SSSC circuit model for three-wire systems coupled with Delta-connected transformer

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Abstract—**This paper presents a new circuit model of SSSC based on state equations in three-wired systems. SSSC is a series compensator of FACTS family. It injects an almost sinusoidal voltage with variable amplitude and is equivalent to an inductive or a capacitive reactance in series with the transmission line. The analysis of a power electronics system is complex, owing to its switching behavior. Since such a system has its special complexities, the need for simpler model is essential, though a more practical one is sometimes appropriate. With simple changes, this paper's model shows different states of SSSC, both transient and steady states, in the range of ideal to practical as confirmed by MATLAB simulation. This model is suitable for circuit simulation software such as MATLAB or PSPICE and is proper to control process and control methods. This model can be used as a starting point for further investigations on control methods in the future.**

Keywords: SSSC model, SSSC Circuit Model, state equations, SSSC transient.

I. INTRODUCTION

The rapid development of power electronics technology provides exciting opportunities to develop new power systems equipment for better utilization of existing systems. During the last decade a number of control devices under the term Flexible AC Transmission Systems (FACTS) technology have been proposed and implemented. The FACTS devices can be used for power flow control, loop flow control, load sharing among parallel corridors, voltage regulation, and enhancement of transient stability and mitigation of system oscillations. FACTS have become an essential and integral part of modern power systems. Modeling and digital simulation plays an important role in the analysis, design, testing and commissioning of such controllers.

Static Synchronous Series Compensator (SSSC) is a series compensator of FACTS family. It injects an almost sinusoidal voltage with variable amplitude. It is equivalent to an inductive or a capacitive reactance in series with the transmission line. The heart of SSSC is a VSI (voltage source inverter) that is supplied by a DC storage capacitor [1]. With no external DC link, the injected voltage has two parts: the main part is in quadrature with the line current and emulates an inductive or capacitive reactance in series with the transmission line, and a small part of the injected voltage is in phase with the line current to cover the losses of the inverter [2]. When the injected voltage is leading the line current, it will emulate a capacitive reactance in series with the line, causing the line current as well as power flow through the line to increase. When the injected voltage is lagging the line current, it will emulate an inductive reactance in series with the line, causing the line current as well as power flow through the line to decrease [3].

SSSC is superior to other FACTS equipment and the benefits of using SSSC are listed in [4]. Paper [5] and [6] have designed the power flow controller of SSSC based on ANN method and Fuzzy self-tuning PID individually, and the results show they all improve the self-adapting and the robustness, and accelerate the speed of power flow adjustment. Paper [7] has built SMIB system with SSSC and TCSC by using the SIMPOWERSYSTEM toolbox of MATLAB, and then the transient characteristic is simulated. The effect of damping power oscillation has been compared between different switch modes of SSSC. Besides, the switch strategy of internal fault and external fault is summarized.

These papers focus on the modeling and controlling SSSC and propose many models suitable for power flow and other applications. Almost all of them have used the phasor equations for calculations and extracting the model which beside other applications is applied for the control in SSSC controllers. Phasor equations based on steady state conditions explain system behavior at steady state but can not show the system behavior at transient state. To control system, its precise behavior is needed in transient and steady states, and fault conditions. Therefore, the new model of SSSC is needed to show the transient behavior and to have enough details about SSSC. On the other hand, the model should have the ability to be simplified for other applications such as power flow or steady state conditions and can vary between complicated and simplified models with nominal changes in it. The time equations expound the transient and steady states and fault conditions in behavior of system. If we can get the SSSC circuit model from time equations, this model can expound behavior of SSSC at transient and steady states and other conditions.

In this paper, first, the state equations of SSSC, Thévenin Equivalent Circuit of network and the switching function are explained. Based on these equations, the SSSC model and its simple PI control are proposed. The next part demonstrates the

978-1-4244-4813-5/10/$25.00

MATLAB simulations' results. The SIMPOWERSYSTEM toolbox (Version V4.3 (R2006b)) of simulink is used for simulations.

