Frequency Estimation of Distorted Signals in Power Systems Using Particle Extended Kalman Filter

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***Abstract*— In this paper, frequency of distorted signal in power system has been estimated with particle extended Kalman filter. Base of particle algorithm, extended Kalman filter and particle extended Kalman filter are mentioned. For selecting state variables, a nonlinear time-variant sinusoidal signal is developed then a particle extended Kalman filter is applied to detect the frequency variations. Several tests are performed to show the performance of PEKF algorithm. Comparison of PEKF with EKF reveals the PEKF preference. Also, these tests prove the fast speed, good accuracy and robustness against noise. These advantages illustrate that PEKF is more suitable for power system applications.**

***Keywords*—frequency, state space model, tracking, particle extended Kalman filter (PEKF)**

I. INTRODUCTION

The using of frequency in power system protection and power quality has caused that in the resent years was studied more about it. Frequency estimation in power system has face with several unwanted distortion contamination, such as random noise, dc component, and higher order harmonics, and also from different forms of frequency variations. Therefore, a frequency estimation method should have ability of tracing in the noisy and distributed environment. Moreover above condition, accuracy and speed are factors for comparison of estimation methods.

Most of the researches on power system applications during in last two decades have put more emphasis on frequency estimation. Several methods are available on the frequency measurement for power. The simplest method is the measuring times between two zero crossing that indicate to half period of signal. Noise and distribution cause Zero-crossing in power system does not have desirable work [1]-[2]. Moreover, Several algorithms have been developed in the past few decades based on discrete Fourier transform (DFT) [1],[3] least-square error technique [4],[5], adaptive notch filter (ANF) [6],[7], orthogonal components filtered algorithm [8],[9] and phase lock loop (PLL) [10]-[12] as frequency estimation methods. A comparative study among these different trackers has been outlined in [13], [14].

Kalman filter is a well-known observer for tracking the state variables of system. Literature [15]-[17] used extended Kalman filter (EKF) to improve the tracking performance.

In this paper a novel method has been proposed for tracking frequency. Particle extended Kalman filter (PEKF) is used to improve frequency estimation at the present of noise and severe variation. Also, as heavier changes, robustness of the proposed method is studied in this paper. Hence, robustness against distribution, accuracy and speed of the proposed method could be scale to prove good performance of the method. In order to illustrate performance of the proposed method, first, the sinusoidal signal is developed on space state. Then state variables were selected for applying the particle extended Kalman filter.

II. PARTICLE EXTENDED KALMAN FILTER

Particle filters perform Monte Carlo based estimation based on point mass approximations of probability densities. Each state probability space is represented by a large number of weighted points (known as particles) which, taken together, approximate the uncertainty of the current system state. One approach that has been proposed for improving particle filtering is to combine it with another filter such **as** the EKF. In this approach, each particle is updated at the measurement time using the EKF, and then resampling is performed using the measurement. This is like running a bank of ***N*** Kalman filters (one for each particle) and then adding a resampling step after each measurement. For clarifying the particle extended filter, the system and measurement equations are given **as** follows:

$$x_k = f_{k-1}(x_{k-1}, w_{k-1}) \tag{1}$$

$$z_k = h_k(x_k, v_k) \tag{2}$$

Where w_k and v_k are independent white process and observation noises with probability distribution matrixes $N(0,Q)$ and $N(0,R)$ respectively. Also x_k and z_k are the hidden state variables and the measurement respectively. Both

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$f(.)$ and $h(.)$ could be non-linear functions that are assumed known.

A. Particle filter

The particle filter was invented to numerically implement the Bayesian estimator. The particle filter can be summarized in the algorithm 1:

Algorithm 1: Particle Filter

1. Initialization:
   - For $i = 1, 2, \ldots, N$
     Assuming that the probability distribution function (PDF) of the initial state $P(x_0)$ is known, N initial particles from the prior $P(x_0)$ are generated randomly. These particles are denoted $x_{0,i}^-$.
   - End For
2. For $k = 1, 2, \ldots$
   - For $i = 1:N$
     - Using the known process equation and the known pdf of the process noise, a priori particles $x_{k,i}^-$ are achieved as follows:
     - Assign the relative likelihood of each particle as follows:

$$q_i \sim \frac{1}{\sqrt{2\pi R}} \exp\left(\frac{-\left[z^* - h(x_{k,i}^-)\right]^T R^{-1} \left[z^* - h(x_{k,i}^-)\right]}{2} \right) \quad (4)$$

   - End For
   - For $i = 1:N$
     - Normalizing step: Normalize the likelihoods that are achieved in the previous step as follows:
   - End For

$$q_i = \frac{q_i}{\sum_{i=1}^{N} q_i} \quad (5)$$

   - Resampling step: a posteriori particles $x_{k,i}^+$ have been generated on the basis of the relative likelihoods q_i .

