A Critical Overview on Zero sequence component compensation in Distorted and Unbalanced Three-Phase Four-Wire Systems

M. Tavakoli Bina, *Senior Member, IEEE*, and E. Pasha Javid

Abstract**—The generalized theory of instantaneous powers has been used in power system applications for compensation of instantaneous powers and harmonic cancellation. Nevertheless, this theory results practically desirable solution in three-phase three-wire systems. However, for a three-phase four-wire system, when the power system is unbalanced or distorted, then these definitions introduce unsatisfactory cancellation of the source zero sequence current. This paper analyzes and simulates this problem to show the issues related to the generalized theory of instantaneous powers. Further, complementary analytical discussions are suggested to remedy the raised problem. Finally, the improved definitions are simulated for a three-phase four-wire system that includes significant zero sequence current. Suggested complementary solutions are then compared with those of conventional generalized definitions, which confirm effectiveness of the proposed solutions in cancellation of zero sequence current of the source.**

Index Terms**— Power definitions, generalized method, zero sequence component, four-wire systems.**

I. Introduction

VARIOUS compensation methods have been suggested for non-sinusoidal conditions in three-phase systems to be implemented by active filters. A variety of strategies are introduced for seeking suitable references for active filters [1]. Among them, the generalized theory of instantaneous powers has shown interesting definitions that are usefully related and formulated [2]-[3]. This method presents compensating reference currents for active filters to supply reactive power, harmonics and zero sequence of the load. This method functions perfectly as long as the power system voltage consists of no zero sequence components.

A complementary strategy is introduced in [4], which introduces a solution for three-wire systems based on modeling an optimization problem. Optimal solutions are simple, and can be fully implemented for isolated power systems. Further, the solutions are modified to be applied to unbalanced four-wire systems in which the load bus contains zero sequence voltage. This paper aims at a critical overview on the developed method by presenting analytical approach to get to the root of the problem. Hence, the power definitions are applied to a typical power system simulated with MATLAB-SIMULINK to show both advantages and disadvantages of the compensating methods. Furthermore, some suggestions are proposed to remedy the disadvantages, while these are simulated and compared with those of the original method.

M. Tavakoli Bina is with K. N. Toosi University of Technology, PO Box 16315-1355, Tehran, Iran (e-mail: tavakoli@ieee.org).

E. Pasha Javid (Master student) is with K. N. Toosi University of Technology, (e-mail: pashajavid@ee.kntu.ac.ir).

II. How Power Definitions Are Derived?

There are currently different definitions available in three-phase power systems [3]-[7]. These are based on instantaneous strategies that are used for compensating load power excluding its average active power. Definitions look dissimilar, but some of them are basically derived by the same root. In principle, they introduce similar compensation issues for three-phase four-wire systems, which here this fact is described.

A. Basic optimized solution

Consider a power system of Fig. 1 including the source, the load and the compensator, where the load terminal voltages are $\mathbf{v}(t) = [v_a(t) \quad v_b(t) \quad v_c(t)]^t$. Also, let us assume the three-phase load instantaneous current ($\mathbf{i}(t) = [i_a(t) \quad i_b(t) \quad i_c(t)]^t$) having two parts; active current ($\mathbf{i_p}(t) = [i_{pa}(t) \quad i_{pb}(t) \quad i_{pc}(t)]^t$) and remaining part that do not contribute to active power ($\mathbf{i_q}(t) = [i_{qa}(t) \quad i_{qb}(t) \quad i_{qc}(t)]^t$), this is called *inactive* current. An optimization problem can be constructed, aiming at minimizing the sum of squares of active parts in the form of active power. One constraint is also added to emphasize that inactive part has no share in active power as follows:

$$\text{Minimize} \sum_{k=a,b,c} (i_k(t) - i_{qk}(t))^2$$

$$\text{Subject to: } i_{qa}(t)v_a(t) + i_{qb}(t)v_b(t) + i_{qc}(t)v_c(t) = 0 \quad (1)$$