II. SSSC STATE SPACE EQUATIONS AND THÉVENIN EQUIVALENT CIRCUIT OF NETWORK

Assume that the converter voltage is synthesized using a PWM or SVM control, and the control loop focuses on α. The open-loop equations are obtained in a standard form that can later be modified for closed-loop control:

$$\dot{x}(t) = f(x(t), s(t), u(t)) \tag{1}$$

Where x(t) is the state vector:

$$x(t) = [i_A(t), i_B(t), V_{dc}(t)]^T \tag{2}$$

u(t) is the input vector:

$$u(t) = \left[V_{sa}(t) - V_{ra}(t), V_{sb}(t) - V_{rb}(t), V_{sc}(t) - V_{rc}(t)\right]^T \tag{3}$$

And s(t) is the vector of switching function.

$$s(t) = \left[s_a(t), s_b(t), s_c(t)\right]^T \tag{4}$$

The state space model of (1) includes a discrete switching function along with the main voltage-supplied source. Consider the SSSC shown in Fig 1. There are two topological modes for every leg. The state equations for the two modes can be obtained separately and then, introducing the switching function $s(t) \in \{-1,1\}$, define the output voltages of converter and i_{dc} with switching function as below:

$$\begin{bmatrix} V'_{ab}(t) \\ V'_{bc}(t) \\ V'_{ca}(t) \end{bmatrix} = \frac{V_{dc}(t)}{2} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} s_a(t) \\ s_b(t) \\ s_c(t) \end{bmatrix} \tag{5}$$

$$i_{dc} = C\frac{dV_{dc}}{dt} = \left(\frac{2s_a - s_b - s_c}{2}\right) i_A + \left(\frac{s_a + s_b - 2s_c}{2}\right) i_B \tag{6}$$

Fig 1 shows the three-phase circuit of SSSC, network and connection between them. Voltage sources (Vs and Vr), resistances (Rl1 and Rl2) and inductors (Ll1 and Ll2) in figure 1 are the thévenin equivalent circuit of network and transmission line. The transformers connect together with delta connection and transformers' turn ratio assume to one. A summary of the equations to get the final equations for SSSC model is explained below:

$$i_A + i_B + i_C = 0 \tag{7}$$

$$V'_{ab} = -(R + L\frac{d}{dt})i_a + V'_{xa} + (R + L\frac{d}{dt})i_b \tag{8}$$

$$R_{l1} + R_{l2} = R_l \tag{9}$$

$$L_{l1} + L_{l2} = L_l \tag{10}$$

$$V_{xa} = V_{sa} - (R_l + L_l\frac{d}{dt})i_A - V_{ra} \tag{11}$$

$$\left|V'_{xa}\right| = \left|V_{xa}\right| \tag{12}$$

For other phases do the same as phase A. Thus, from (5) to (12) and other phases' equations we obtain:

$$((3R + R_l) + (3L + L_l)\frac{d}{dt})i_A = -V'_{ab} + (V_{sa} - V_{ra}) \tag{13}$$

$$((3R + R_l) + (3L + L_l)\frac{d}{dt})i_B = -V'_{bc} + (V_{sb} - V_{rb}) \tag{14}$$

Substituting $V'_{ab}, V'_{bc}, V'_{ca}$ from (5) to (13) and (14) gives:

$$((3R + R_l) + (3L + L_l)\frac{d}{dt})i_A = -(\frac{s_a - s_b}{2})V_{dc} + (V_{sa} - V_{ra}) \tag{15}$$

$$((3R + R_l) + (3L + L_l)\frac{d}{dt})i_B = -(\frac{s_b - s_c}{2})V_{dc} + (V_{sb} - V_{rb}) \tag{16}$$

