The 6th International Power Electronics Drive Systems and Technologies Conference (PEDSTC2015)
3-4 February 2015, Shahid Beheshti University, Tehran, Iran

Design of a Three-Level Hysteresis Controller for a Four-Leg Voltage Source Inverter in $\alpha\beta 0$ Frame

Adel Nazemi
Department of Electrical Engineering
K.N Toosi University of Technology
Tehran- Iran
a.nazemi@ee.kntu.ac.ir

Omid Salari
Department of Electrical Engineering
K.N Toosi University of Technology
Tehran- Iran
omid.salari@ee.kntu.ac.ir

Mohamad Tavakoli Bina
Department of Electrical Engineering
K.N Toosi University of Technology
Tehran- Iran
Tavakoli@eetd.kntu.ac.ir

***Abstract*— This paper studies Design and simulation of a three-level Hysteresis current controller for a four-leg inverter connected to a distribution system in αβo frame. It is necessary to transfer ABC frame to αβo frame in order to avoid practical issues of implementing hysteresis modulation. This paper concentrates on choosing the best switching states for the fourth leg of the Inverter to reduce low order harmonics and losses. Finally Matlab-Simulink was used for validating the proposed idea.**

Keywords—Four leg Inverter; Three level Hysteresis controller; Controller in αβo Frame;

I. Introduction

Nowadays because of increment of nonlinear and variable loads in power systems, and their harmonic effects on power quality of the grid, researchers have been tried to mitigate these effects by using power electronics [1]. Hysteresis band method has been utilized widely in control of power converters, because of its simplicity in hardware implementation, fast dynamic response and protection against over currents [2].On the other hand with increment in numbers of Hysteresis band levels, the possibility of reduction in switching frequency is provided which leads to decrement of switching losses[3]. In reference [4] a comparison between controlling methods based on Hysteresis controllers and PI based controllers has been studied.

In the three phase three wire system supplying a nonlinear load, using controllers in *ABC* frame consequences to incorrect results, therefore transferring the controller from *ABC* frame to αβ coordinate system is a compulsory solution to this problem. Utilizing the fourth wire as a path for flowing zero sequence currents in Four-leg grid connected Inverters has been studied in various applications such as STATCOM, Uninterruptable Power Supplies, and active filters, also could be a good solution to aforementioned controlling problem in *ABC* frame. Since in a four-leg Inverter there are three independent current variables, consequently Hysteresis controller could be implemented in both *ABC* and $\alpha\beta o$ frames. Controlling methods proposed in [5] and [6] utilized transferring the coordinating system, however these methods could be implemented in *ABC* frame and transferring the coordinating axis leads to difficulty in implementation of the system.

For the first time the idea of transferring the controller based on hysteresis method to αβ Frame was proposed by Kazmierkowski in 1991 for a three-leg inverter [8]. However to generate controlling signals in $\alpha\beta o$ Frame and to produce appropriate pulses for driving a voltage source inverter lots of papers have been published. During the next years proposed methods for optimization of switching states in Four-leg Inverters for such applications where published in [9-14]. In [5] current controller in βα frame is implemented for a Four - leg Inverter utilizing pq theory based on average of the reference power. Digital implementation of hysteresis controller has been proposed in [7] which introduces reduction in number of voltage and current sensors. In [8] and [12] controllers based on voltage vector, amplitude and time of voltage vector error have been introduced. In references [10]-[11] hysteresis controllers using adaptive algorithms with the possibility of prediction and correction of hysteresis band width have been proposed. Reference [14] proposes a 4*4 matrix to transfer controlling system to dq frame in order to decouple controlling parameters in the new coordination system .

Using Four-leg Inverters with the fourth leg connected to the null wire of the load could be effective in elimination of low order harmonics. Transferring the controller to $\alpha\beta o$ frame and selection of the best switching states for switches of the fourth leg in order to reduce low order harmonics and relating switching states of the fourth leg in *ABC* frame to $\alpha\beta o$ frame are investigated in this paper. In other words switching of the fourth leg is done without existing problems in ABC frame and without switching states of the main three legs of the inverter the fourth leg's switching state could be obtained.