978-981-05-9423-7 © 2007 RPS

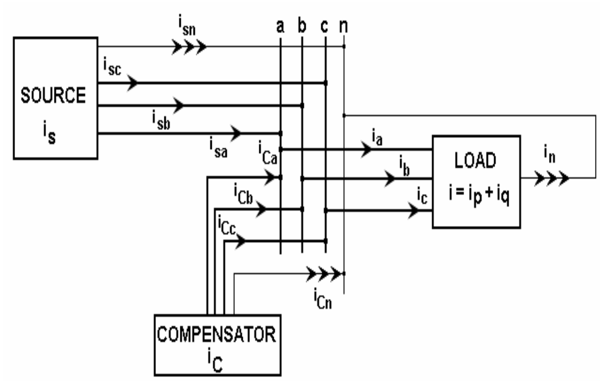

Figure 1: A typical interconnection of the source, the load and the compenstor across a three-phase four-wire power system bus.

Solving (1) using the well-known Lagrange multiplier method leads to the following optimal solution:

$$\begin{cases} \begin{bmatrix} i_{pa}(t) \\ i_{pb}(t) \\ i_{pc}(t) \end{bmatrix} = \dfrac{i_a(t)v_a(t)+i_b(t)v_b(t)+i_c(t)v_c(t)}{v_a(t)^2+v_b(t)^2+v_c(t)^2} \begin{bmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{bmatrix} \\ \begin{bmatrix} i_{qa}(t) \\ i_{qb}(t) \\ i_{qc}(t) \end{bmatrix} = \begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{bmatrix} - \begin{bmatrix} i_{pa}(t) \\ i_{pb}(t) \\ i_{pc}(t) \end{bmatrix} \end{cases} \tag{2}$$

These results have already appeared in literature [5]-[8], where the term $i_a(t)v_a(t)+i_b(t)v_b(t)+i_c(t)v_c(t)$ in (2) can be replaced with the load active power definition ($P(t)$) as:

$$\begin{cases} \mathbf{i_p}(t) = \dfrac{P(t)}{v_a(t)^2+v_b(t)^2+v_c(t)^2}\mathbf{v}(t) = \dfrac{P(t)}{\mathbf{v}(t)\cdot\mathbf{v}(t)}\mathbf{v}(t) \\ \mathbf{i_q}(t) = \mathbf{i}(t) - \mathbf{i_p}(t) \end{cases} \tag{3}$$

Let us call (3) as the optimal solution (*OS*) throughout this paper, which provides reference currents for the compensator to follow. The *OS* is simple to implement, but it is only suitable for balanced three-wire systems. Even when a three-wire system contains negative sequence voltage, compensation with the *OS* results in distorted waveforms at the source-end.

B. Derivation of the generalized theory of instantaneous powers

Interesting definitions were introduced in [2]-[3] as the generalized theory of instantaneous powers (*GTIP*) as below:

$$\begin{cases} P(t) = \mathbf{v}(t)\cdot\mathbf{i}(t), & \mathbf{q}(t) = \mathbf{v}(t)\times\mathbf{i}(t) \\ \mathbf{i_p}(t) = \dfrac{P(t)}{\mathbf{v}(t)\cdot\mathbf{v}(t)}\mathbf{v}(t), & \mathbf{i_q}(t) = \dfrac{\mathbf{q}(t)\times\mathbf{v}(t)}{\mathbf{v}(t)\cdot\mathbf{v}(t)} \end{cases} \tag{4}$$

We implemented the *GTIP* of (4) as a compensation rule for a three-phase four-wire active filter. The resultant source currents after compensation were unconvincing, showing distorted waveform at the source-end as well as flow of significant zero sequence current through the source neutral-wire. Simulation results also confirm the stated drawbacks, made us to seek the reasons for these issues.