From (6), (15) and (16) the state equations are concluded as below:

$$\begin{bmatrix} \dot{i}_A \\ \dot{i}_B \\ \dot{V}_{dc} \end{bmatrix} = \begin{bmatrix} -(\frac{3R + R_l}{3L + L_l}) & 0 & -(\frac{s_a - s_b}{2(3L + L_l)}) \\ 0 & -(\frac{3R + R_l}{3L + L_l}) & -(\frac{s_b - s_c}{2(3L + L_l)}) \\ \frac{2s_a - s_b - s_c}{2C} & \frac{s_a + s_b - 2s_c}{2C} & 0 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ V_{dc} \end{bmatrix} + \frac{1}{(3L + L_l)} \begin{bmatrix} +1 & 0 \\ 0 & +1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_{sa} - V_{ra} \\ V_{sb} - V_{rb} \end{bmatrix} \tag{17}$$

To get the circuit model of SSSC, first define two independent voltage sources:

$$f_1((u(t)) = [V_{sa}(t) - V_{ra}(t)] \tag{18}$$

$$f_2((u(t)) = [V_{sb}(t) - V_{rb}(t)] \tag{19}$$

Along with two voltage-controlled voltage sources (VCVS) are defined as follows:

$$g_1(s(t)) = (\frac{s_a(t) - s_b(t)}{2})V_{dc} \tag{20}$$

$$g_2(s(t)) = (\frac{s_b(t) - s_c(t)}{2})V_{dc} \tag{21}$$

Three state equations (15), (16) and (6) describe the inductor currents and capacitor voltage. The first two equations mean that the inductor currents depend on the voltage difference between the independent sources (18), (19) and dependent sources (20), (21). The third equation (6) shows that the current of capacitor C is composed of two current controlled current sources (CCCS) h1 and h2, each as a function of switching functions and line currents.

$$h_1(s(t),i_A(t)) = \left(\frac{2s_a(t)-s_b(t)-s_c(t)}{2}\right)i_A(t) \qquad (22)$$

$$h_2(s(t),i_B(t)) = \left(\frac{s_a(t)+s_b(t)-2s_c(t)}{2}\right)i_B(t) \qquad (23)$$

According to above equations we have:

$$((3R+R_l)+(3L+L_l)\frac{d}{dt})i_A + g_1(s(t)) = f_1(u(t)) \qquad (24)$$

$$((3R+R_l)+(3L+L_l)\frac{d}{dt})i_B + g_2(s(t)) = f_2(u(t)) \qquad (25)$$

$$C\frac{dV_{dc}}{dt} = h_1(s(t),i_A(t)) + h_2(s(t),i_B(t)) \qquad (26)$$

The resulting equivalent circuit model is shown in Fig 2, and is suitable for circuit simulators such as MATLAB or PSPICE. Note that the capacitor circuit should be connected to the inductor circuits using two very big impedances Z for PSpice simulation, which leaves negligible effect on the circuit behavior.

