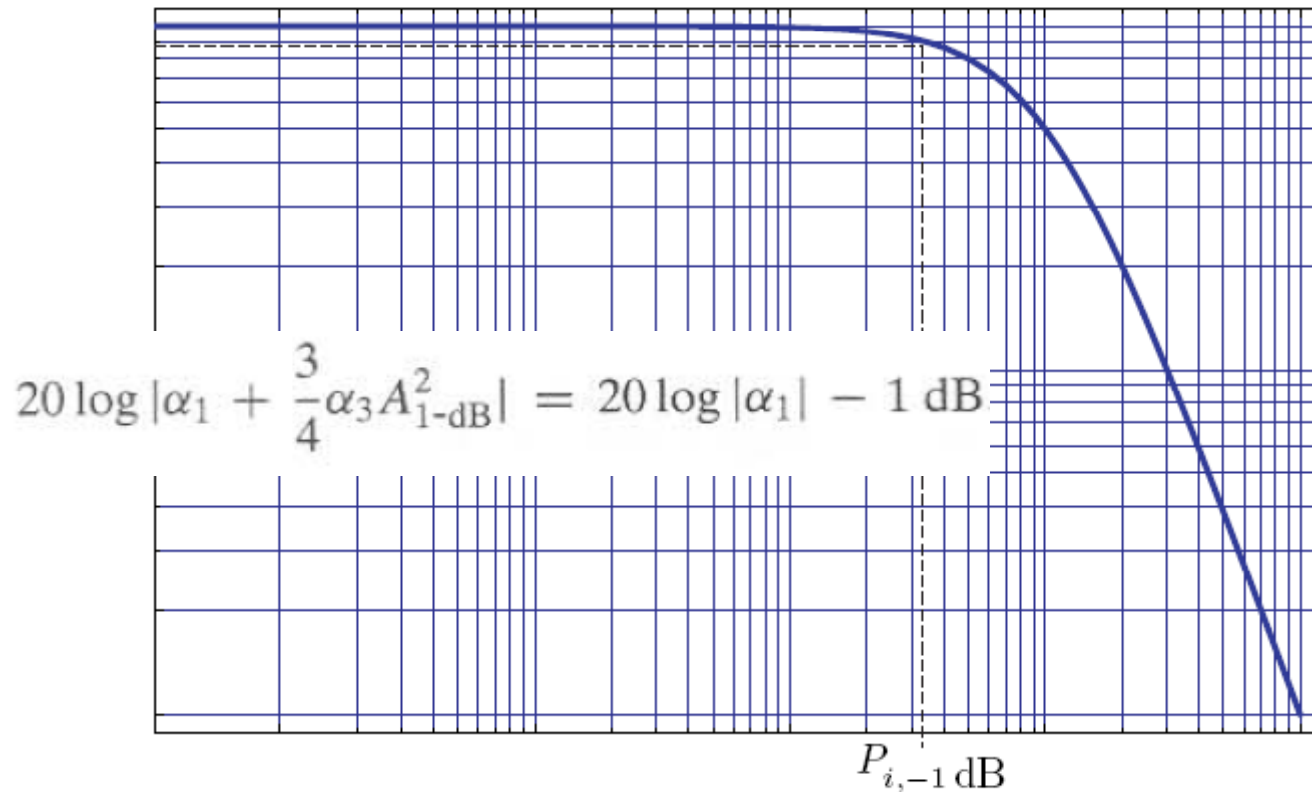
Gain Compression (III)



Extrapolation of the 1 dB compression point

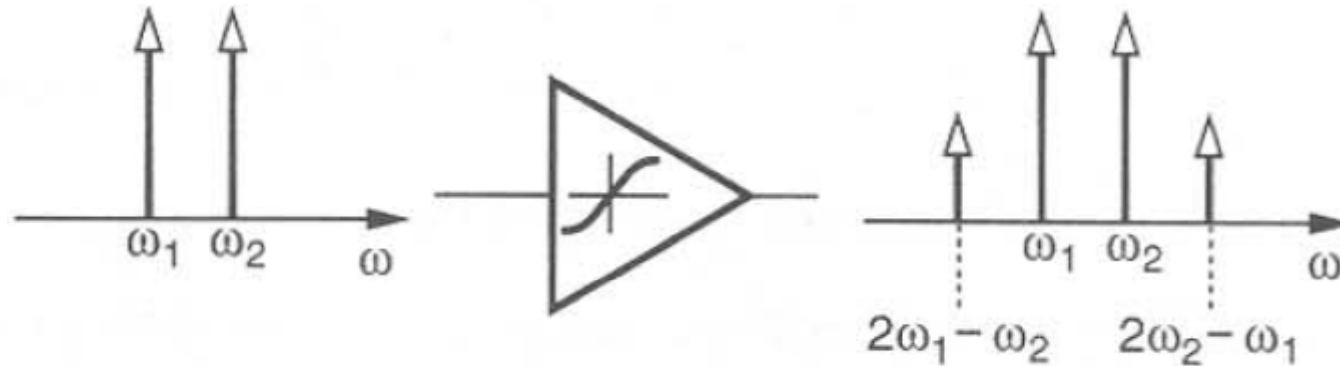
Gain Compression (IV)

Gain(dB)



If we plot the gain (log scale) as a function of the input power, we identify the point where the gain has dropped by 1 dB. This is the 1 dB compression point. It's a very important number to keep in mind.

Intermodulation (I)

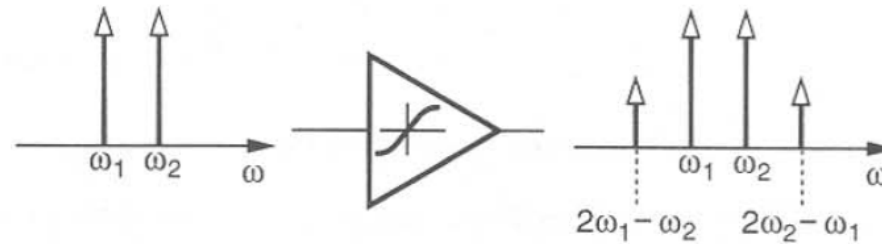


$$x(t) = A_1 \cos \omega_1 t + A_2 \cos \omega_2 t$$

$$y(t) = \alpha_1(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t) + \alpha_2(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^2 + \alpha_3(A_1 \cos \omega_1 t + A_2 \cos \omega_2 t)^3$$

Expanding the left side and discarding dc terms and harmonics, we obtain the following intermodulation products:

Intermodulation (II)

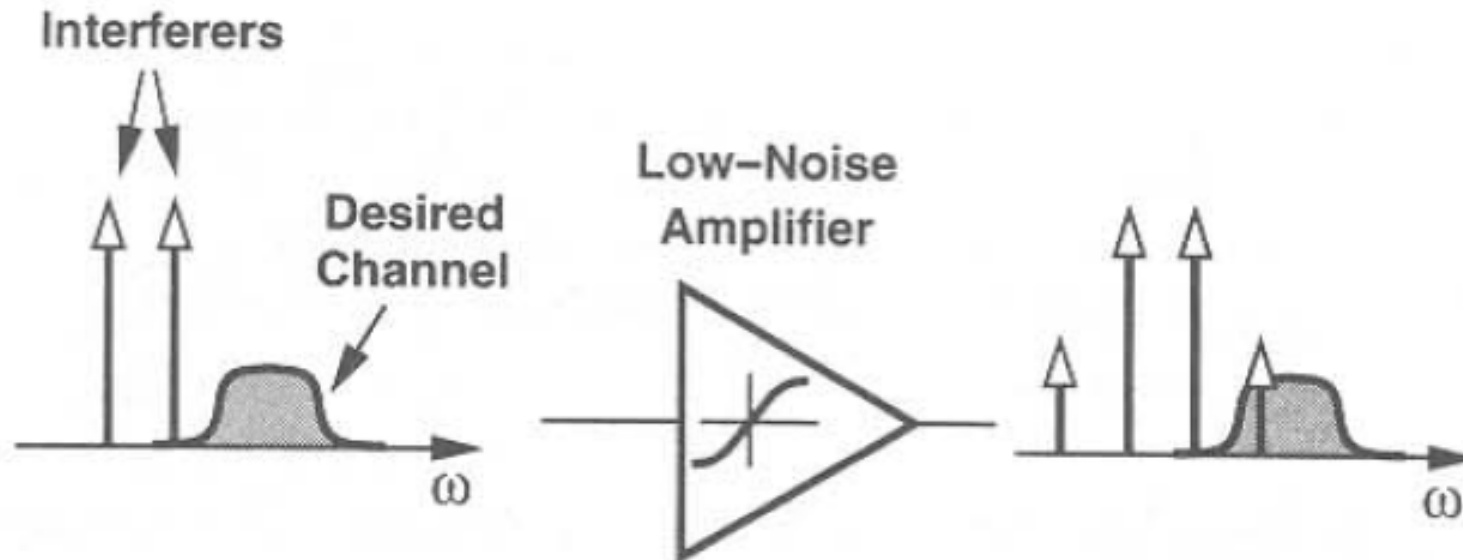


$$\begin{aligned} \omega = \omega_1 \pm \omega_2 : & \alpha_2 A_1 A_2 \cos(\omega_1 + \omega_2)t + \alpha_2 A_1 A_2 \cos(\omega_1 - \omega_2)t \\ = 2\omega_1 \pm \omega_2 : & \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 + \omega_2)t + \frac{3\alpha_3 A_1^2 A_2}{4} \cos(2\omega_1 - \omega_2)t \\ = 2\omega_2 \pm \omega_1 : & \frac{3\alpha_3 A_2^2 A_1}{4} \cos(2\omega_2 + \omega_1)t + \frac{3\alpha_3 A_2^2 A_1}{4} \cos(2\omega_2 - \omega_1)t \end{aligned}$$