Taking a closer look into the *GTIP* definitions indicates that they can be derived using the *OS*. Interestingly, the active current of the load ($\mathbf{i_p}(t)$) defined by the OS in (3) and the GTIP in (4) are identical. From the OS, inactive current $\mathbf{i_q}(t)$ in (3) can be expanded as follows:

$$\mathbf{i_q}(t) = \mathbf{i}(t) - \mathbf{i_p}(t) = \mathbf{i}(t) - \frac{P(t)}{\mathbf{v}(t)\cdot\mathbf{v}(t)}\mathbf{v}(t) = \frac{\mathbf{i}(t)[\mathbf{v}(t)\cdot\mathbf{v}(t)] - [\mathbf{v}(t)\cdot\mathbf{i}(t)]\mathbf{v}(t)}{\mathbf{v}(t)\cdot\mathbf{v}(t)} = \frac{[\mathbf{v}(t)\times\mathbf{i}(t)]\times\mathbf{v}(t)}{\mathbf{v}(t)\cdot\mathbf{v}(t)} \tag{5}$$

Comparing (5) with (4) verifies that by defining $\mathbf{q}(t) = \mathbf{v}(t)\times\mathbf{i}(t)$ the *GTIP* gives exactly identical solutions as those of the *OS*. Therefore, it is expected similar issues for the *GTIP* like those of the *OS*.

III. SUGGESTIONS TO IMPROVE THE OS

Considering Fig. 1, the currents of compensator ($\mathbf{i_C}(t) = [i_{Ca}(t) \quad i_{Cb}(t) \quad i_{Cc}(t)]^t$) need to be determined in a way that the resultant source currents ($\mathbf{i_s}(t) = \mathbf{i} - \mathbf{i_C} = [i_{sa}(t) \quad i_{sb}(t) \quad i_{sc}(t)]^t$) follow the pre-planed components. A method in [4] suggests replacing $\mathbf{v}(t)$ with $\mathbf{v}(t) - \mathbf{v_0}(t)$ ($\mathbf{v_0}(t)$ is the zero sequence voltage of the load terminal) in (3) to overcome the issues concerned with the source-end zero sequence current. We follow this suggestion by extending the replacement for all voltage terms in (4). Also, it is assumed that the source current after compensation turns into the following relationship, since the load active current ($\mathbf{i_p}(t)$) is necessary to remain unchanged:

$$\mathbf{i_s}(t) = \frac{\overline{P(t)}}{(\mathbf{v}(t)-\mathbf{v_0}(t))\cdot(\mathbf{v}(t)-\mathbf{v_0}(t))}(\mathbf{v}(t)-\mathbf{v_0}(t)) \tag{6}$$

Where $\overline{P(t)}$ is the load average power. One advantage of using (6) is that by summing up all three source currents, the neutral current at the source-end approaches zero ($i_{sn} = 3i_{s0} = \sum_{k=a,b,c} i_{sk} = 0$). Also, comparing the source active power ($P_s(t)$) with the load active power ($P(t)$) verifies that:

$$P_s(t) = \mathbf{v}(t) \cdot \mathbf{i_s}(t) = \frac{\overline{P(t)}[v_a(t)^2 + v_b(t)^2 + v_c(t)^2 - 3v_0(t)^2]}{\mathbf{v}(t) \cdot \mathbf{v}(t) + \mathbf{v}_0(t) \cdot \mathbf{v}_0(t) - 2\mathbf{v}(t) \cdot \mathbf{v}_0(t)} = \overline{P(t)} \tag{7}$$

This clearly indicates that the active power supplied by the source is the same as the average active power delivered to the load. The rest of the needed active power for the load (load ripple active power) can be balanced through the compensator. Note that the compensator current $\mathbf{i_C}(t)$ is also different from $\mathbf{i_q}(t)$ of the load in (3) and (4), and can be simplified to:

$$\mathbf{i_C}(t) = \frac{[(\mathbf{v}(t) - \mathbf{v_0}(t)) \times \mathbf{i}(t)] \times (\mathbf{v}(t) - \mathbf{v_0}(t))}{(\mathbf{v}(t) - \mathbf{v_0}(t)) \cdot (\mathbf{v}(t) - \mathbf{v_0}(t))} + [(\widetilde{P}(t) - 3P_0(t)] \frac{(\mathbf{v}(t) - \mathbf{v_0}(t))}{(\mathbf{v}(t) - \mathbf{v_0}(t)) \cdot (\mathbf{v}(t) - \mathbf{v_0}(t))}$$

(8)

Where $\widetilde{P}(t)$ is the load ripple active power and $P_0(t) = v_0(t) i_0(t)$ is the load zero sequence active power that would be nonzero when both zero sequence voltage and current are nonzero. Equations (6) and (8) state that all load-bus voltage samples ($\mathbf{v}(t)$) need to be replaced with $\mathbf{v}(t) - \mathbf{v_0}(t)$. Meanwhile, the *GTIP* definition for reactive power can no longer be used for derivation of the compensating current; rather the combination of (8) would be a better choice.