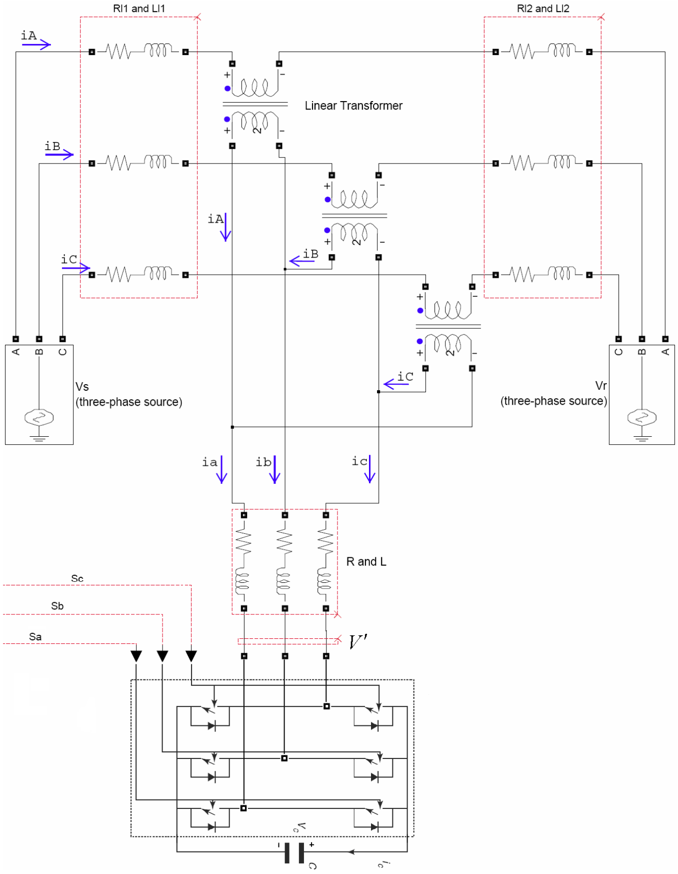


Figure 1. three-phase circuit of SSSC, network and connection between them

The heart of SSSC is VSI converter while the important part of VSI is its switches and switching method. If having an ideal converter, it can make ideal sinusoidal wave form without any harmonics. The farther from ideal converter, the range and amplitude of harmonics increase. In practical mode, the large range of harmonics alongside the main harmonic are made. These can be shown by switching function. If all harmonics made in practical situations are neglected, $s_a(t)$, $s_b(t)$ & $s_c(t)$ can be substituted with sinusoidal wave form. If real simulation is needed and all harmonics are taken into account, $s_a(t)$, $s_b(t)$ & $s_c(t)$ can be substituted with real wave forms of switches as show in Fig 3.

$s_a(t)$, $s_b(t)$ & $s_c(t)$ include the wide range of functions, which by exerting each of them a behavior of SSSC vis-à-vis a situation is attained. For example, if $s_a(t)$, $s_b(t)$ & $s_c(t)$ are substituted with the practical switching function (the function includes the pulses made by PWM, SVM (Fig 3) or other practical pulse makers), the model shows the practical model of SSSC. If $s_a(t)$, $s_b(t)$ & $s_c(t)$ are substituted with the average of practical switching functions (explained in [8]), the model shows the average behavior of SSSC and eliminates some harmonics depending on averaging period. If $s_a(t)$, $s_b(t)$ & $s_c(t)$ are substituted with the ideal switching functions, that is, an ideal converter exists to make the exact sinusoidal wave at output of converter, the model shows behavior of SSSC in ideal conditions by disregard of all harmonics and distortions. Or if $s_a(t)$, $s_b(t)$ & $s_c(t)$ are substituted with the fixed harmonic sinusoidal wave, the model shows the behavior of SSSC that is related to this harmonic, so the effects of one harmonic on SSSC can be elaborated. For example, if $s_a(t)$, $s_b(t)$ & $s_c(t)$ are substituted with main harmonic plus the third harmonic, the effects of third harmonic on transient state and power of SSSC can be analyzed.

Therefore, this model can show the SSSC behavior in various conditions. When simple model of SSSC is needed, simplified conditions and limitations are used, or else, when complete model of SSSC is needed, complicated conditions and limitations are used.

III. SSSC CONTROLLER

The Alfa method is applied for control of SSSC [8]. In this method with changes of the Alfa (Alfa is the angle of converter output voltage, in Fig 1 the angle of V' point) in a finite range, the active or inactive power (injected or received) in the desired range can be controlled. This proposed model can be used as a starting point for exact calculation of Alfa in the next paper.

IV. MATLAB SIMULATION RESULTS

This section shows simulation results for 1) SSSC practical model (Fig 4, 5 & 6), and 2) SSSC ideal model with the 40th harmonic (the amplitude of the 40th harmonic is equal to 30 percent of main harmonic made by converter) (Fig 7). For practical model the results of three different conditions are shown: 1) SSSC works in capacitive mode, 2) SSSC work in inductive mode, and 3) transiency of SSSC, changing from inductive mode to capacitive mode. And for each of them the inactive power of SSSC is shown that is injected into or received from network at transformers' terminals, the lines current and capacitor voltage(V_{dc}).

