Complete Harmonic-Domain Modeling and Performance Evaluation of an Optimal-PWM-Modulated STATCOM in a Realistic Distribution Network

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***Abstract*—Power systems use STATCOM for compensating purposes that is subjected to the high switching frequencies. Various PWM techniques make selective harmonic elimination possible, which effectively control the harmonic content of voltage source inverters. On the other hand, distribution systems have to supply unbalanced nonlinear loads, transferring oscillations to the DC-side of the converter in a realistic operating condition. Thus, additional uncharacteristic harmonics are modulated through the STATCOM at the point of common coupling (PCC). This requires more attention when switching angles are calculated offline using the optimal-PWM technique. This paper suggests a harmonic-domain model in order to realistically evaluate the injected harmonics at the PCC. This model properly takes into account the DC capacitor effect, effects of other possible varying parameters such as voltage unbalance as well as network harmonics, and effects of operating conditions on the STATCOM harmonic performance. The model is programmed, and can be easily run with other algorithms such as Monte Carlo simulation. Further, a semi-stochastic method is proposed to predict and simulate the three-phase voltage unbalance, leading to an analytical tool for prediction of harmonic performance of STATCOM. The predictive method is developed based on the measured data obtained from a low-voltage distribution network. Finally, the modeled STATCOM is linked with the distribution substation, applying the proposed voltage unbalance modeling to evaluate aggregate harmonics of the load.**

I. Introduction

A STATCOM consists of a voltage source converter (VSC), a dc capacitor and a coupling transformer. It may be seen as a basic building block with which several power system objectives can be achieved. A variety of applications include improving power quality problems in low-voltage distribution networks, voltage regulation and reactive power control. Reference signals, containing these objectives, are then modulated by PWM switching frequencies that are much higher than the synchronous frequency.

In practice, as with any VSC-based applications, STATCOM will act as a source of producing harmonics for power systems. It could also interact with possible harmonic distortions and unbalances of the power network (e.g. distributed systems are coined with many power electronic loads). These interactions would be complex, making the analysis of steady-state harmonic levels and full assessment of their dynamic behavior challenging tasks. This is necessary since STATCOM can be better designed, and power quality problems are more efficiently treated. Thus, it is required to pursue evaluation of these harmonic interactions through a suitable combination of practical measurements and analytical studies [1], [2].

Various modulation techniques and topologies are suggested for STATCOM to effectively remove the generated low-order *characteristic* harmonics, including multi-module PWM techniques, selective harmonic elimination and multilevel topologies [3], [4]. An optimal pulse-width modulation (OPWM) uses pre-calculated switching angles based on assuming an ideal fixed DC bus voltage. This method presents several advantages in comparison to the conventional carrier-based sinusoidal PWM schemes [5].

On the other hand, load-terminal harmonics and unbalance of distribution systems impose distortion on both DC and AC sides [2, 12], introducing additional *uncharacteristic* harmonics generated by STATCOM. Considering the OPWM, the pre-calculated chopping angles will not then be optimal under these conditions. Hence, the amount of uncharacteristic harmonics that is injected to a distribution system depends on several factors such as ratings and operating conditions of both STATCOM and distribution network. Therefore, the following steps should be performed to obtain a more reliable harmonic performance evaluation of STATCOM:

- Developing a comprehensive model which takes into account relevant factors and interactions in harmonic generation such as DC capacitor oscillations, capacitance limitation, and unbalance of the grid system. This model will provide accurate and efficient analytical capability.
- Simulating a realistic distribution system under unbalance and distorted situation. The environment should be also capable of integrating deterministic and stochastic approaches, if necessary.
- Making the simulation components efficient and fast to avoid unnecessary iterative procedures that slow down complementary simulations such as Monte Carlo method.