B. Extended kalman filter

Consider a nonlinear system with state and measurement equations as (1), (2). Algorithm 2 illustrates The Extended Kalman filter.

Algorithm 2: Extended Kalman Filter

Step1: Predict the state with initials:

$$\hat{x}_k^- = f(\hat{x}_{k-1}, 0)$$

Step 2: Compute the error covariance:

$$P_k^- = F_k P_{k-1} F_k^T + W_k Q_{k-1} W_k^T$$

Step 3: Compute the Kalman gain:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)$$

Step 4: Update the state estimate:

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0))$$

Step 5: Update the error covariance

$$P = (I - K_k H_k) P_k^-$$

Step 6: Return to step 1 with updated measurements

Where z is the measurement vector, $\hat{x}_k^-$ and P_k^- are the approximate state and covariance, F and H are the Jacobian matrices of partial derivatives of f and h with respective to $\hat{x}_k^-$. At each frame, the filter predicts the current state of the system and correct this estimated state using measurement of the system. A Kalman gain (K_k) is computed to find the optimum feedback gain that minimizes the error covariance between the priory and the posteriori estimation. The updated estimate will then be used to predict the state for the next frame. Manner achievement of F, H, W and V is shown as below:

$$F = \frac{\delta f}{\delta x}(\hat{x}_{k-1}, u_k, 0) \quad (6)$$

$$W = \frac{\delta f}{\delta w}(\hat{x}_{k-1}, u_k, 0) \quad (7)$$

$$H = \frac{\delta h}{\delta x}(\tilde{x}_k, 0) \quad (8)$$

$$V = \frac{\delta h}{\delta v}(\tilde{x}_k, 0) \quad (9)$$

C. *Particle extended kalman filter*

In a system that is nonlinear, the extended Kalman filter can be used for state estimation, but the particle filter may give better results at the price of additional computational effort. Combining EKF with particle filter results a method that involves better performance. Particle extended Kalman algorithm is summarized in algorithm 3:

Algorithm 3: Particle Extended Kalman Filter

1. Initialization: $k = 0$
   - For $i = 1, 2, \dots, N$
     - Randomly generate particles $x_{0,i}^{+}$ around the prior pdf $P(x_0)$
   - End For
2. For $k = 1,2, \dots$ do the following.
   - For $i = 1{:}N$
     - Compute the Jacobian matrices $F_{k-1,i}$ and $Q_{k-1,i}$ of the process model to obtain priori particles and covariance $P_{k,i}^{-}$ using the known process equation:

$$x_{k,i}^{-} = f_{k-1}(x_{k-1,i}^{+}, w_{k-1}^{i}) \tag{10}$$

$$F_{k-1,i} = \left.\frac{\partial f}{\partial x}\right|_{x=x_{k-1,i}^{+}} \tag{11}$$

$$P_{k,i}^{-} = F_{k-1,i} P_{k-1,i}^{+} F_{k-1,i}^{T} + Q_{k-1} \tag{12}$$

     - Compute the Jacobian matrix $H_{k,i}$ of the measurement model and then update the a *priori* particles and covariance to obtain a *posteriori* particles and covariance:

$$H_{k,i} = \left.\frac{\partial h}{\partial x}\right|_{x=x_{k,i}^{-}} \tag{13}$$

$$K_{k,i} = P_{k,i}^{-} H_{k,i}^{T} (H_{k,i} P_{k,i}^{-} H_{k,i}^{T} + R_k)^{-1} \tag{14}$$

$$x_{k,i}^{+} = x_{k,i}^{-} + K_{k,i}\left[z_k - h(x_{k,i}^{-}) \right] \tag{15}$$

$$P_{k,i}^{+} = (I - K_{k,i} H_{k,i}) P_{k,i}^{-} \tag{16}$$

     - Compute the relative likelihood of each particle $x_{k,i}^{+}$ by (4).
   - End For
   - For $i = 1{:}N$
     - Scale the likelihoods is obtained **as** shown in (5).
   - End For
   - Resampling step: on the basis of the relative likelihoods q_i refine $x_{k,i}^{+}$ and $P_{k,i}^{+}$.