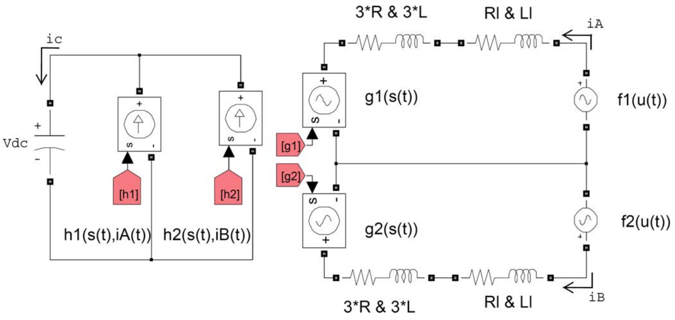


Figure 2. equivalent circuit model of SSSC, suitable for circuit simulator such as MATLAB

The parameters of network and SSSC used for simulations are: Vsa=100sin(wt), Vsb=100sin(wt-2π/3), Vsc=100sin(wt+2π/3), Vra=95sin(wt-5π/180), Vrb=95sin(wt-5π/180-2π/3), Vrc=95sin(wt-5π/180+2π/3), Rl1=Rl2=0.005Ω, Ll1=Ll2=0.06mH, R=0.005Ω, L=0.1mH, C=1.2mF, Vdc(0)=20v, ω=100π .

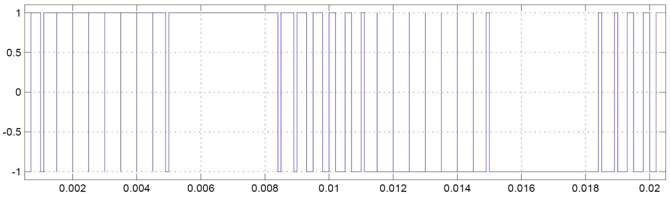


Figure 3. produced pulse by 2-levels SVM for sinusoidal wave form, switching period=20 kHz

To calculate three-phase instantaneous power, the way explained in [9] and defined in below is used. For a three-phase power system, instantaneous voltages v_a, v_b, v_c and instantaneous currents i_a, i_b, i_c are expressed as instantaneous space vectors, V and i, that is:

$$\mathbf{V} = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}, \mathbf{i} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \qquad (27)$$

The instantaneous active power of a three-phase circuit, p, can be given by $p = \mathbf{V} \bullet \mathbf{i}$ where "$\bullet$" denotes the dot (internal) product, or scalar product of vectors. Instantaneous space vector q is defined as $\mathbf{q} = \mathbf{V} \times \mathbf{i}$ where "$\times$" denotes the cross (exterior) product of vectors or vector product. Vector q is designated as the instantaneous inactive power vector of the three-phase circuit, and the magnitude or the length of **q**, q, is designated as the instantaneous inactive power, that is, $q = \|\mathbf{q}\| = \|\mathbf{V} \times \mathbf{i}\|$.