The first step is a challenging topic that has been devoted a noticeable amount of research work over the last few years [6-14]. A detailed overview of harmonic modeling methods can be found in [15]-[17]. Generally speaking, there are three philosophies in modeling of devices, network and their interactions. The methods are embedded in time-domain, frequency-domain, and harmonic-domain. The time-domain formulation consists of relevant differential equations representing the dynamic behavior of the interconnected power system components. The resultant set of equations is normally solved using numerical methods as done in typical simulating software

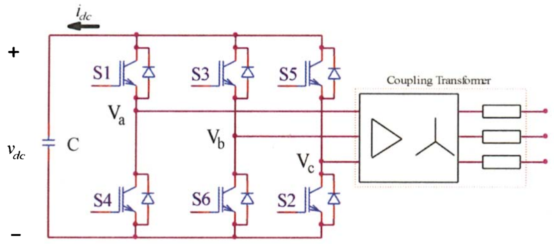

Figure 1: Three-phase voltage source converter connected to the power system through a transformer.

such as PSCAD/EMTDC, PSpice, and SABER. Harmonic information is then obtained using the fast Fourier transform (FFT) in steady state [2]. This requires considerable computation steps even for relatively small systems [18]. Other problems attached to the time-domain algorithms for harmonic studies are the difficulty of describing distributed or frequency-dependent parameters [15], their inflexibility related to the advanced stochastic algorithms, and the needed delay to converge steady state.

Meanwhile, a model is suggested in [10, 11] based on the switching function, introducing a direct solution of the steady state in a closed form. The presented algorithm requires a periodic steady state to be quantified in the time-domain. Thus, this constraint implies similar situation when FFT analysis is performed on the resulting waveforms from the time-domain simulations. Also, non-linearity cannot be well-performed by the closed-form solution of the steady state.

Another direct solution for the effect of an individual harmonic (or frequency) is discussed in [15] in the frequency-domain, excluding the harmonic interaction between the network and the nonlinear compensator. When the harmonic injection from each source depends on those of other sources along with the state variables of the system, accurate results can only be obtained in harmonic-domain. The harmonic-domain is a restricted frequency-domain, while all non-linear interactions are modeled [19]. While some research works take the effects of control system into account [7, 14] in the harmonic-domain, another method works on harmonic power flow [6]. These are not relevant to evaluation of steady state harmonic performance of STATCOM. In [8], modeling of STATCOM is carried out by representing switching functions by their Fourier spectra. This approach potentially offers an accurate solution, considering all harmonics and their interactions. This model is extended and improved in this paper using harmonic-domain techniques.

This paper introduces a more realistic evaluation of harmonic penetration of STATCOM connected across a distribution substation. The structure and modeling of STATCOM for harmonic analysis is presented, where harmonic performance of STATCOM using the OPWM is investigated by means of a harmonic-domain model. A detailed semi-stochastic modeling is then proposed at the PCC for the voltage unbalance. The suggested techniques are extensions of the proposed methods in [20, 21]. Then, this realistic representation of the PCC is applied to the model of STATCOM. Resultant outcomes show the maximum uncharacteristic harmonic injection by the OPWM-STATCOM. This can be easily performed with other modulation techniques under a realistic operating

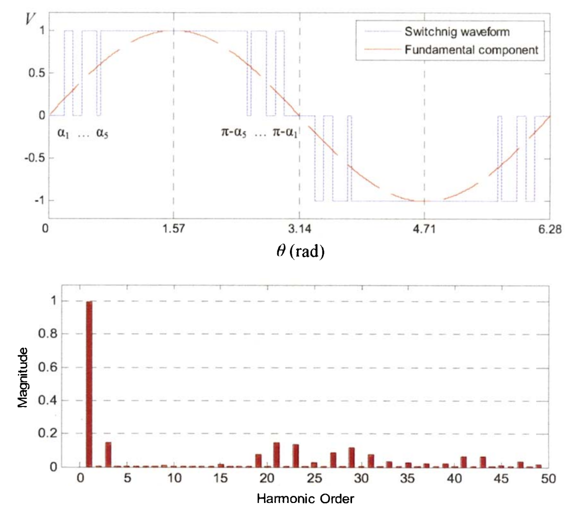

Figure 2: A typical selective harmonic elimination (5^{th}, 7^{th}, 11^{th}, and 13^{th}) based on quarter-cycle symmetry: the waveform and its harmonic spectra.

condition. Further, a real case study is arranged in which the voltage unbalance and background harmonics are simulated at a distribution substation. This substation is chosen from part of Tehran north-east distribution system. Also, a STATCOM will be connected across this real substation. It is eventually examined uncharacteristic harmonics produced by STATCOM that is subjected to the interaction with the realistic model of the PCC. The obtained conclusions are finally discussed and evaluated.

II. STATCOM STRUCTURE AND MODELING

Here it is presented briefly the OPWM, followed by a proposal on modeling of STATCOM for harmonic analysis and performance evaluation.