and these fundamental components

$$\begin{aligned} \omega = \omega_1, \omega_2 : & \left(\alpha_1 A_1 + \frac{3}{4} \alpha_3 A_1^3 + \frac{3}{2} \alpha_3 A_1 A_2^2 \right) \cos \omega_1 t \\ & + \left(\alpha_1 A_2 + \frac{3}{4} \alpha_3 A_2^3 + \frac{3}{2} \alpha_3 A_2 A_1^2 \right) \cos \omega_2 t. \end{aligned}$$

Intermodulation (III)



Corruption of a signal due to intermodulation between two interferers.

Second order Intermodulation

- The term $\cos(\omega_1 \pm \omega_2)t$ is the second-order intermodulation term
- The intermodulation distortion IM_2 is defined when the two input signals have equal amplitude $A_1=A_2=A$

$$IM_2 = \frac{\text{Amp of Intermod}}{\text{Amp of Fund}} \quad IM_2 = \frac{\alpha_2}{\alpha_1} A$$

- Note the relation between IM_2 and HD_2

$$IM_2 = 2HD_2 = HD_2 + 6\text{dB}$$

Practical Effect of IM2

- This term produces distortion at a lower frequency $\omega_1 - \omega_2$ and at a higher frequency $\omega_1 + \omega_2$
- Example: Say the receiver bandwidth is from 800MHz – 2.4GHz and two unwanted interfering signals appear at 800MHz and 900MHz.
- Then we see that the second-order distortion will produce distortion at 100MHz and 1.7GHz. Since 1.7GHz is in the receiver band, signals at this frequency will be corrupted by the distortion.
- A weak signal in this band can be “swamped” by the distortion.
- Apparently, a “narrowband” system does not suffer from IM_2 ? Or does it ?

Low IF Receiver

- In a low-IF or direct conversion receiver, the signal is down-converted to a low intermediate frequency f_{IF}
- Since $\omega_1 - \omega_2$ can potentially produce distortion at low frequency, IM_2 is very important in such systems
- Example: A narrowband system has a receiver bandwidth of 1.9GHz - 2.0GHz. A sharp input filter eliminates any interference outside of this band. The IF frequency is 1MHz
- Imagine two interfering signals appear at $f_1 = 1.910\text{GHz}$ and $f_2 = 1.911\text{GHz}$. Notice that $f_2 - f_1 = f_{IF}$
- Thus the output of the amplifier/mixer generate distortion at the IF frequency, potentially disrupting the communication.

Third order Intermodulation

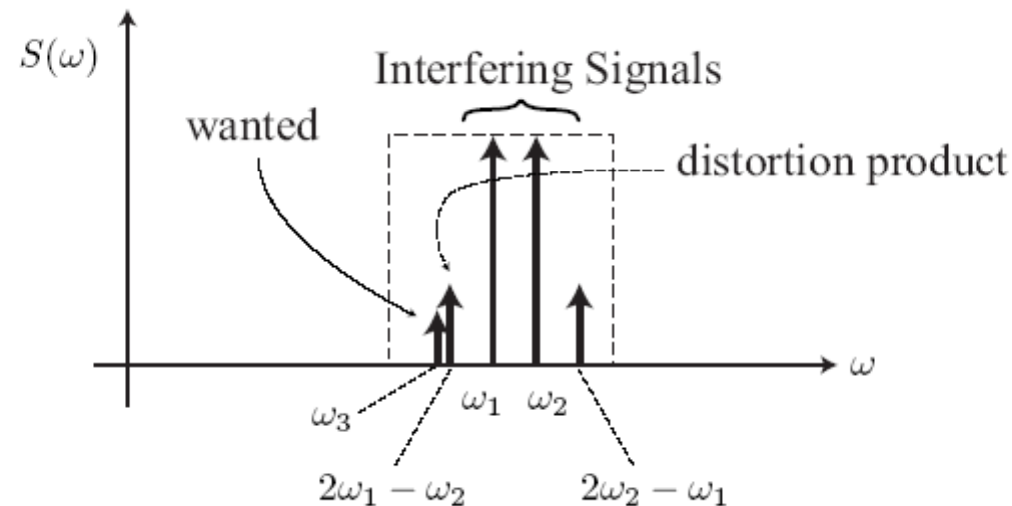
- We define IM_3 in a similar manner for $A_1=A_2=A$

$$IM_3 = \frac{\text{Amp of Third Intermod}}{\text{Amp of Fund}} \quad IM_3 = \frac{3 \alpha_3}{4 \alpha_1} A^2$$