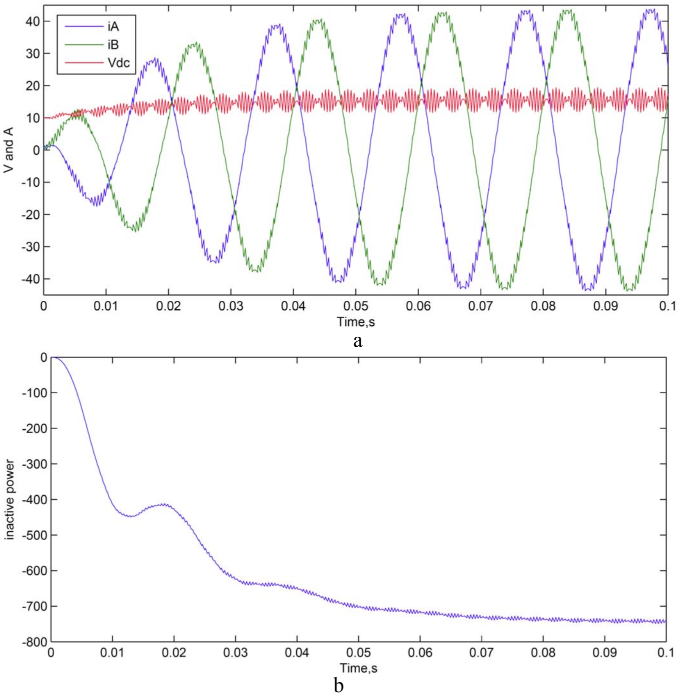
In Fig 4 & 5, at the start of simulation, the SSSC neither injects inactive power into the network nor receives it from the network (initial conditions). At the next stage, controller adjusts Alfa angle so that the SSSC injects 750 VAr into the network at capacitive mode and receives 750 VAr from the network at inductive mode.

In Fig 6 & 7, at the start of simulation, controller adjusts Alfa angle so that the SSSC receives 750 VAr from the network at inductive mode (initial conditions). At 0.2 second, the controller changes its target and adjusts Alfa angle so that the SSSC inject 750 VAr into the network at capacitive mode.

These results show that the capacitor voltage tends to increase for α=20.72° (capacitive mode) and decrease for α=37.48° (inductive mode) as compared with the capacitor voltage when the SSSC neither injects inactive power into network nor receives it from the network. Fig 7 shows the effect of the 40th harmonic at SSSC and injecting into or receiving inactive power from network. This harmonic (40th) is one of the famous harmonics produced in converters during the switching. Various states of SSSC by help of changing the input switching function can be studied in this model.

V. CONCLUSION

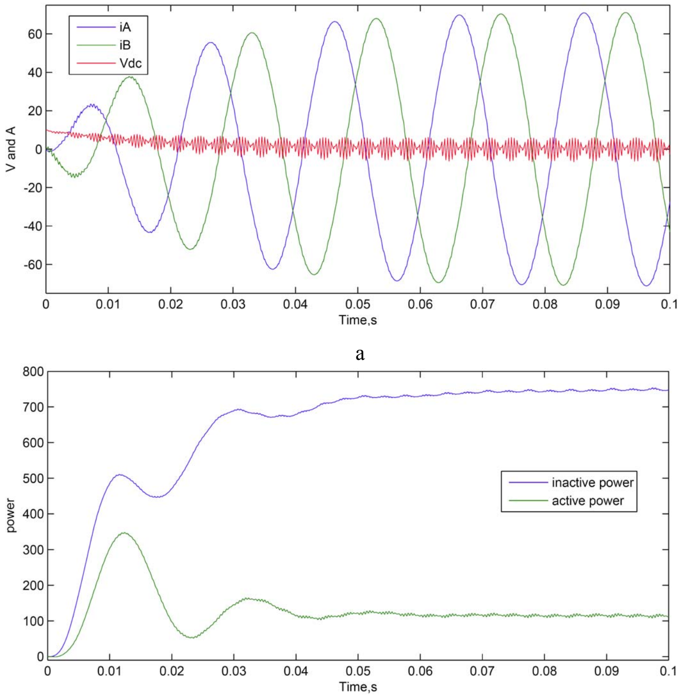
This paper proposes the new SSSC circuit model based on time equations in three-wired system with delta connection for series transformers. This model includes the switching functions, and with nominal changes in these functions, simulates different states of SSSC, that is, transient and steady states. This model is suitable for circuit simulation software and control methods.