A. The OPWM modulation technique

A three-phase VSC, using IGBT switches, is shown in Fig. 1. Like other PWM schemes, the DC-link voltage is modulated by the converter using the programmed harmonic elimination technique, i.e. the OPWM. Both switching instants and durations are pre-calculated off-line such that certain chosen harmonics are eliminated; also, a desired value is assigned to the fundamental component of the output. A typical waveform is shown in Fig. 2 in which it is programmed to eliminate non-triple odd harmonics up to 17. The quarter-wave symmetry is commonly used to reduce the complexity of the resulting equations. A general technique is also proposed without this assumption in [22]. Normally, a set of solutions for a certain range of fundamental voltage magnitudes is pre-calculated and stored in a look-up table for on-line implementations. Detailed discussions on implementing the OPWM can be found in [23].

B. STATCOM Modeling for Harmonic Analysis

Frequency-dependent approximate representation of AC/DC converters already used for small levels of distortion using transfer function approach [15]. Approximations can be avoided by modeling the converter in the time-domain in the expense of the solution speed. It is noticeable, however, that advanced probabilistic

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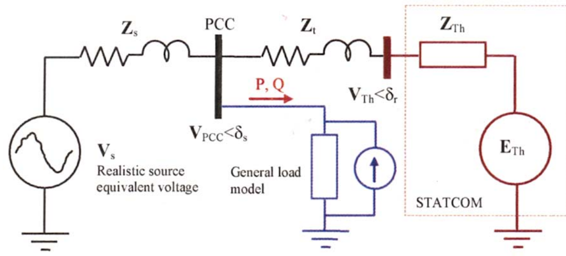

Figure 3: Suggested three-phase modeling for a STATCOM in the harmonic-domain that is connected to a distribution substation.

analysis and Monte Carlo simulation needs fast and flexible procedures.

Hence, building on the work of [8], this paper proposes a Thévenin equivalent model for STATCOM in the harmonic-domain. This model, shown in Fig. 3, provides a fast procedure with sufficient accuracy in addition to the flexibility in programming capabilities. Considering a pulse train for the converter's switches, assume S_a, S_b and S_c are the three-phase switching functions. Three combinations $S_{ab}= S_a\text{-}S_b$, $S_{bc}= S_b\text{-}S_c$ and $S_{ca}= S_c\text{-}S_a$ are defined based on the three switching functions of the PWM converter (e.g. see Fig. 2). Then, the following equations can be obtained in harmonic-domain:

$$\mathbf{V}=\begin{bmatrix}\mathbf{V}_{an}\\ \mathbf{V}_{bn}\\ \mathbf{V}_{cn}\end{bmatrix}=\frac{\mathbf{V}_{dc}}{3}\begin{bmatrix}1 & 0 & -1\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}\mathbf{S}_{ab}\\ \mathbf{S}_{bc}\\ \mathbf{S}_{ca}\end{bmatrix} \tag{1}$$

Where the converter outputs $\mathbf{V}_{an}$, $\mathbf{V}_{bn}$, $\mathbf{V}_{cn}$ are the phase voltages, all represented as complex harmonic vectors, and $\mathbf{S}_{ab}$, $\mathbf{S}_{bc}$, $\mathbf{S}_{ca}$ are convolution matrices constituted of the harmonic coefficients of the S_{ab}, S_{bc} and S_{ca} at a certain harmonic. Also, $\mathbf{V}_{dc}$ is the harmonic-domain vector of the DC-side voltage. Considering Fig. 1, the DC current can be introduced as:

$$\mathbf{I}_{dc}=\mathbf{S}_a\mathbf{I}_a+\mathbf{S}_b\mathbf{I}_b+\mathbf{S}_c\mathbf{I}_c \tag{2}$$

Where converter currents $\mathbf{I}_a$, $\mathbf{I}_b$ and $\mathbf{I}_c$ are the complex harmonic vectors of the line currents, and $\mathbf{I}_{dc}$ is the harmonic-domain vector of the DC-side voltage. Equation (2) could also be presented in matrix form of

$$\mathbf{I}_{dc}=\begin{bmatrix}\mathbf{I}_a & \mathbf{I}_b & \mathbf{I}_c\end{bmatrix}\begin{bmatrix}1 & 0 & -1\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}\mathbf{S}_{ab}\\ \mathbf{S}_{bc}\\ \mathbf{S}_{ca}\end{bmatrix} \tag{3}$$