II. Four-Leg Inverter

A. calculation *of Inverter equations*

The case study system in this paper consists of a three-phase four-wire grid supplying a nonlinear load (with the neutral point) and Four-leg Inverter is used to mitigate power quality problems in the point of common coupling (PCC) to the grid. Fig.1 indicates the schematic of the considered system. To study the Four-leg Inverter the system equations in *ABC* frame should be calculated. These equations can be described through the followings (s_f is the switching state of the fourth leg) [15]:

$$\frac{d}{dt}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = -\frac{R}{L}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \frac{V_{dc}}{2L}\begin{bmatrix} s_f - s_a & 0 & 0 \\ 0 & s_f - s_b & 0 \\ 0 & 0 & s_f - s_c \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{L}\begin{bmatrix} Ean \\ Ebn \\ Ecn \end{bmatrix} \quad (1)$$

Where

S_f: Switching state of the fourth leg.

S_a, S_b, S_c: Switching states of phases a, b, c.

E_{an}, E_{bn}, E_{cn} : Line to Neural Voltage of Phases.

Inverter equations at PCC in αβ Frame are as the following:

$$\frac{d}{dt}\begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} = -\frac{R}{L}\begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} + \frac{V_{dc}}{2L}\left(\sqrt{\frac{2}{3}}\right)\begin{bmatrix} S_f - S_a & -\frac{S_f - S_b}{2} & -\frac{S_f - S_c}{2} \\ 0 & \frac{\sqrt{3}}{2}(S_f - S_b) & -\frac{\sqrt{3}}{2}(S_f - S_c) \\ \frac{S_f - S_a}{\sqrt{2}} & \frac{S_f - S_b}{\sqrt{2}} & \frac{S_f - S_c}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{L}\begin{bmatrix} E_\alpha \\ E_\beta \\ E_\gamma \end{bmatrix} \quad (2)$$

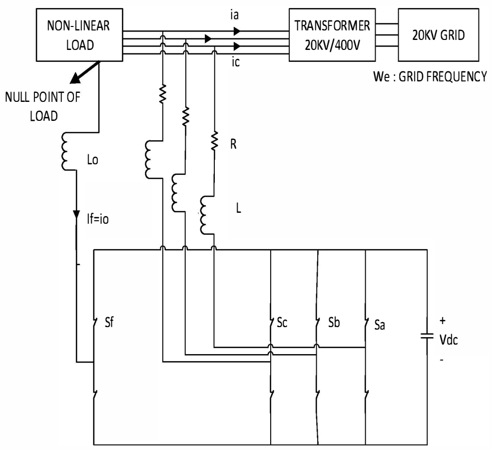

Fig.1. Schematic circuit of the Four-leg Inverter and case study.

To obtain state-space model of the Inverter reference [14] could be used. As it can be understood from the above equation currents i_α and i_β are not related to switching states of the fourth leg and vice versa for null current of the load therefore zero sequence currents are related to switching states of other three legs. Now if a small time interval is considered and V_k is the output voltage of the inverter related to the k_{th} switching state, Eq. 2 can be written as the followings (In dq0 frame):

$$\Delta i_q = \frac{1}{L}(V_{kq} - E_q) - ki_d - k'i_q \quad (3)$$

$$\Delta i_d = \frac{1}{L}(V_{kd} - E_d) + ki_q - k'i_d \quad (4)$$

$$\Delta i_0 = \frac{1}{L}(V_{k0} - E_0) \quad (5)$$

Where k and k' are constants related to w_e(grid frequency), inductance of grid, and DC link voltage (This two parameters in $\alpha\beta o$ frame are equals to zero). Based on three equations obtained above, Inverter could be controlled in two different methods .In first method, system is controlled using scalar method based on the sign of the current error. However, the second method utilizes vector control and the resultant vector from difference between instantaneous grid voltage and the Inverter desired voltage.