a

b

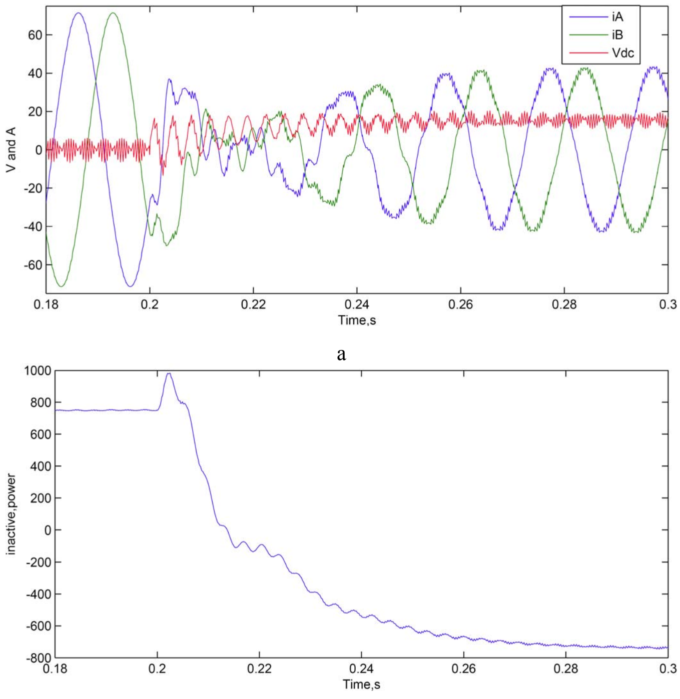
Figure 4. simulation results for practical model of SSSC, operating in capacitive mode (α=20.72°). a) Lines current and capacitor voltage. b) The inactive power of SSSC that injected into network



a

b

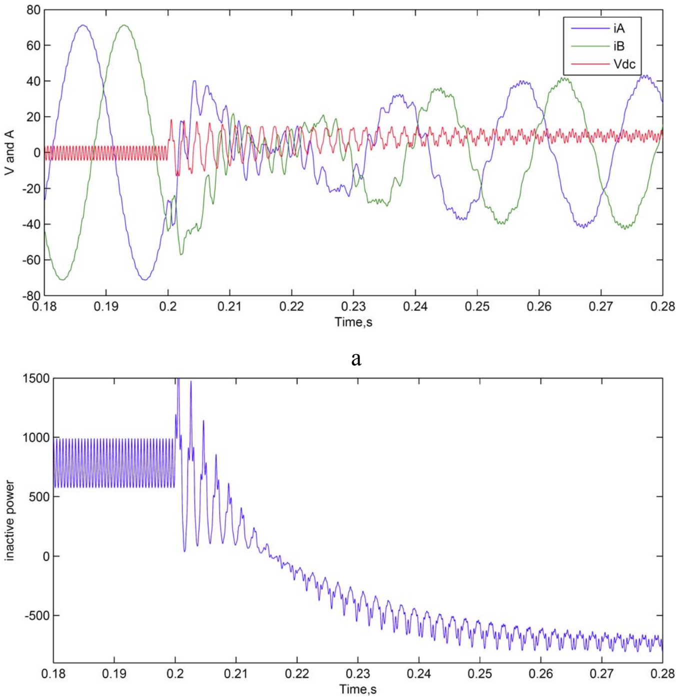
Figure 5. simulation results for practical model of SSSC, operating in inductive mode (α=37.48°). a) Lines current and capacitor voltage. b) The inactive and active power of SSSC that received from network



a

b

Figure 6. simulation results for transient state of SSSC from inductive mode (α=37.48°) to capacitive mode (α=20.72°) by practical model. a) Lines current and capacitor voltage. b) The inactive power of SSSC that injected into and received from network



a

b

Figure 7. simulation results for transient state of SSSC from inductive mode (α=37.48°) to capacitive mode (α=20.72°) by ideal SSSC with 40th harmonic model. a) Lines current and capacitor voltage. b) The inactive power of SSSC that injected into and received from network

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