An integral formula relates both DC quantities in the time-domain (see Fig. 1) as follows:

$$v_{dc}(t)=\frac{1}{C}\int_0^t i_{dc}(t)dt+v_{dc}(0^+). \tag{4}$$

Assuming the DC-side harmonics are originated from harmonic content of the $i_{dc}(t)$, $\mathbf{E}_{dc}$ is the pure DC voltage

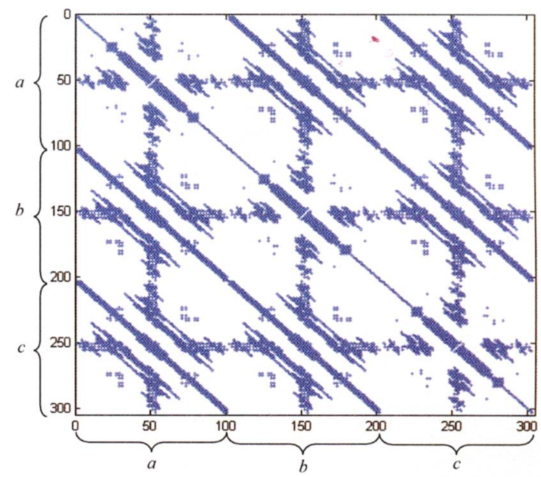

Figure 4: STATCOM equivalent impedance matrix structure implying the phases a, b and c components cross coupling for harmonics bigger than 0.02 P.U. up to 50th.

and $\mathbf{Z}_{cap}$ is the DC-link impedance. Then, the following relationship can be established based on (4):

$$\mathbf{V}_{dc}=\mathbf{Z}_{cap}\mathbf{I}_{dc}+\mathbf{E}_{dc} \tag{5}$$

Substituting $\mathbf{I}_{dc}$ and $\mathbf{V}_{dc}$ from (3) and (5) in (1), both the Thévenin equivalent voltage ($\boldsymbol{E}_{th}$) and impedance ($\boldsymbol{Z}_{th}$) of the converter can be represented as below (shown in Fig. 3 in which $\boldsymbol{V}_{th}$ is the resultant equivalent converter output voltage):

$$\mathbf{V}=\mathbf{Z}_{th}\mathbf{I}+\mathbf{E}_{th} \tag{6}$$

$$\mathbf{Z}_{th}=\mathbf{S}^t\mathbf{Z}_{cap}\mathbf{S}\,,\ \mathbf{E}_{th}=\begin{bmatrix}\mathbf{E}_{tha} & \mathbf{E}_{thb} & \mathbf{E}_{thc}\end{bmatrix}^t=\mathbf{E}_{dc}\mathbf{S}$$

$$\mathbf{S}=\frac{1}{3}\begin{bmatrix}\mathbf{S}_{ab}-\mathbf{S}_{ca} & \mathbf{S}_{bc}-\mathbf{S}_{ab} & \mathbf{S}_{ca}-\mathbf{S}_{bc}\end{bmatrix}^t \tag{6a}$$

This is the converter model of STATCOM that is connected through an equivalent inductance to the PCC. The phase components cross coupling structure of the $\mathbf{Z}_{th}$ has been plotted in Fig. 4 for harmonic interactions up to the 50th harmonic that are bigger than 0.02 P.U. It is noticeable that the matrix excludes control system transfer elements because of steady state analysis (all sub-matrices are diagonal).

III. Harmonic Behavior of STATCOM

The harmonic performance of STATCOM can now be established using the developed model in (6) under ideal operating conditions. Also, the OPWM scheme assumes ideal switching transitions (having no turn-on or turn-off delays). Further, assume the balanced system parameters are $|V_s|$ = 1 P.U., Z_s = 0.0043 + j0.1561 P.U., Z_t = 0.012 + j0.5612 P.U. at 50 Hz. Simulating the presented model of (6) results in the voltage and current waveforms shown in Fig. 5 (Harmonics up to the 100th are considered).