III. SIGNAL MODEL

In power system, an observed signal is contaminated with harmonics and noise. Therefore the signal is modeled by:

$$V(t) = \sum_{n=1}^{N} A_n \sin(n\omega t + \phi_n) + \varepsilon_k \tag{17}$$

Where,

A_n	amplitude of the $n'th$ harmonic
ϕ_n	phase of the $n'th$ harmonic
n	harmonic order
N	highest harmonic order
$t = kT_s$	T_s Sampling time and k sampling instant
ω	radian frequency
ε_k	additive noise ($\sim N(0, R)$)

Since the amplitude of harmonic components are low compared to fundamental, the signal with no harmonics is considered. Hence the signal is simplified as below:

$$V(t) = A \sin(\omega k T_s + \phi) + \varepsilon \tag{18}$$

Since in this study main objective is tracking of the fundamental frequency, for applying PEKF in estimation problem, states variables are formed as below:

$$x_1(k) = \omega k T_s \tag{19}$$

$$x_2(k) = A \tag{20}$$

$$x_3(k) = \sin(\omega k T_s + \phi) \tag{21}$$

$$x_4(k) = \cos(\omega k T_s + \phi) \tag{22}$$

By consideration state variables above, $f(x_k)$ and $h(x_{k+1})$ in (1), (2) are written as follows:

$$f(x_k) = \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_4(k)\sin(x_1(k)) + x_3(k)\cos(x_1(k)) \\ x_4(k)\cos(x_1(k)) - x_3(k)\sin(x_1(k)) \end{pmatrix} \tag{23}$$

$$h(x_k) = x_2(k) x_3(k) \tag{24}$$

Jacobian matrices for linearizing (23), (24) are obtained from (6) and (8) as follows:

$$F_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ M_1 & 0 & \cos(x_1(k)) & \sin(x_1(k)) \\ M_2 & 0 & -\sin(x_1(k)) & \cos(x_1(k)) \end{pmatrix} \quad (25)$$

$$M_1 = x_4(k)\cos(x_1(k)) - x_3(k)\sin(x_1(k))$$

$$M_2 = -x_4(k)\sin(x_1(k)) - x_3(k)\cos(x_1(k))$$

Measurement equation is shown as follows:

$$H_k = [0 \;\; x_3(k) \;\; 0 \;\; x_2(k)] \quad (26)$$

By using (18)-(26), PEKF is applied for tracking frequency as it is shown simulation section.

IV. Simulation Results

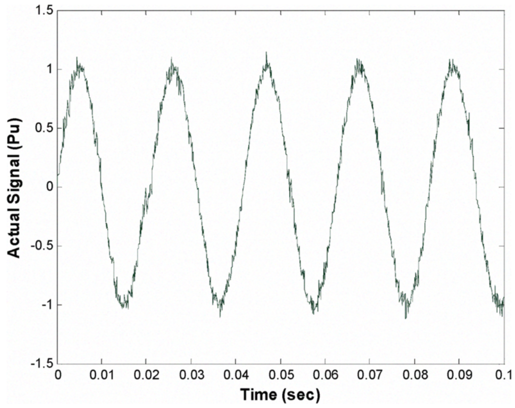
In simulation, different noise level and step change in the fundamental frequency is considered to test the performance of particle extended Kalman filter for tracking frequency. For all tests, fundamental frequency and amplitude are considered 50 Hz and 1 Pu, respectively. Hence state variables x_1 and x_2 are initialized 50 Hz and 1 Pu. Also, step change in frequency for all tests is applying at 0.2 second. For PEKF algorithm, 100 particles in each sample time are considered.

a) Test Signal 1: 2 Hz step-down changes in frequency with low noise: The frequency of the signal is suddenly decreased from 50 Hz to 48 Hz. Signal-to-noise ratio (SNR) for this case is 50 dB. Fig. 1 illustrates actual signal and the tracking of frequency with both methods EKF and PEKF. As it is reveals that PEKF has estimated frequency fast and accurately than EKF. Very fast estimation of frequency with PEKF is considerable in Fig (1)-(b). Hence, this test confirms the fast speed of PEKF method.

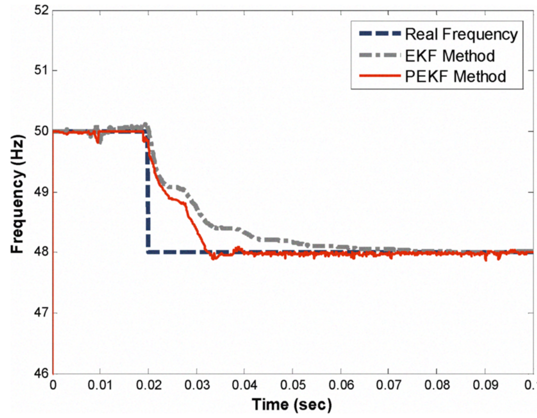
b) Test Signal 2: 1 Hz step-up change in frequency with high noise: The frequency of the signal change from 50 Hz to 51 Hz with 10 dB measurement noise. The contaminated signal with high level noise and estimated frequency with PEKF and EKF have been shown in Fig. 2. In this test, PEKF has been better performance than EKF, too. However, severe noise effect on both performance. Therefore, ability of tracking in present of strong noise is confirmed by this test.

c) Test Signal 3: Severe step-down (8Hz) in frequency with high noise: At 0.2 sec, in present of high level noise (10 dB SNR), frequency of actual signal varied from 50 Hz to 42 Hz. Results of this test are shown Fig. 3. It is shown that PEKF succeeds to estimate new frequency. However EKF method has lacking ability to track the frequency. This test reveals that PEKF robust against severe change.