- Note the relation between IM_3 and HD_3

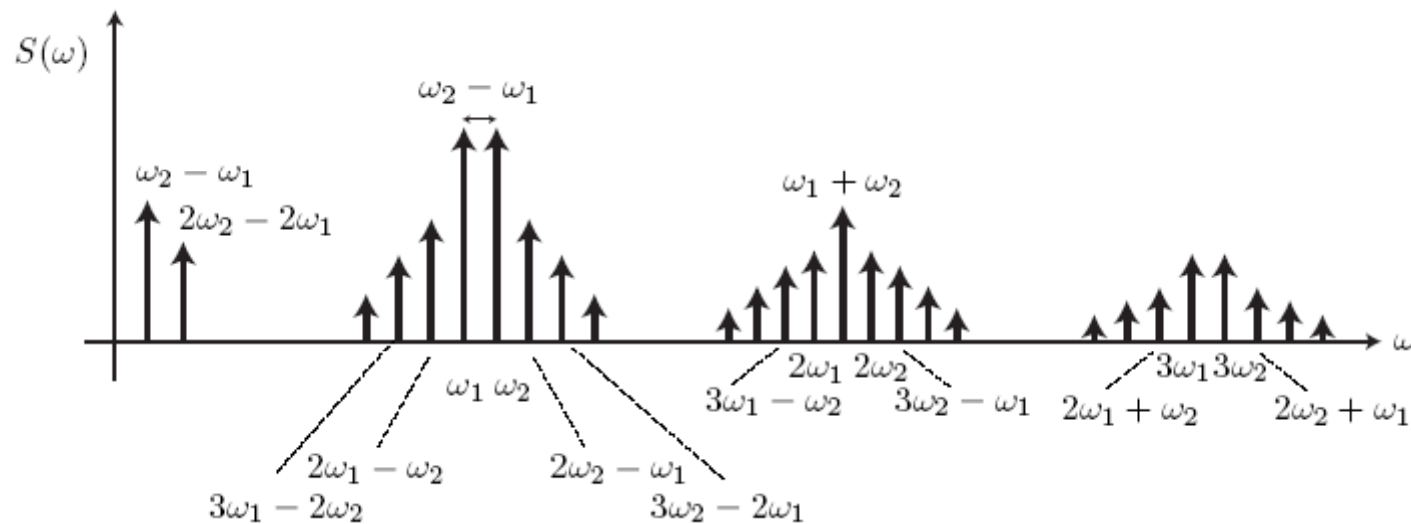
$$IM_3 = 3HD_3 = HD_3 + 10\text{dB}$$

In-band IM_3 Distortion



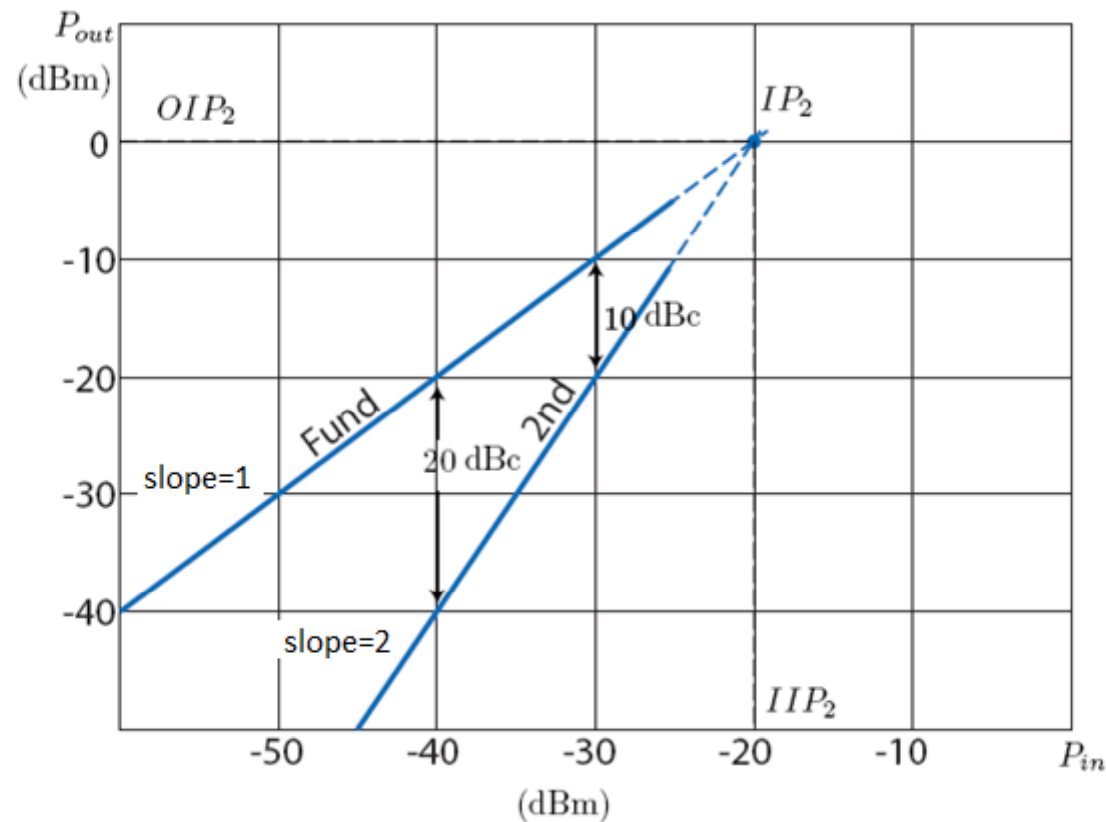
- Now we see that even if the system is narrowband, the output of an amplifier can contain in band intermodulation due to IM_3 .
- This is in contrast to IM_2 where the frequency of the intermodulation was at a lower and higher frequency. The IM_3 frequency can fall in-band for two in-band interferences

Complete Two-Tone Response



- We have so far identified the harmonics and IM_2 and IM_3 products
- A more detailed analysis shows that an order n non-linearity can produce intermodulation at frequencies $j\omega_1 \pm k\omega_2$ where $j + k = n$
- All tones are spaced by the difference $\omega_2 - \omega_1$

Intercept Point IP_2

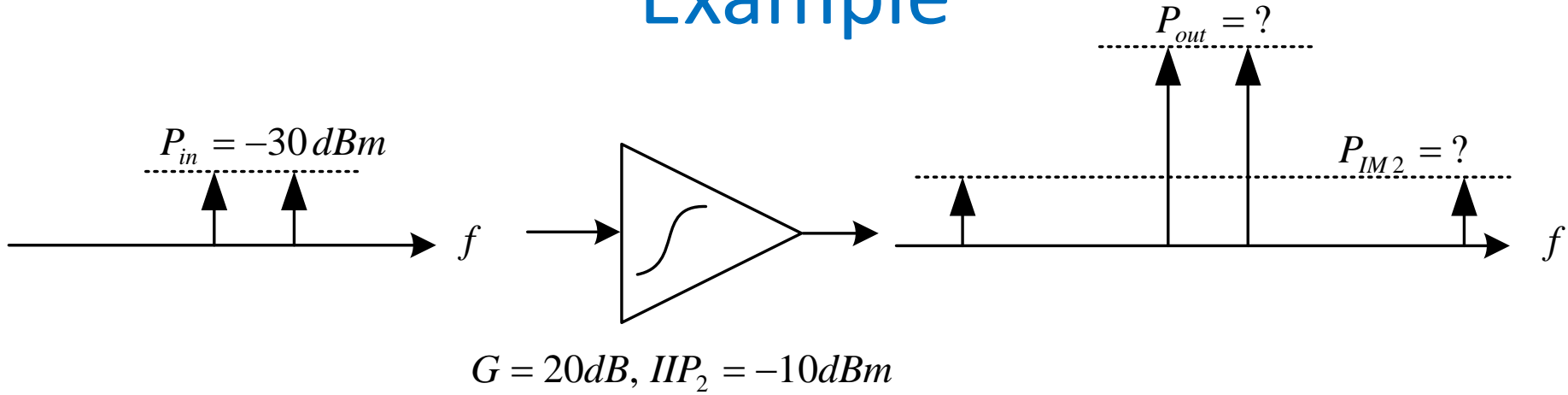


- The extrapolated point where $IM_2 = 0$ dBc is known as the second order intercept point IP_2 .

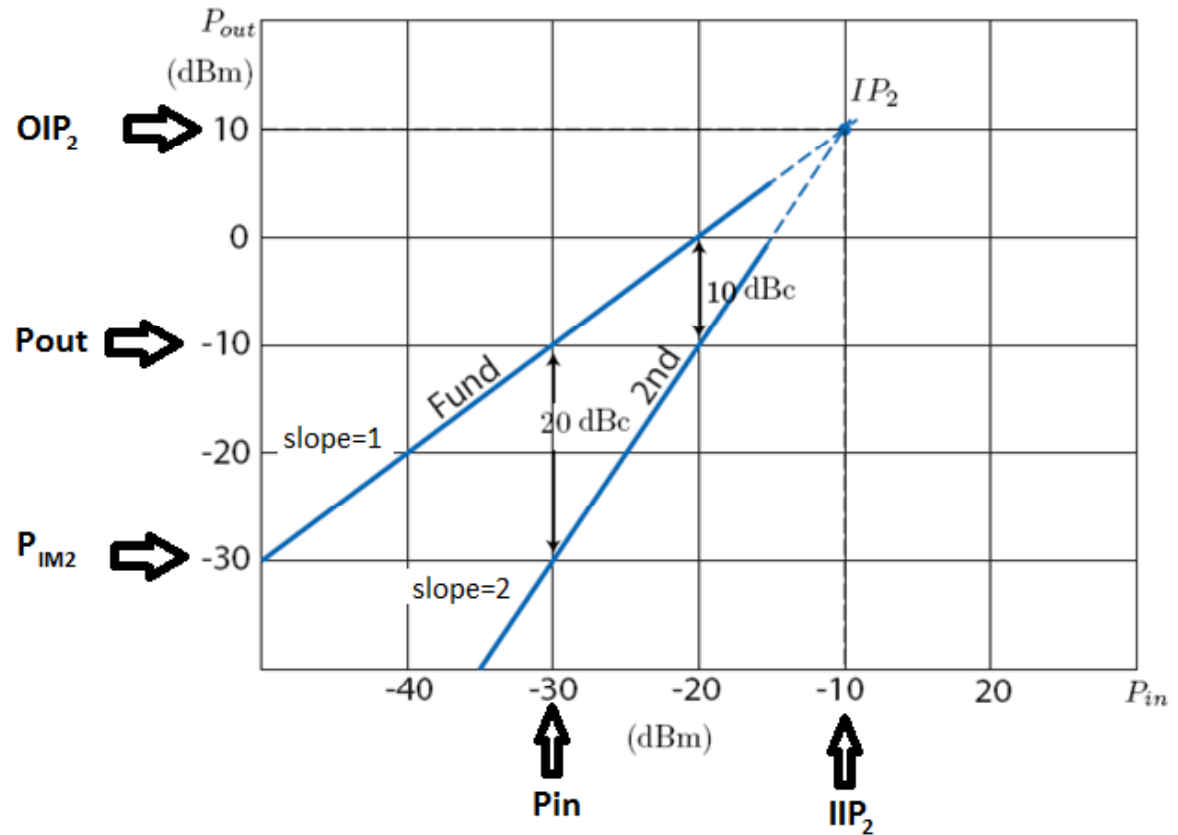
Properties of Intercept Point IP_2

- Since the second order IM distortion products increase like s_i^2 , we expect that at some power level the distortion products will overtake the fundamental signal.
- The extrapolated point where the curves of the fundamental signal and second order distortion product signal meet is the Intercept Point (IP_2).
- At this point, then, by definition $IM_2 = 0$ dBc.
- The input power level is known as IIP_2 , and the output power when this occurs is the OIP_2 point.
- Once the IP_2 point is known, the IM_2 at any other power level can be calculated. Note that for a 10dB back-off from the IP_2 point, the IM_2 improves 10dB.