In fact, the fundamental and harmonic contents are obtained directly from the equivalent circuit of Fig. 1. Then, the time-domain waveforms in Fig. 5 are analyzed through an inverse Fourier transform. This is different

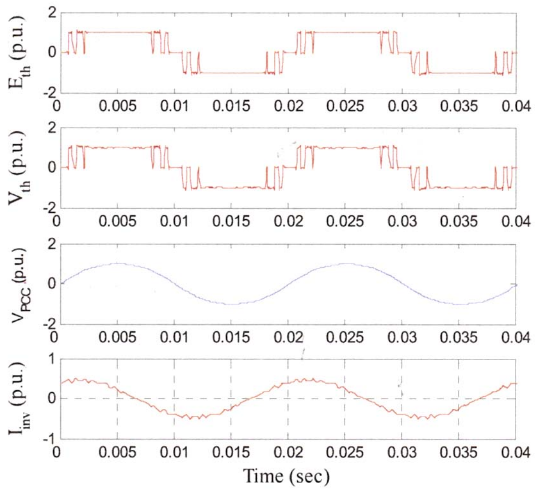

Figure 5: Simulations of the model (6) under balanced condition when STATCOM supplies 1 P.U. reactive power to the PCC; pictures from the top one are the Thevenin voltages ($\boldsymbol{E_{th}}$ and $\boldsymbol{V_{th}}$), the PCC voltage at phase a (V_{PCC}), and the converter current at phase a (I_{inv}) all in P.U.

from a normal procedure when a time-domain simulation is taken place. The OPWM is optimized such that selective harmonics (5th, 7th, 11th, 13th and 17th) are eliminated. Figure 6 shows the harmonic spectra of the converter phase voltage and current. Note that the STATCOM current spectra exclude the third harmonics.

Recently, a detailed study on the harmonic performance of STATCOM is introduced in [2] using an OPWM switching pattern under realistic operating conditions. While that method can be examined with the proposed theoretical modeling in harmonic-domain (see (6)), but a more realistic case is investigated here in which an additional system examination is performed with regard to the degrees of unbalance and distortion. This could be useful when commissioning study of a distribution system focuses on the levels of uncharacteristic harmonic produced by a STATCOM at a distribution substation.

IV. Modeling and Simulation of System Unbalance

Operation of STATCOM under three-phase unbalanced voltages affects noticeably the penetration of uncharacteristic harmonics which are not usually targeted by the engaged modulation technique [2]. The proper prediction of the level of voltage unbalance could be useful in applying further measures to the OPWM, influencing the harmonic performance of STATCOM. Here an improved and enhanced version of simulation approach in [21] is utilized for statistical prediction of voltage unbalance.

A. Case study: System Description and Measuements

A data logger is installed at the distribution substation of Mavad that is located in north-east of Tehran. The measured data is gathered during a week in September 2002. Figure 7 shows single-line diagram of the local distribution system in which the transformer T3 corresponds to the substation of Mavad (20 kV/400 V, 1 MVA). Figure 8 demonstrates the recorded active and reactive powers of the three phases at this substation for one week. Also, Fig. 9 introduces phase voltages of the

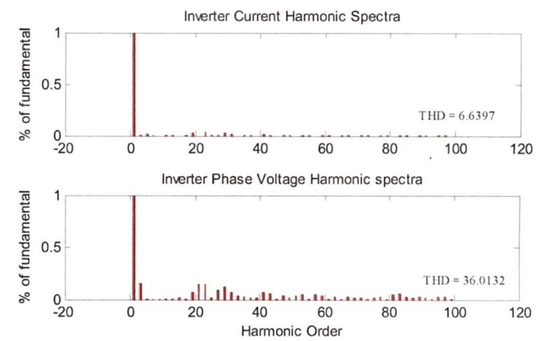

Figure 6: Harmonic spectra of $\boldsymbol{V_{th}}$ and I_{inv}.

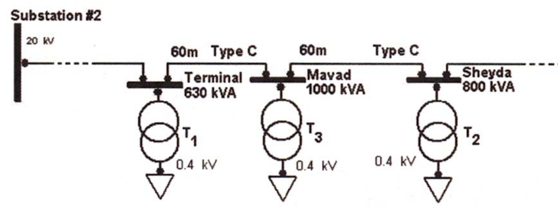

Figure 7: Single-line diagram of the distribution network under study.