IV. Analysis and Simulation

To distinguish the advantages of the proposed improved method over the conventional ones, various simulations are arranged and analyzed. Here it is simulated the suggested improvement of (8), where the outcomes are compared with available methods to examine their advantages as well as disadvantages. Figure 1 is considered and simulated with SIMULINK in which three single-phase rectifiers are introduced as the load like it is shown by Fig. 2. Two cases are examined to describe the difference between the performance of the discussed methods; first, balanced sinusoidal load voltages and second, distorted unbalance load terminal voltages. Three voltages of the first case are sinusoidal 50 Hz with RMS value of 220 V. The following distorted unbalanced voltages are applied to the load terminal for the second case:

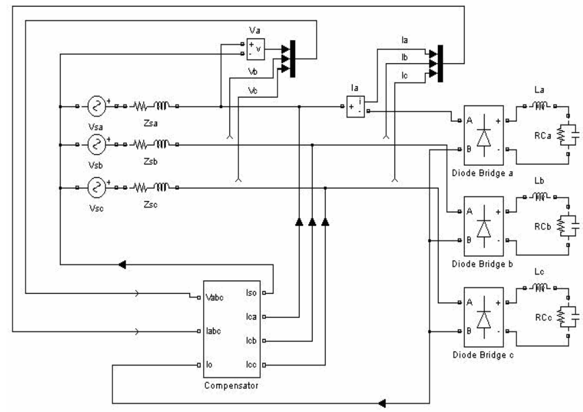

Figure 2: SIMULINK simulation of non-linear unbalanced power system

$$\begin{cases} v_a(t) = 311\sin(\omega t) + 40\sin(3\omega t - 18^\circ) \\ v_b(t) = 280\sin(\omega t - 72^\circ) + 35\sin(3\omega t - 90^\circ) \\ v_c(t) = 280\sin(\omega t + 90) + 20\sin(3\omega t - 51^\circ) \end{cases} \tag{9}$$

Also, the single-phase non-linear load parameters are as below:

$$\begin{cases} R_a = 2.0\,\Omega & L_a = 1\,mH & C_a = 1\,mF \\ R_b = 3.87\,\Omega & L_b = 1\,mH & C_a = 1\,mF \\ R_c = 4.84\,\Omega & L_c = 1\,mH & C_a = 1\,mF \end{cases} \tag{10}$$

Note that the load for the two cases is identical, while the load terminal voltages are assumed to be balanced for the case 1.

A. Case 1: Balanced sinusoidal load terminal voltage

Considering Figs. 1 and 2, simulation of the first case is arranged with unsymmetrical loads of (10), but balanced load terminal voltages. This cannot be implemented in practice, while nearly all compensating cases use these fictitious conditions. During the compensation process, since the source is inherently balanced, this situation causes balanced source currents flow to the load terminal despite unsymmetrical load assumption.

Figure 3 shows the simulation results for the first case. In this picture, Figs. 3(a)–(b) depict the load terminal balanced voltages and asymmetrical distorted load currents, respectively. It can be seen that the load currents are considerably distorted, showing significant load zero sequence current on the fourth wire as it is illustrated by Fig. 3(f). Note that both the source and the load zero sequence currents are identical before the compensator is