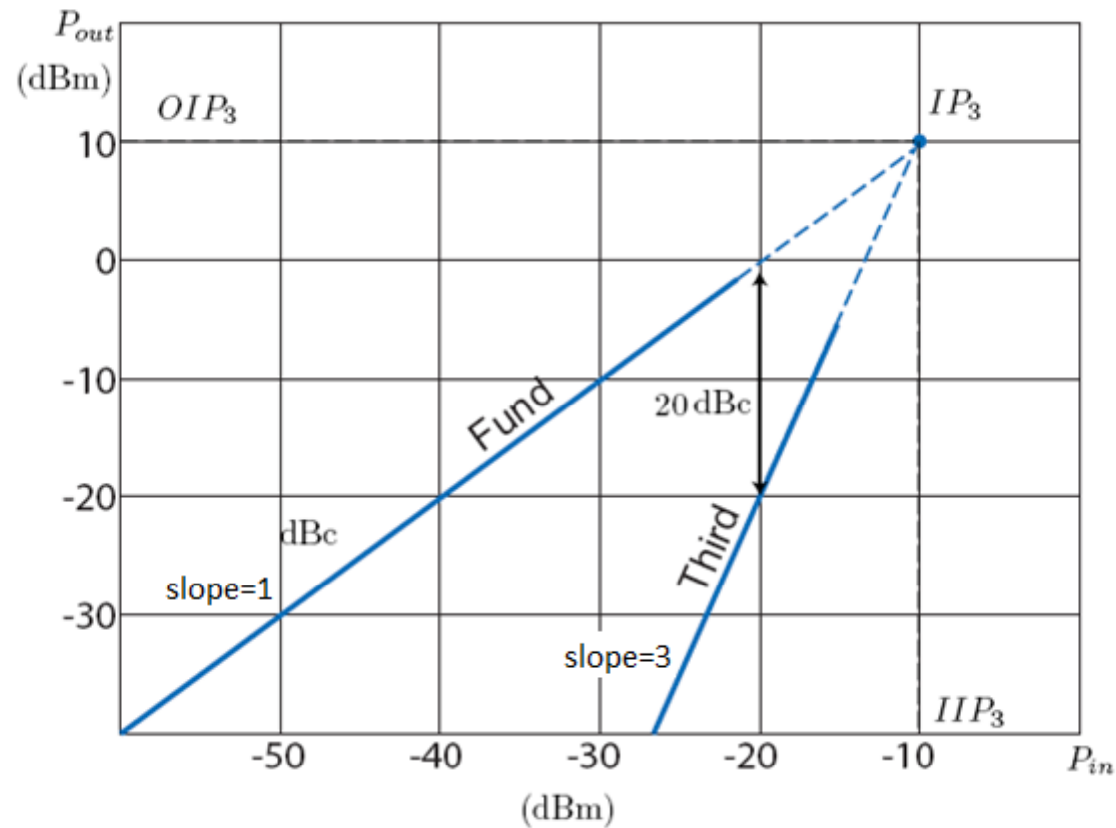
Example



Solution: $P_{out} = -10 \text{ dBm}$
 $P_{IM2} = -30 \text{ dBm}$



Intercept Point IP_3

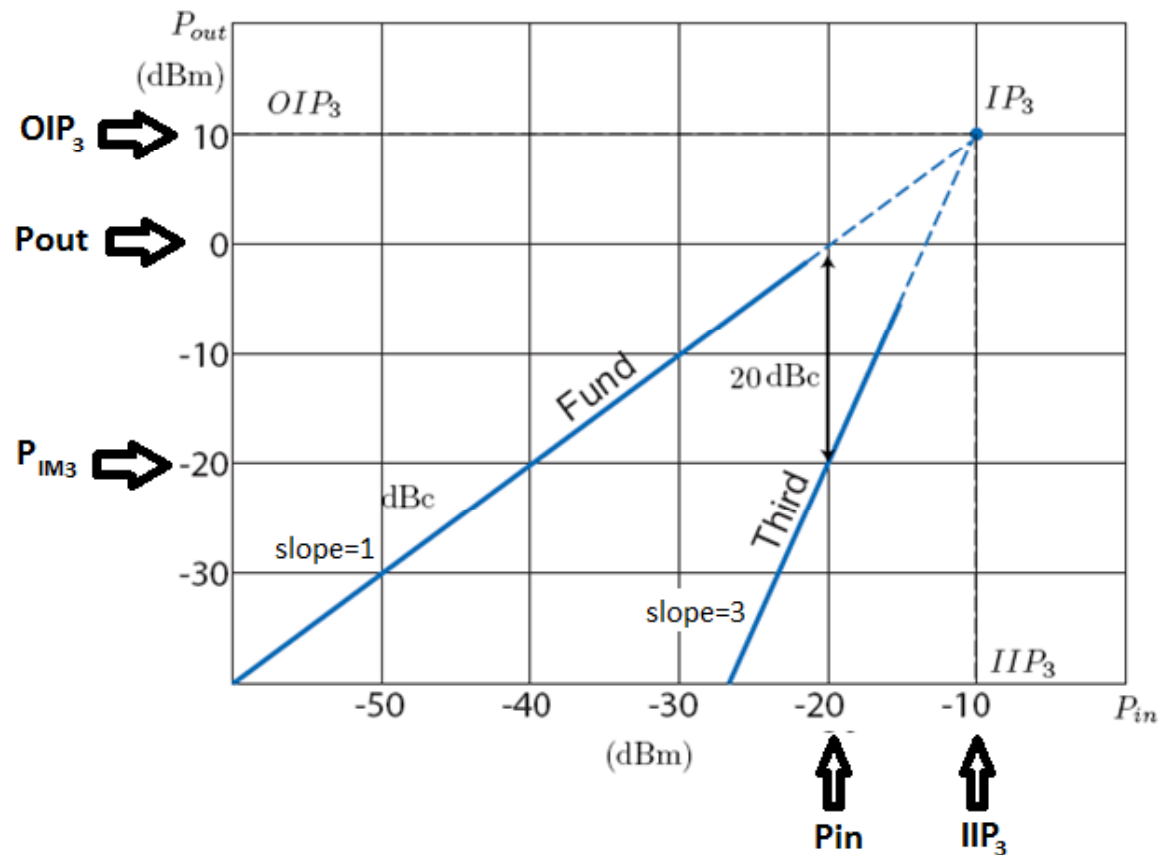
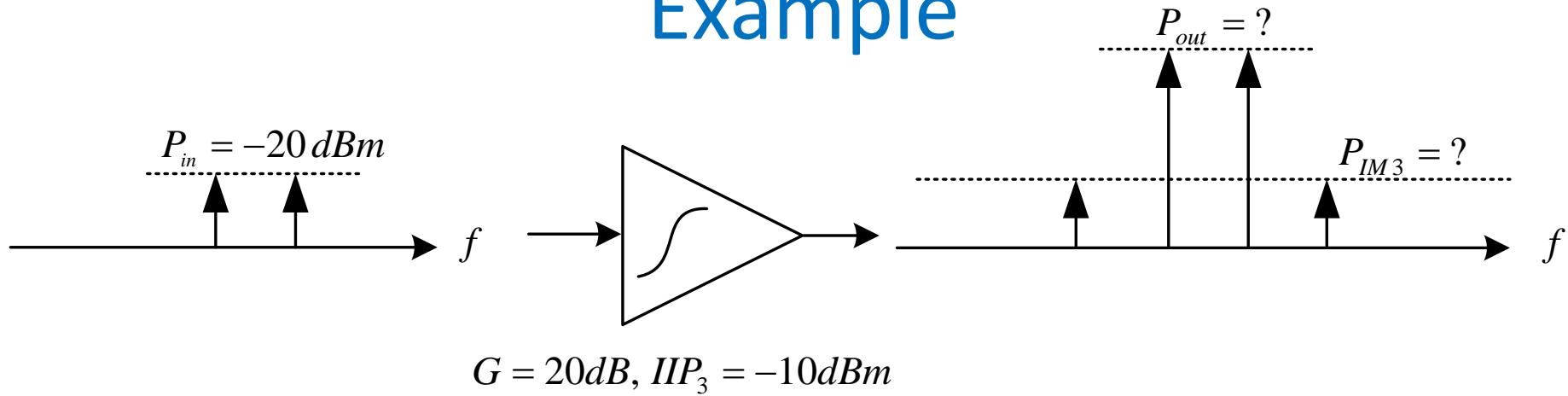


- The extrapolated point where $IM_3 = 0$ dBc is known as the third-order intercept point IP_3 .