These tests confirm the high robustness of particle extended Kalman filter against noise and variation of state. Tracking time and steady state error are mentioned in Table I for both methods PEKF and EKF.



(a)



(b)

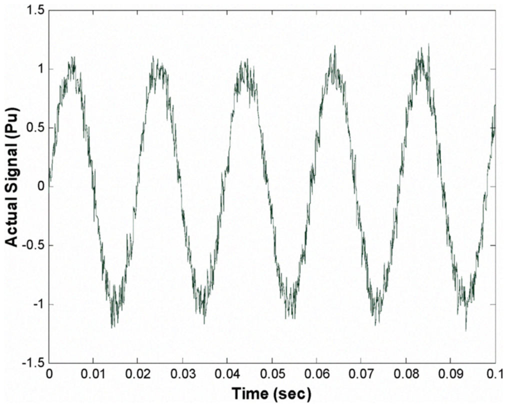
Fig. 1. a) Actual Signal for Test 1
b) Estimated Frequency with PEKF and EKF and Real Frequency

V. Conclusion

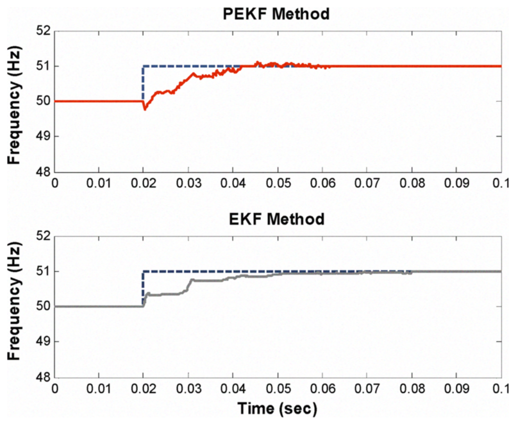
In this paper is used particle extended Kalman filter for tracking the frequency of power system. . State space model of the nonlinear time-varying system is developed using exact analytical equations. For showing the performance of particle extended Kalman filter, several tests as different level noise and variation of the frequency have been applied, then compared with extended Kalman filter. Simulation results show that PEKF has better accuracy and speed than EKF. Also, robustness against noise and severe variation of state are the other special features of PEKF method.

TABLE I
Tracking time and steady state error for three test signals

Test Signals	Tracking time		Steady state error (Hz)	
	PEKF	EKF	PEKF	EKF
Test Signal 1	~0.5cycle	~ 3cycle	.00001	0.0003
Test Signal 2	~ 1cycle	~ 2cycle	0	0
Test Signal 3	~ 1.5cycle	Not Tracking	0.001	∞

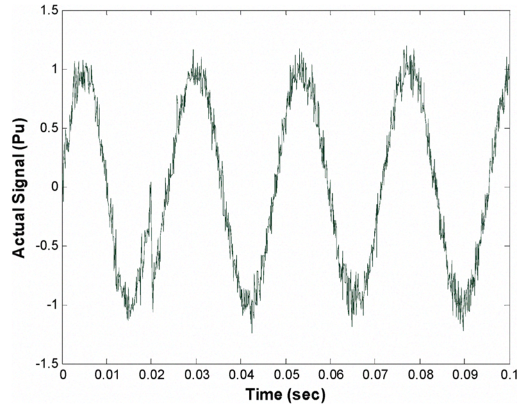


(a)

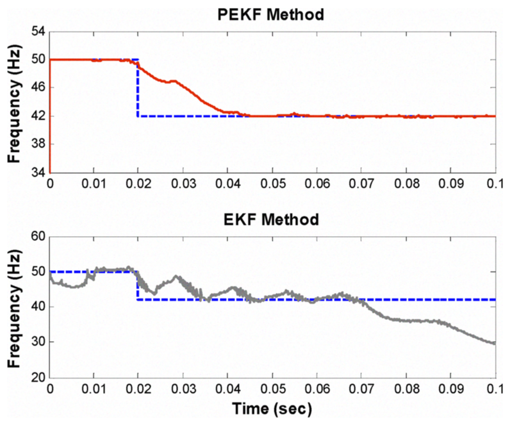


(b)

Fig. 2. a) Actual Signal for Test 2
b) Estimated Frequency with PEKF and EKF and Real Frequency



(a)



(b)

Fig. 3. a) Actual Signal for Test 3
b) Estimated Frequency with PEKF and EKF and Real Frequency

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