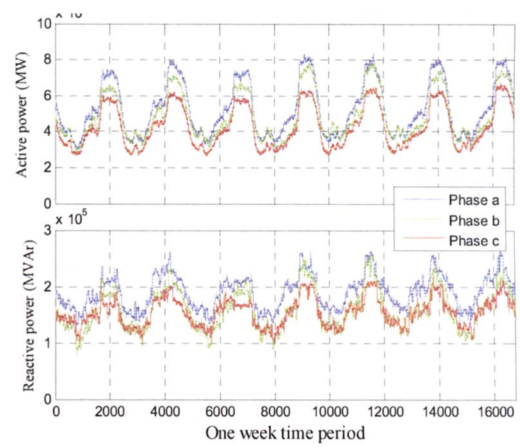

Figure 8: Recorded three-phase active and reactive powers by data logger that is installed at 400 V substation of Mavad.

substation transformer (at 400 V-end), which are obtained using a three-phase load flow program in MATLAB.

B. Voltage Unbalance Simulation

The following procedure is suggested to be used here for prediction and simulating voltage unbalance:

1) Active and reactive powers are split into two parts: *probabilistic* and *deterministic* components. This is performed using the *wavelet* transform. Also, the analysis is managed separately for two categories of *working days* and *week-end* like those of the load forecasting algorithms.

2) The *approximation* parts (obtained from the wavelet transform) are then directly predicted as the *average daily power* curve.

3) The *detail* parts of the wavelet transform are modeled using a *Gaussian linking* function which

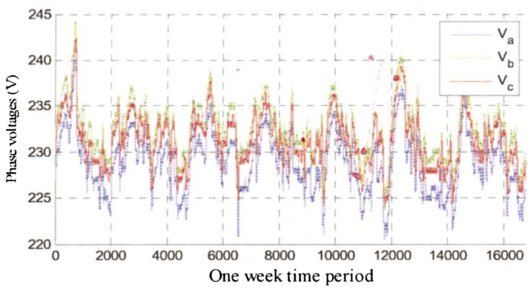

Figure 9: Measured phase voltages of the transformer T3 (Mavad) during a week.

generates six correlated random variables corresponding to *active and reactive powers* of the three phases.

4) Time-domain active and reactive powers (modeled during the previous steps) are reconstructed by applying the inverse wavelet transform.

5) The reconstructed powers are then used to predict the *three-phase voltages* through a three-phase load flow program using the *Monte Carlo* simulation. The IEC voltage unbalance factor (VUF) is calculated from the obtained voltages in step 4.

This procedure provides a more realistic insight into the VUF at the PCC. It is also compatible with the Monte Carlo simulation program and statistical evaluations. Wavelet transform presents suitable filtering characteristic. This enables the procedure to introduce the power as a deterministic component (approximation parts) with a quite stable mean and standard variation as shown in Fig. 10 (a). Also, the procedure introduces a nearly Gaussian distributed probabilistic component (detail parts) as shown in Figs. 10 (b)-(c). Assuming the Daubechies (db5) mother-wavelet, up to the fourth filtering level is used to determine approximations and details. It is noticeable that the probabilistic component in Fig. 10 (b) can be modeled using a Gaussian linking function that describes dependencies among variables, providing a way to correlate multivariate data [24, 25]. In a practical distribution system, active and reactive powers of the three phases show moderate correlation characteristics, which can be modeled using the Gaussian function in large networks.

The suggested procedure is applied to the studied distribution substation (Mavad). Simulations are shown in Fig. 11 over a week. The voltage unbalance percent (VUF%) of the substation of Mavad is calculated at the low-voltage side (400 V) of the transformer T3 using MATLAB. Simulations are depicted in Fig. 11(a) for a one week period, varying within [0.38%, 1.58%]. To validate the suggested algorithm, the probability distribution functions are obtained from both the proposed unbalance algorithm and field measurements. Both PDFs are shown in Fig. 11(b). Comparing the exact collected data (dotted line) with those of the proposed method (solid line), it can be seen that the suggested algorithm provides an accurate simulation of unbalance variation at the PCC.