Properties of Intercept Point IP_3

- Since the third order IM distortion products increase like s_i^3 , we expect that at some power level the distortion products will overtake the fundamental signal.
- The extrapolated point where the curves of the fundamental signal and third order distortion product signal meet is the Intercept Point (IP_3).
- At this point, then, by definition $IM_3 = 0$ dBc.
- The input power level is known as IIP_3 , and the output power when this occurs is the OIP_3 point.
- Once the IP_3 point is known, the IM_3 at any other power level can be calculated. Note that for a 10 dB back-off from the IP_3 point, the IM_3 improves 20 dB.

Example



Solution: $P_{out} = 0 \text{ dBm}$
 $P_{IM3} = -20 \text{ dBm}$

Calculated $IIP2/IIP3$

- We can also calculate the IIP points directly from our power series expansion. By definition, the $IIP2$ point occurs when

$$IM_2 = 1 = \frac{a_2}{a_1} A$$

- Solving for the input signal level

$$IIP_2 = A = \frac{a_1}{a_2}$$

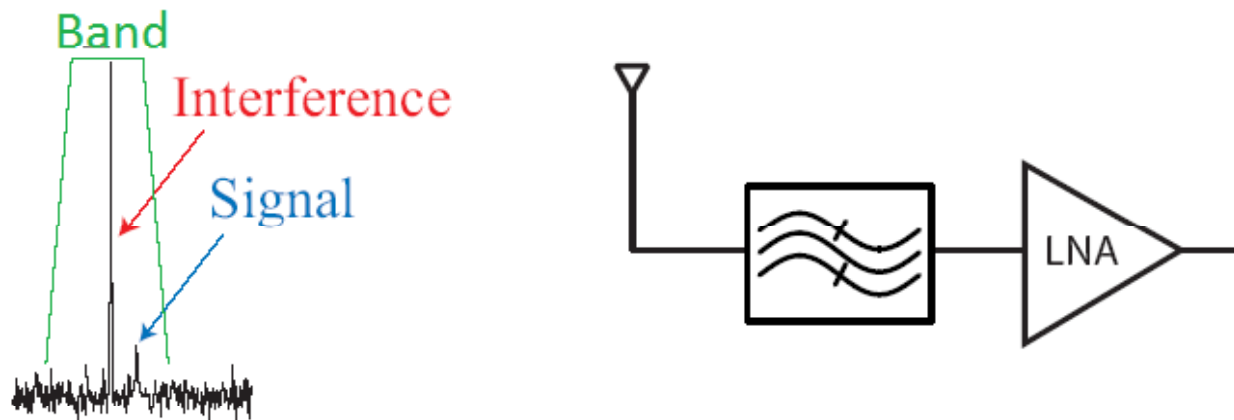
- In a like manner, we can calculate IIP_3

$$IM_3 = 1 = \frac{3 a_3}{4 a_1} A^2 \quad IIP_3 = A = \sqrt{\frac{4}{3} \left| \frac{a_1}{a_3} \right|}$$

Relation Between P_{1-dB} and IIP3

$$P_{1-dB} = IIP3 - 9.6 \text{ dB}$$

Blocker or Jammer

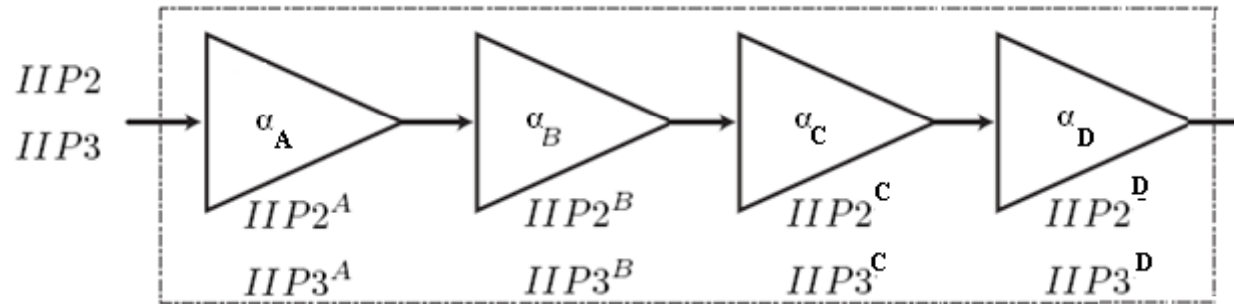


- Consider the input spectrum of a weak desired signal and a “blocker”

$$S_i = \underbrace{S_1 \cos \omega_1 t}_{\text{Blocker}} + \underbrace{s_2 \cos \omega_2 t}_{\text{Desired}}$$

- We shall show that in the presence of a strong interferer, the gain of the system for the desired signal is reduced. This is true even if the interference signal is at a substantially difference frequency. We call this interference signal a “jammer”.

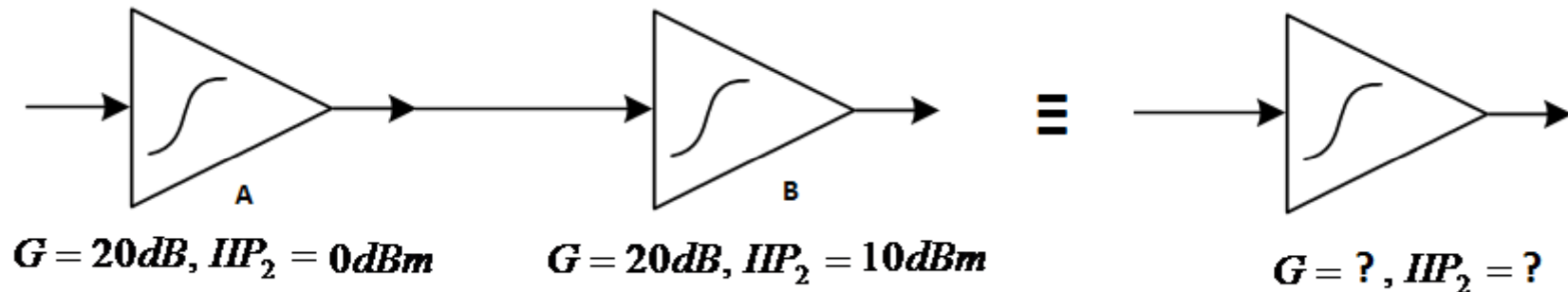
Cascade



$$\frac{1}{A_{IP2}} = \frac{1}{A_{IP2,A}} + \frac{1}{\frac{A_{IP2,B}}{\alpha_A}} + \frac{1}{\frac{A_{IP2,C}}{\alpha_A \alpha_B}} + \frac{1}{\frac{A_{IP2,D}}{\alpha_A \alpha_B \alpha_C}}$$

$$\frac{1}{(A_{IP3})^2} = \frac{1}{(A_{IP3,A})^2} + \frac{1}{\left(\frac{A_{IP3,B}}{\alpha_A}\right)^2} + \frac{1}{\left(\frac{A_{IP3,C}}{\alpha_A \alpha_B}\right)^2} + \frac{1}{\left(\frac{A_{IP3,D}}{\alpha_A \alpha_B \alpha_C}\right)^2}$$

Example



$$\frac{1}{A_{IP_2}} = \frac{1}{A_{IP_2,A}} + \frac{1}{\frac{A_{IP_2,B}}{\alpha_A}}$$