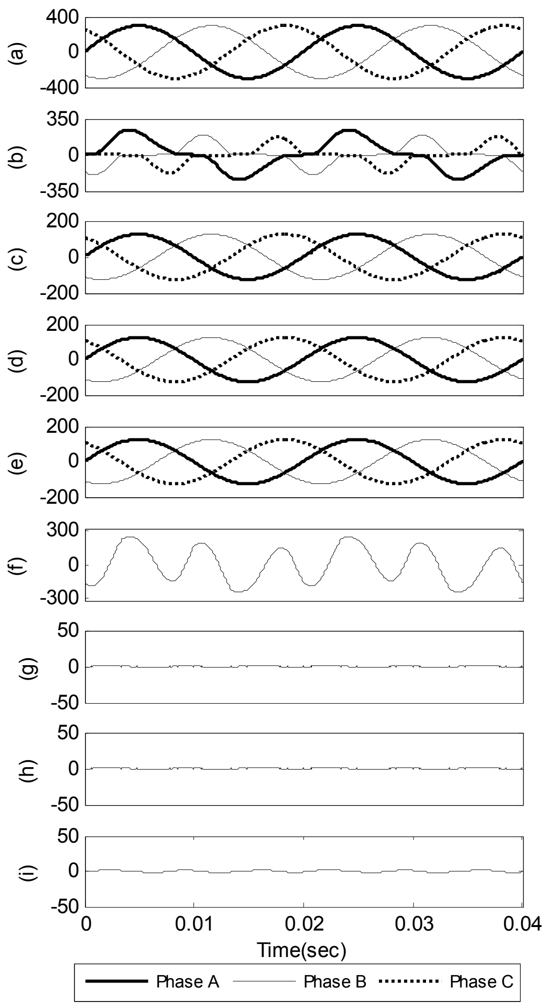

Figure 3: Simulation results for the case 1: (a) three phase balanced load voltages (V), (b) asymmetrical distorted load currents (A), (c)–(e) three-phase source currents (A) after compensation using the *OS* method, the *GTIP* method and the suggested improvement on the *OS*, (f) the load neutral current (A), (g)–(i) the source neutral current (A) after compensation using the *OS* method, the *GTIP* method and the suggested improvement on the *OS*.

coupled to the power system bus.

Also, the compensation method is initially based on the *OS*. Figure 3(c) provides the source currents after compensation. It is noticeable that source currents are absolutely balanced as it is expected by considering balanced terminal voltages. Further, Fig. 3(g) introduces the source neutral wire zero sequence current after compensation, which is almost equal to zero. It should be emphasized that the *OS* compensating rule of (3) has no role in forcing the load zero sequence current to flow through the compensator. Instead, *untrue pre-assumption of the balanced voltages for the load terminal* in simulation makes the source to supply balanced currents.

Moreover, the *GTIP* compensation rule of (5) is simulated with the same simulation background. Figure 3(d) shows the source currents after compensation, and Fig. 3(h) presents the source zero sequence currents flows on the fourth wire. Simulation results show that the GTIP gives identical waveforms like those of the OS. This has already been examined mathematically in Section II.

Finally, the proposed improvement on the *OS* (given by (8)) is simulated (described by (6)–(8)). Figure 3(e) introduces the source current after compensation, and Fig. 3(i) shows the zero sequence current flows on the neutral wire. Simulation results are absolutely identical for the three methods under the case 1. This clearly indicates that case 1 cannot distinguish the performance of the three methods. Thus, a more realistic case should be taken into account for finding advantages as well as the disadvantages of compensating methods.

B. Case 2: Unbalanced and distorted load terminal voltages

Load terminal voltages are distorted and unbalance like waveforms of (9) for the case 2. This situation is normally true for power systems, especially for distribution systems. Meanwhile, the distortion might be somehow exaggerated to distinguish performance of the discussed compensation methods in cancellation of zero sequence current of the fourth wire.

Simulation results for the second case are given by Figure 4, where Figs. 4(a)–(b) depict the load terminal distorted unbalanced voltages and asymmetrical distorted load currents, respectively. Both pictures show significantly distorted waveforms, resulting in large load zero sequence current on the fourth wire as it is illustrated by Fig. 4(f) before applying any compensating rules.

Next, the compensation of the load is simulated with the *OS*. Simulation results are presented by Fig. 4(c), where the source currents are neither sinusoidal nor balanced. This confirms the unsuitability of the OS in full compensation of the load. This can be verified by taking a closer look into the current of the fourth wire in Fig. 4(g) that oscillates within $[-160,160]$ A, nearly close to the peak of the load current.