V. STATCOM HARMONIC PERFORMANCE UNDER UNBALANCE OPERATION

A realistic voltage unbalance case is studied using the foregoing suggestion at a distribution substation. It is also shown that the distribution system may operate under

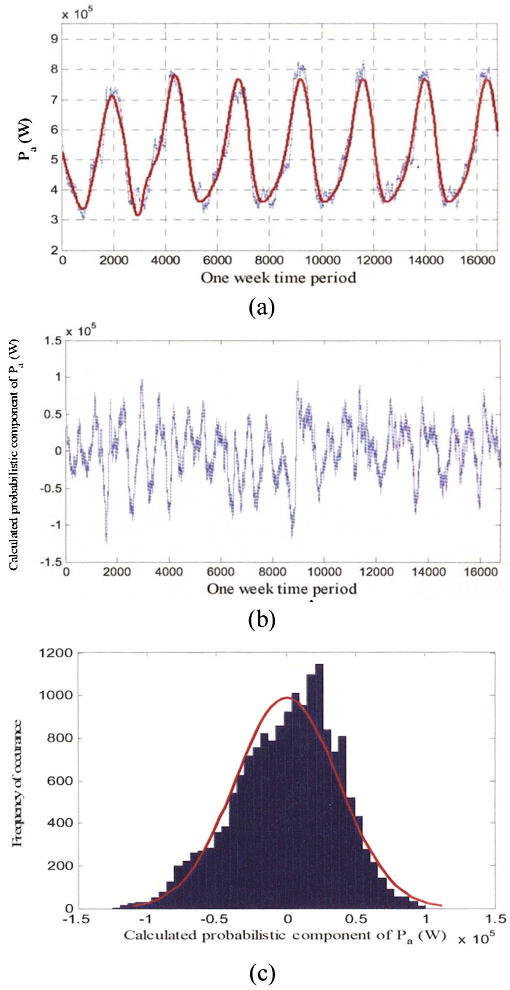

Figure 10: Application of the wavelet transform to active power, (a) real (blue trace) and approximated (red trace) logged data for one week, (b) the difference between the approximated and the exact data, and (c) the distribution of active power vs. Gaussian distribution function.

certain voltage unbalance percent (see Fig. 11(a)). It can now be studied the effects of voltage unbalance on the harmonic performance of the OPWM-STATCOM. The proposed model for STATCOM (see (6) along with Fig. 3) is linked with the Monte Carlo simulation of the calculated unbalance percent (VUF%) in Fig. 11(b).

Assume the capacitance of the DC-link capacitor of STATCOM is 1 mF. Then, the resultant THD% is calculated for all data as shown in Fig. 12. Background harmonics are represented by voltage harmonic source obtained from field measurements. It can be seen from Fig. 12 that a realistic voltage unbalance would not dramatically modifies the uncharacteristic THD of STATCOM AC current (THDS) except for situations that STATCOM supplies or absorbs relatively small amounts of reactive power. Figure 12(a) provides a qualitative evaluation of the proposed method. Also, to validate quantitatively the suggested method, Figs. 12 (b)-(e) compare the probability distributions of the THDS for realistic VUF% when four different reactive power are supplied by STATCOM. It can be seen from Figs. 12 (b)-(e) that reactive powers of -0.8 P.U and 0.9 P.U. along with realistic VUF% give a relatively small range of the THDS variations around 1% and 0.96%, respectively.

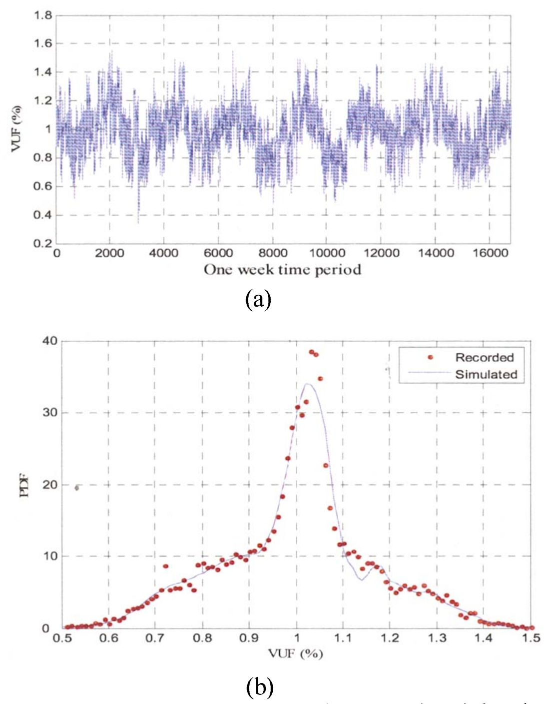

Figure 11: (a) The VUF% variation over the one week period at the low-voltage side of T3 obtained using the proposed simulation method, and (b) comparing the PDFs of VUF% obtained from field measurements (dotted line) and the proposed algorithm (solid line).