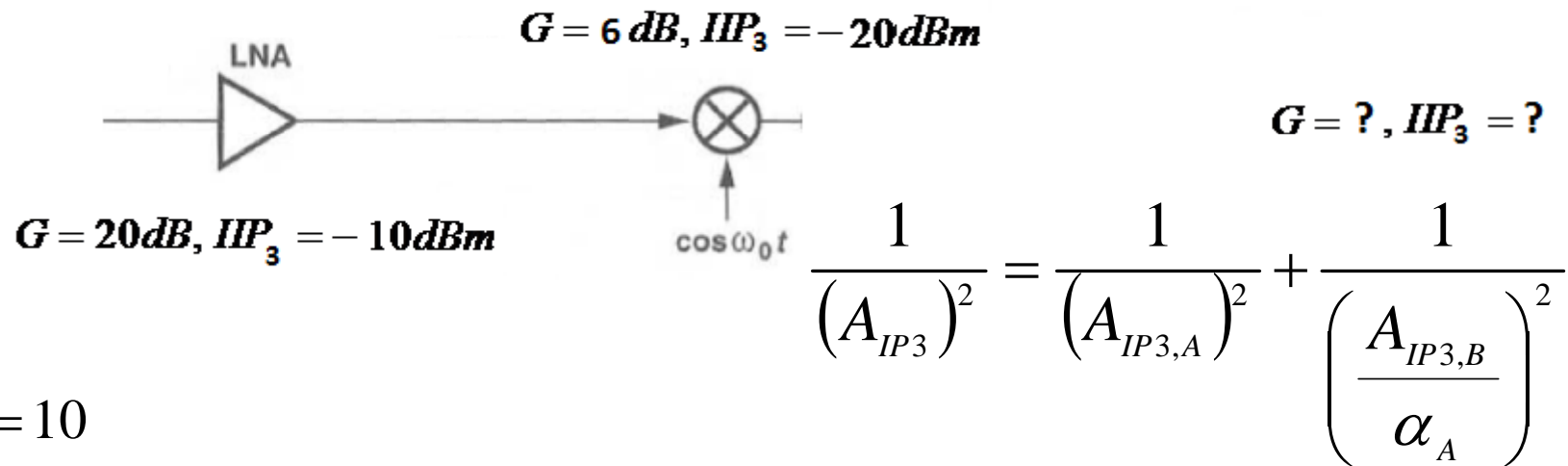
$$\alpha_A = 10$$

$$IIP_2 = 20\log(A_{IP_2}) + 10 \implies \begin{cases} A_{IP_2,A} = 0.316 \\ A_{IP_2,B} = 1 \end{cases}$$

$$\frac{1}{A_{IP_2}} = \frac{1}{0.316} + \frac{1}{\frac{1}{10}} \implies A_{IP_2} = 0.076 \implies IIP_2 = -12.4\text{ dBm}, G = 40\text{dB}$$

so clearly the second amplifier dominates the distortion.

LNA/Mixer Example



$$\alpha_A = 10$$

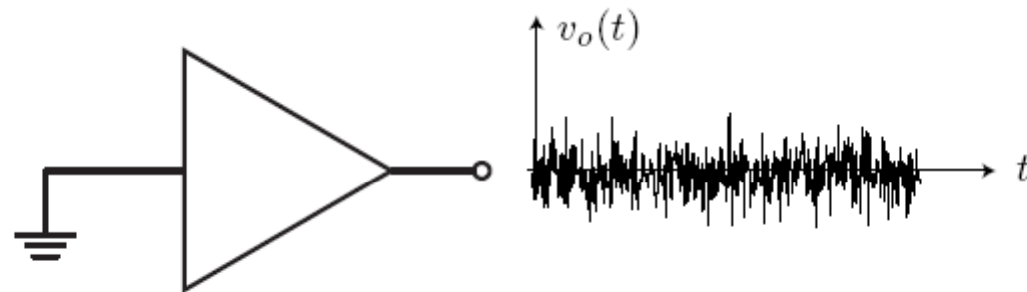
$$IIP_3 = 20 \log(A_{IP3}) + 10 \implies \begin{cases} A_{IP3,A} = 0.1 \\ A_{IP3,B} = 0.0316 \end{cases}$$

$$\frac{1}{(A_{IP3})^2} = \frac{1}{(0.1)^2} + \frac{1}{\left(\frac{0.0316}{10}\right)^2} \implies \frac{1}{(A_{IP3})^2} = 100 + 100144 \implies A_{IP3} = 0.00316$$

$$\implies IIP_3 = -40 \text{ dBm}, G = 26 \text{ dB}$$

The mixer will dominate the overall IIP_3 of the system.

Introduction to Noise



- All electronic amplifiers generate noise. This noise originates from the random thermal motion of carriers and the discreteness of charge.
- Noise signals are random and must be treated by statistical means. Even though we cannot predict the actual noise waveform, we can predict the statistics such as the mean (average) and variance.

Noise Power

- The average value of the noise waveform is zero

$$\overline{v_n(t)} = \langle v_n(t) \rangle = \frac{1}{T} \int_T v_n(t) dt = 0$$

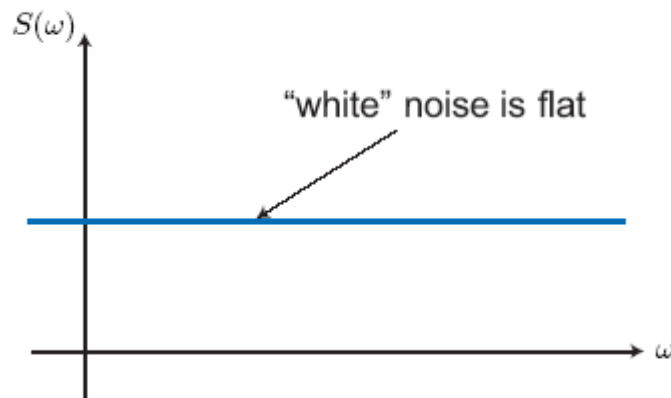
- The mean is also zero if we freeze time and take an infinite number of samples from identical amplifiers.
- The variance, though, is non-zero. Equivalently, we may say that the signal power is non-zero

$$\overline{v_n(t)^2} = \frac{1}{T} \int_T v_n^2(t) dt \neq 0$$

- The RMS (root-mean-square) voltage is given by

$$v_{n,rms} = \sqrt{\overline{v_n(t)^2}}$$

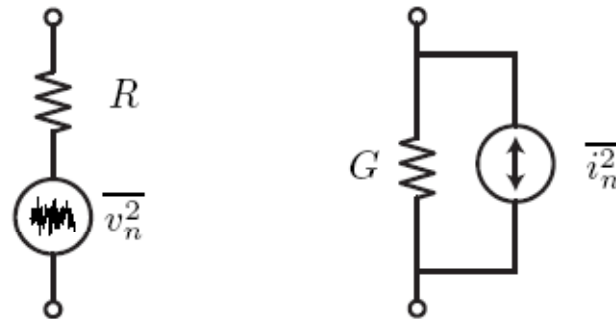
Power Spectrum of Noise



The power spectrum of the noise shows the concentration of noise power at any given frequency. Many noise sources are "white" in that the spectrum is flat (up to extremely high frequencies)

- In such cases the noise waveform is totally unpredictable as a function of time. In other words, there is absolutely no correlation between the noise waveform at time t_1 and some later time $t_1 + \delta$, no matter how small we make δ .

Thermal Noise of a Resistor



- All resistors generate noise. The noise power generated by a resistor R can be represented by a series voltage source with mean square value $\overline{v_n^2}$

$$\overline{v_n^2} = 4kTRB$$

- Equivalently, we can represent this with a current source in shunt

$$\overline{i_n^2} = 4kTGB$$

Resistor Noise Example

- Suppose that $R = 10\text{k}\Omega$ and $T = 20^\circ\text{C} = 293\text{K}$.

$$4kT = 1.62 \times 10^{-20}$$

$$\overline{v_n^2} = 1.62 \times 10^{-16} \times B$$

$$v_{n,rms} = \sqrt{\overline{v_n(t)^2}} = 1.27 \times 10^{-8} \sqrt{B}$$

- If we limit the bandwidth of observation to $B = 10^6\text{MHz}$, then we have

$$v_{n,rms} \approx 13\mu\text{V}$$

- This represents the limit for the smallest voltage we can resolve across this resistor in this bandwidth

Diode Shot Noise

- A forward biased diode exhibits noise called *shot noise*. This noise arises due to the quantized nature of charge.
- The noise mean square current is given by

$$\overline{i_{d,n}^2} = 2qI_{DC}B$$

- The noise is white and proportional to the DC current I_{DC}
- Reversed biased diodes exhibit excess noise not related to shot noise.