Further, the *GTIP* method, introduced by (4), is applied as the compensation rule to the SIMULINK simulation study of Fig. 2. Simulation results are introduced by Fig. 4(d). It can be seen that source currents in this picture are identical with those of the *OS* presented in Fig. 4(c). Also, Fig. 4(h) shows the zero sequence current flowing through the source neutral wire, which is again identical to those of the *OS* (depicted by Fig. 4(g)).

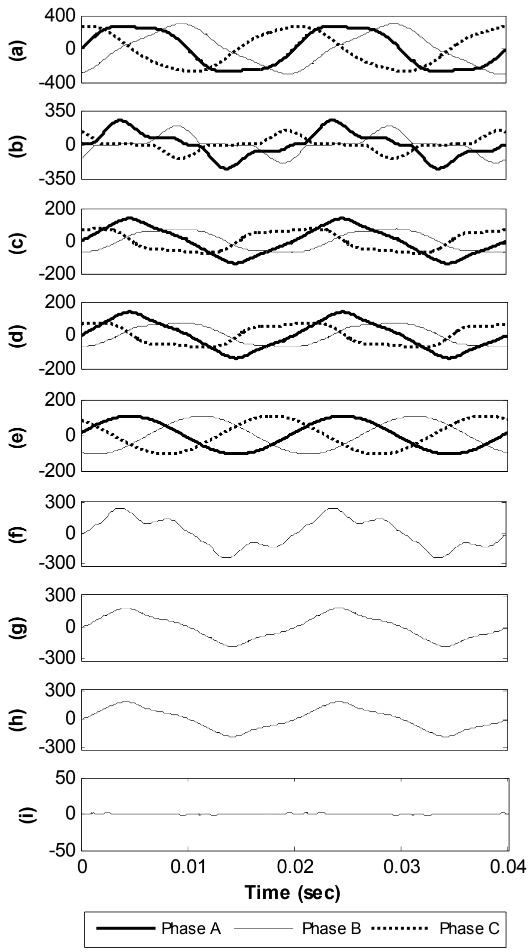

Figure 4: Simulation results for the case 2: (a) unbalanced distorted load voltages (V), (b) asymmetrical distorted load currents (A), (c)–(e) three-phase source currents (A) after compensation using the *OS* method, the *GTIP* method and the suggested improvement on the *OS*, (f) the load neutral current (A), (g)–(i) the source neutral current (A) after compensation using the *OS* method, the *GTIP* method and the suggested improvement on the *OS.*

With reference to the mathematical formulation in Section II, simulation results confirm that both the *OS* and the *GTIP* provide the same solutions for the compensation of load reactive power and harmonics. This is true for all cases including balanced sinusoidal cases or unbalance distorted situations.

At last, the proposal of Section III (equations (6)–(8)) is simulated for the case 2. The source currents after compensation are provided by Fig. 4(e). For this simulation, the negative sequence voltage of the load terminal is detected and subtracted from the load terminal voltage in (6)–(8) alongside the zero sequence voltage. Resultant source currents are closely balanced and sinusoidal. Fig. 4(i) depicts the resulting zero sequence current flowing through the source neutral wire after compensation. This current is significantly attenuated at about a few amps.

Table I summarizes the THD of the source currents after compensation that are gathered from simulation results for all the discussed compensating methods. Also, Table II compares the RMS values of the source's fourth wire current after compensation. It can clearly be observed the advantages of the proposed method in Section III over both the *GTIP* and the *OS*. The following key points also can be expressed based on analytical and simulation results:

- Both the *OS* and the *GTIP* provide identical solutions in sinusoidal, non-sinusoidal, balanced or unbalanced networks.
- The load terminal voltage cannot be considered balanced, when the performance of a compensating rule is needed to be verified.
- The proposed improvement on the *OS* not only cancels successfully the current flowing through the source's neutral wire, but also can actively filters out the load harmonic component by including negative sequence voltage of the load terminal in (6)-(8).

TABLE I
THE THD OF THE THREE COMPENSATING METHODS IN THE TWO CASES.