However, for reactive powers equal to 0.04 and -0.1 P.U. along with the same realistic VUF% result in a considerable range of the THDS variations around 25% and 15%, respectively. It should also be noted that the THDS is affected by changing the DC-link capacitance. Also, the power system equivalent impedance influences the THDS under variation of the voltage unbalance percent. This analysis can be easily extended to include other realistic conditions. It is theoretically possible to modify the OPWM procedure to remove uncharacteristic harmonics under unbalanced conditions. However, the analysis become more complex, it is not purely deterministic and could be difficult to implement online.

VI. Conclusion

This paper suggests a comprehensive analysis related to the penetration level of harmonics by STATCOM into power systems. Hence, starting with introduction of a harmonic-domain model for STATCOM, an equivalent Thevenin circuit is established. This simplifies simulation of the developed model, while it links certain harmonics to their complex values in a three-dimensional representation. A case study is arranged in which a data logger gathers required data from a 20 kV/400 V distribution substation located in Tehran for a one week period. Analyzing the exact measured voltage unbalance of this substation, a probabilistic method is suggested to predict the unbalance percent. Then, the Monte Carlo method is linked with the STATCOM model. The resultant model is used to evaluate harmonic performance of STATCOM under the suggested unbalanced prediction algorithm. Various simulations are performed to verify the suggested model along with the proposed unbalance predictive algorithm.

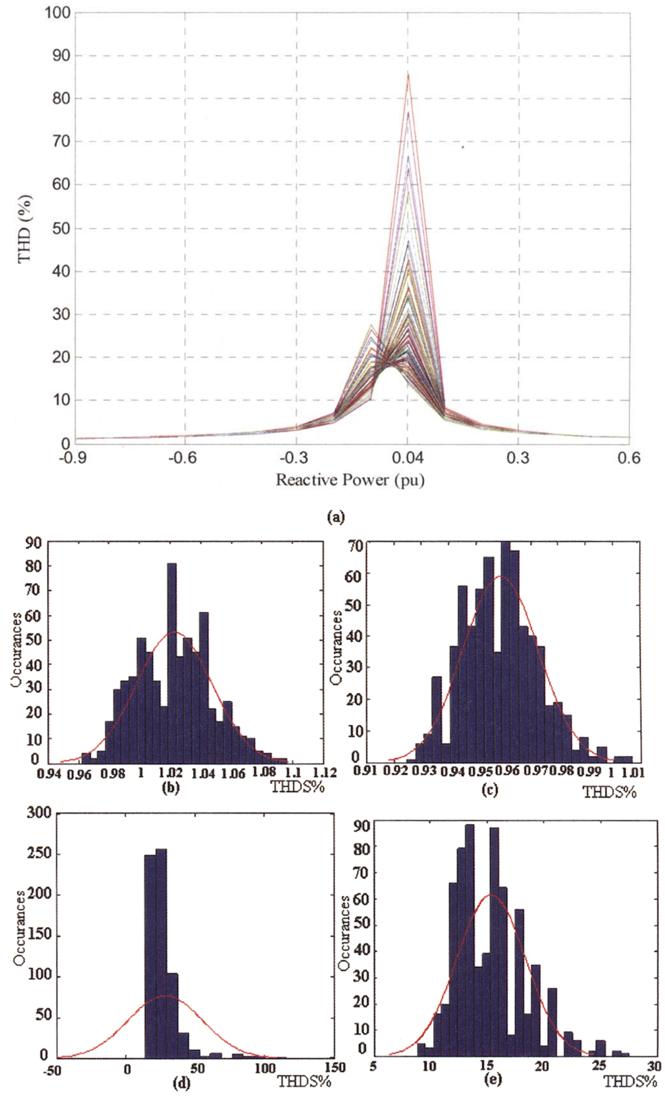

Figure 12: (a) Penetrated THDS due to the 5th, 7th, 11th, 13th, 17th and 19th harmonics under various VUF% and reactive power obtained from Monte Carlo Simulation. (b)-(e) distributions of the THDS corresponding to the reactive power loadings of -0.8 P.U., 0.9 P.U., 0.04 P.U. and -0.1 P.U., respectively.

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