Noise in a BJT

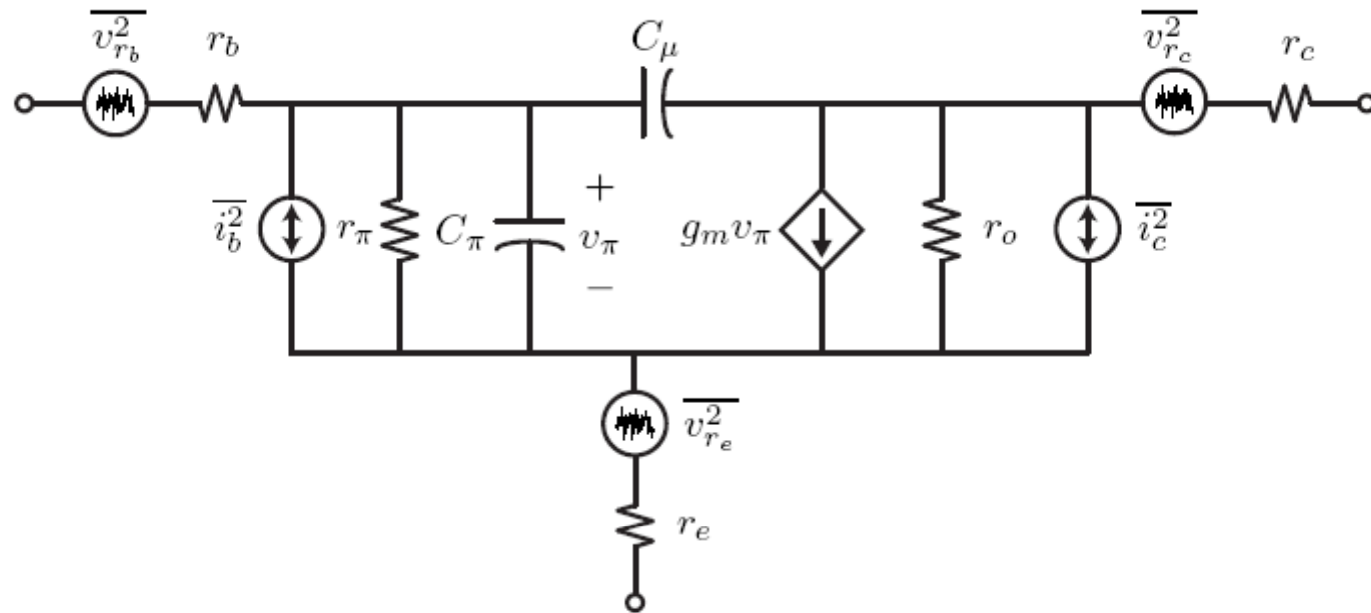
- All physical resistors in a BJT produce noise (r_b, r_e, r_c). The output resistance r_o , though, is *not* a physical resistor. Likewise, r_π , is not a physical resistor. Thus these resistances do not generate noise
- The junctions of a BJT exhibit shot noise

$$\overline{i_{b,n}^2} = 2qI_B B$$

$$\overline{i_{c,n}^2} = 2qI_C B$$

- At low frequencies the transistor exhibits “Flicker Noise” or $1/f$ Noise.

BJT Hybrid- Π Model



- The above equivalent circuit includes noise sources. Note that a small-signal equivalent circuit is appropriate because the noise perturbation is very small

FET Noise

- In addition to the extrinsic physical resistances in a FET (r_g, r_s, r_d), the channel resistance also contributes thermal noise
- The drain current noise of the FET is therefore given by

$$\overline{i_{d,n}^2} = 4kT\gamma g_{ds0}\delta f + K \frac{I_D^a}{C_{ox}L_{eff}^2 f^e} \delta f$$

- The first term is the thermal noise due to the channel resistance and the second term is the “Flicker Noise”, also called the $1/f$ noise, which dominates at low frequencies.
- The factor $\gamma = \frac{2}{3}$ for a long channel device.
- The constants K , a , and e are usually determined empirically.

FET Channel Resistance

- Consider a FET with $V_{DS} = 0$. Then the channel conductance is given by

$$g_{ds,0} = \frac{\partial I_{DS}}{\partial V_{DS}} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

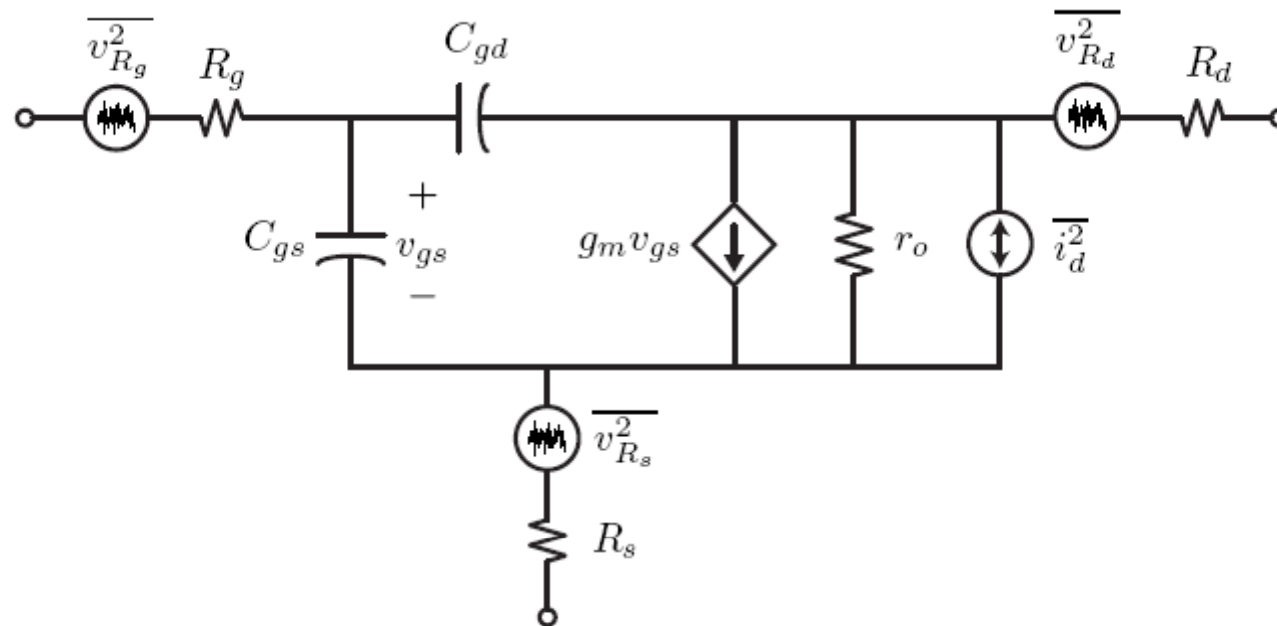
- For a long-channel device, this is also equal to the device transconductance g_m in saturation

$$g_m = \frac{\partial I_{DS}}{\partial V_{GS}} = \mu C_{ox} \frac{W}{L} (V_{GS} - V_T)$$

- For short-channel devices, this relation is not true, but we can define

$$\alpha = \frac{g_m}{g_{d0}} \neq 1$$

FET Noise Equivalent Circuit

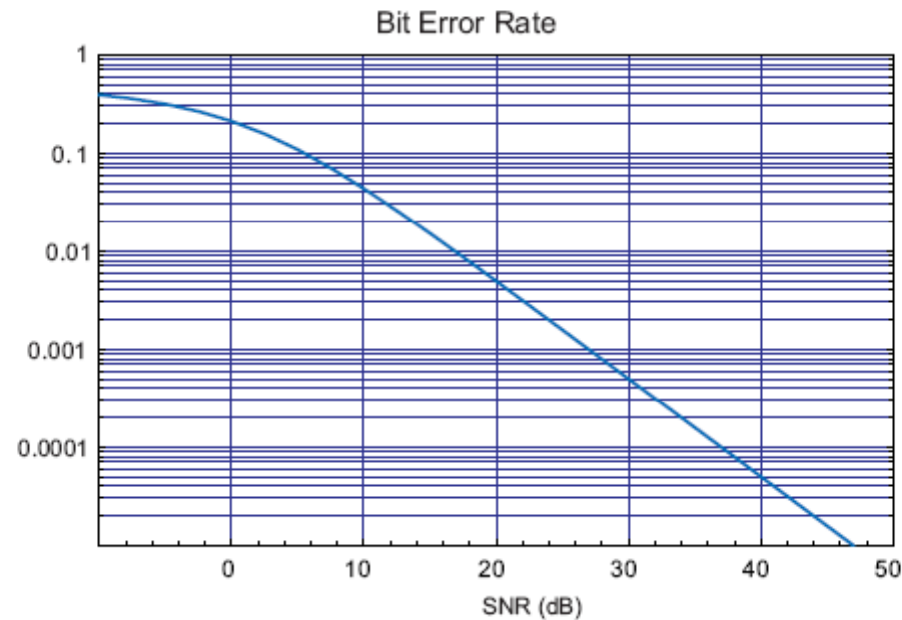


- The resistance of the substrate also generates thermal noise. In most circuits we will be concerned with the noise due to the channel $\overline{i_d^2}$ and the input gate noise $\overline{v_{R_g}^2}$

Degradation of Link Quality

- As we have seen, noise is an ever present part of all systems. Any receiver must contend with noise.
- In analog systems, noise deteriorates the quality of the received signal, e.g. the appearance of “snow” on the TV screen, or “static” sounds during an audio transmission.
- In digital communication systems, noise degrades the throughput because it requires retransmission of data packets or extra coding to recover the data in the presence of errors.