THD %	Phase	Uncompensated	GTIP	OS	Proposed improvement
Case 1	a	36.12	0.7	0.7	0.7
	b	63.36	0.9	0.9	0.9
	c	69.5	0.9	0.9	0.9
Case 2	a	39.7	12	12	0.8
	b	62.83	12	12	1
	c	60.3	23.2	23.2	1

TABLE II
THE RMS VALUES OF THE SOURCE NEUTRAL WIRE (THE FOURTH WIRE) FOR THREE COMPENSATING METHODS IN THE TWO CASES.

	Uncompensated	OS	GTIP	Proposed improvement
Case 1	150.5 A	2.05 A	2.05 A	2.05 A
Case 2	142 A	111 A	111 A	1.05 A

V. CONCLUSION

This paper reviews various compensation strategies, introducing capabilities as well as practical issues related to the zero sequence component of the source. Here it is reviewed an optimal solution that provides the basic solution for derivation of the conventional method of generalized theory of instantaneous powers. Moreover, it is shown that these methods are practically problematic for unbalanced or distorted power systems. Hence, the optimized solution is improved by proposing certain modifications to get useful cancellation of the neutral-wire current of the source under unbalance and distorted conditions. The two available named methods as well as the suggested improvements are simulated with MATLAB SIMULINK in which simulation results confirm that both the optimal solution and the generalized theory of instantaneous powers cannot be used satisfactorily for a real system in which voltages are not completely balanced. Nevertheless, simulations of the improved proposal perform successfully under all conditions including balanced, unbalanced, distorted and non-sinusoidal three-phase four-wire networks.

VI. References

[1] B. Singh, K. AL-Haddad, A. Chandra, "A Review of Active Power Filters for Power Quality Improvement", IEEE Trans. On Industrial Electronics, Vol. 46, No. 5, Oct. 1999, pp. 960-971

[2] F. Z. Peng and J. S. Lai, "Generalized instantaneous reactive power theory for three-phase power systems," *IEEE Trans. Instrum. Meas.*, vol. 45, No. 1, pp. 293–297, 1996.

[3] F. Z. Peng, George W. Ott, Jr. and Donald J. Adams, "Generalized instantaneous reactive power theory for three-phase power systems", IEEE Trans. on Power Electronics, Vol. 13, NO. 6, November 1998.

[4] M. Tavakoli Bina, "A New Complementary Approach to Inactive Power Compensation", IEEE PESC, 2003, Electronic Record.

[5] S. Fryze, Effective wattles and apparent power in electrical circuits for the case of non sinusoidal waveform of current and voltage, in elektrotechnische zeitschr., vol. 53, pp. 596-599, 1932.

[6] L. Czarnecki, "Minimization of Unbalanced and Reactive Currents in Three-Phase asymmetrical Circuits with Non-sinusoidal Voltage", *IEE Proceedings-B*, Vol.139, No. 4, July 1992.

[7] M. Depenbrock, "The FBD Method, a Generally Applicable Tool for Analyzing Power Relations," *IEEE Trans. on Power Systems*, vol.8, May 1993, pp. 381-387.

[8] M. Depenbrock, V. Staudt, H. Wrede, "A Theoretical Investigation of Original and Modified Instantaneous Power Theory Applied to Four-Wire Systems", IEEE Trans. on Industry Application, Vol. 39, NO. 4, July/August 2003

VII. Biographies

M. Tavakoli Bina was born in Tehran, Iran, on July 14, 1962. He received a B.Sc. degree from the University of Tehran in 1992, and his Ph.D. degree in power electronics and power system interconnection from the University of Surrey in the UK in June 2001. He has been an IEEE member for six years. He is presently holding an associate professor position at the K. N. Toosi University of Technology, researching in the area of designing power electronics devices as well as power system analysis. His latest work corresponds to the design and installation of a research Laboratory in the area of power electronics multilevel converters and FACTS devices.

E. Pasha Javid was born in Sarab, Iran, on April 20, 1982. He received a B.Sc. degree in control engineering from University of Tabriz in 2004. He is now working towards his M.Sc. degree in power engineering and power electronics interconnected systems at K.N. Toosi University of Technology.