BER Plot



- It's typical to plot the Bit-Error-Rate (BER) in a digital communication system.
- This shows the average rate of errors for a given signal-to-noise-ratio (SNR)

SNR

- In general, then, we strive to maximize the signal to noise ratio in a communication system. If we receive a signal with average power P_{sig} , and the average noise power level is P_{noise} , then the SNR is simply

$$SNR = \frac{S}{N}$$

$$SNR(\text{dB}) = 10 \cdot \log \frac{P_{sig}}{P_{noise}}$$

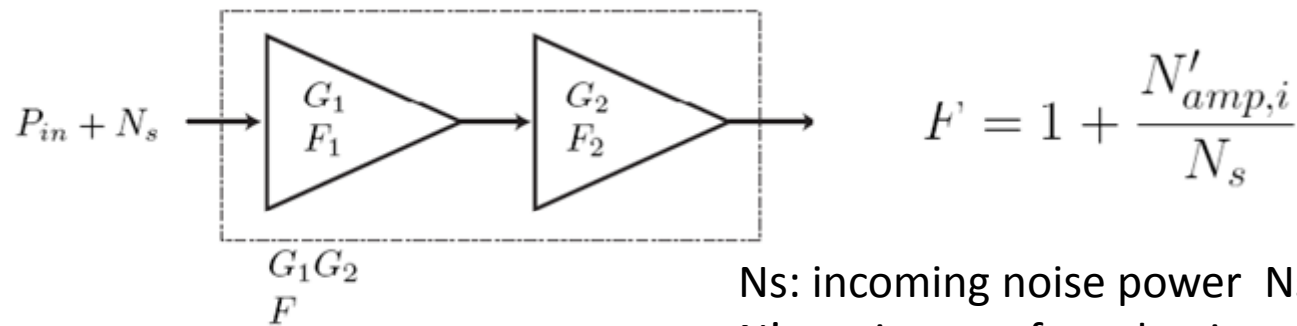
- We distinguish between random noise and “noise” due to interferers or distortion generated by the amplifier

Noise Figure

$$F = \frac{SNR_i}{SNR_o}$$

$$NF = 10 \cdot \log(F) \text{ (dB)}$$

General Cascade Formula



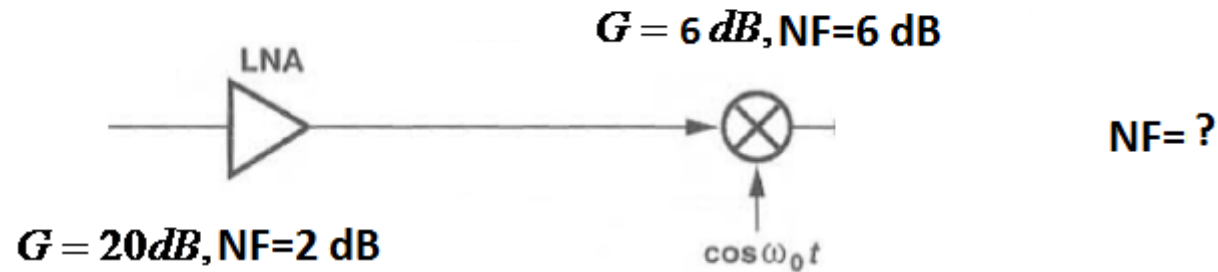
N_s : incoming noise power $N_s = kTB$

$N'_{amp,i}$: input referred noise of the amplifiers

- The general equation is written by inspection

$$F = F_1 + \frac{F_2 - 1}{A_{v1}^2} + \frac{F_3 - 1}{(A_{v1} A_{v2})^2} + \frac{F_4 - 1}{(A_{v1} A_{v2} A_{v3})^2}$$

Example



$$F = F_1 + \frac{F_2 - 1}{A_{v1}^2}$$

$$A_{v1} = 10$$

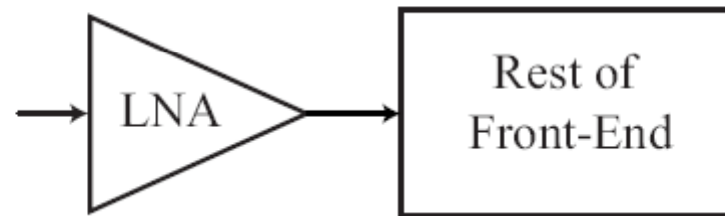
$$\text{NF} = 10 \log(F) \implies \begin{cases} F_1 = 0.158 \\ F_2 = 4 \end{cases}$$

$$F = F_1 + \frac{F_2 - 1}{A_{v1}^2} \implies F = 1.58 + \frac{4 - 1}{10^2} = 1.61$$

$$\implies \text{NF} = 2.08\text{dB}$$

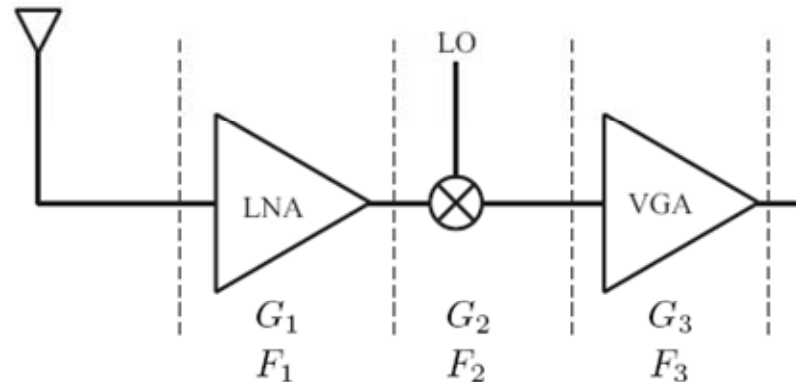
The LNA will dominate the overall NF of the system.

Cascade Formula Interpretation



- We see that in a cascade, the noise contribution of each successive stage is smaller and smaller.
- The noise of the *first* stage is the most important. Thus, every communication system employs a *low noise amplifier* (LNA) at the front to relax the noise requirements
- A typical LNA might have a $G = 20$ dB of gain and a noise figure $NF < 1.5$ dB. The noise figure depends on the application.

NF Cascade Example



- The LNA has $G = 15$ dB and $NF = 1.5$ dB. The mixer has a conversion gain of $G = 10$ dB and $NF = 10$ dB. The IF amplifier has $G = 70$ dB and $NF = 20$ dB.
- Even though the blocks operate at different frequencies, we can still apply the cascade formula if the blocks are impedance matched

$$F = 1.413 + \frac{10 - 1}{(5.6)^2} + \frac{100 - 1}{(5.6)^2(3.16)^2} = 3 \text{ dB}$$

Minimum Detectable Signal (Sensitivity)

- Say a system requires an SNR of 10 dB for proper detection.

If a front-end with sufficient gain has $NF = 10$ dB, let's compute the minimum input power that can support communication:

kTB is the meaning of "Thermal Noise".

$$SNR_o = \frac{SNR_i}{F} = \frac{P_{in}}{N_s} > \mathbf{SNR}_{\min}$$

$$> \mathbf{SNR}_{\min} \cdot F \cdot N_s = 10 \cdot F \cdot kTB$$

- we see that the answer depends on the bandwidth B .

$$P_{in} = \mathbf{SNR}_{\min} + NF - 174 \text{ dBm} + 10 \cdot \log B$$

$$P_{in} = 10 \text{ dB} + NF - 174 \text{ dBm} + 10 \cdot \log B$$

Minimum Signal (cont)

- For wireless data, $B \sim 10\text{MHz}$:

$$P_{in} = 10 \text{ dB} + 10 \text{ dB} - 174 \text{ dB} + 70 \text{ dB} = -84 \text{ dBm}$$

- We see that the noise figure has a dB for dB impact on the minimum detectable input signal. Since the received power drops $> 20 \text{ dB}$ per decade of distance, a few dB improved NF may dramatically improve the coverage area of a communication link.
- Otherwise the transmitter has to boost the TX power, which requires excess power consumption due to the efficiency η of the transmitter.