B. Scalar *Method*

According to the problems of this method, just implementation algorithm of this method is explained. One of the basic problems for this method is the limitations of DC link voltage. In this method DC link voltage should exceed 1.5 times line to line voltage of the grid which leads to increment in DC link voltage. The reason of high value DC link voltage is to omit effects of nonlinear terms in (3) and (4).

Implementation stages of this method are as followings:

- Instantaneous sampling of the grid voltage.
- Calculation of the current error.
- Passing current error through hysteresis band.
- Division of the Trigonometric circle based on the sign of the hysteresis band block output.
- Defining switching conditions based on the difference between instantaneous voltage of the grid and six space vectors in the output of the Inverter.
- Specifying the switching states of the switches based on conditions obtained in the previous stage.

To clarify the issue, in the following, related codes of a section of the Trigonometric region are presented:

```
%%%%%  a1 & n1 & m1 :
resultant in alpha,Beta,gama direction
if da==1
%%%%%  da : Error in alpha direction
 if db==1
%%%%%  db : Error in Beta direction
   if d0==1
%%%%%  d0 : Error in gama direction
            if      0<=a1<=pi/2 && n1>=0 && m1>=0
                    Sa=1;Sb=0;Sc=0;Sf=0;
            elseif 0<=a2<=pi/2 && n2>=0 && m2>=0
                    Sa=1;Sb=1;Sc=0;Sf=1;
            elseif 0<=a3<=pi/2 && n3>=0 && m3>=0
                    Sa=0;Sb=1;Sc=0;Sf=1;
            elseif 0<=a4<=pi/2 && n4>=0 && m4>=0
                    Sa=0;Sb=1;Sc=1;Sf=0;
            elseif 0<=a5<=pi/2 && n5>=0 && m5>=0
                    Sa=0;Sb=0;Sc=1;Sf=1;
            else
                    Sa=1;Sb=0;Sc=1;Sf=0;
            end
else........
```

The above codes just illustrate a small part of the general program and according to the three level hysteresis controller, there would be 18 different modes like the above codes. Here we neglect mentioning all the codes.

C. *Vector* Control

The idea of Inverter switching based on vector control is divided into two parts.

PART ONE: SWITCHING OF THREE LEGS OF THE THREE PHASES.

First of all the grid voltage should be sampled and the resultant vector with respect to the desired output of the inverter should be calculated. In the following some variables are defined and corresponding equations to elaborate the topic would be obtained.

Δi: The difference between actual current and the reference current.

V_k: The space vector voltage at the Inverter output side.

E: Sampled voltage of the grid.

i_e : Error vector of the current

In output of the comparator the value of the current error is calculated as below:

$$\Delta i = i_{real} - i_{ref} \tag{6}$$

Now if the resistance of PCC is neglected, the following equation is obtained simply:

$$\frac{di}{dt} = \frac{1}{L}(V_k - E) \tag{7}$$

Now combining (6) and (7) leads to:

$$\frac{di_e}{dt} = \frac{1}{L}(V_k - V_{ref}) \tag{8}$$

Where:

$$V_{ref} = E + L\frac{di_{ref}}{dt} \tag{9}$$

Every instant the amount and direction of the current error should be calculated using (8) and then its location in six-region Trigonometric circle should be specified. Fig. 3 indicates the above statements schematically.

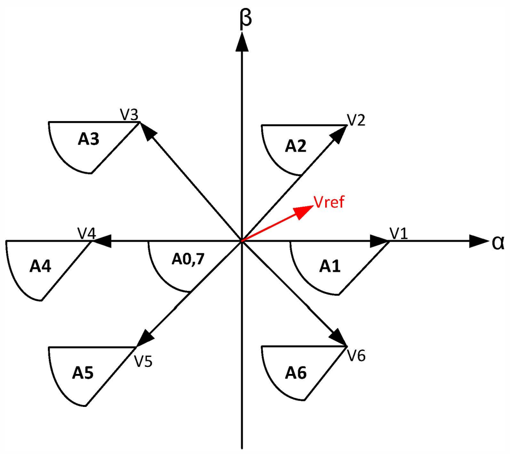

Fig. 2. Calculation of current error vector

In Fig. 2 with the assumption that the reference vector sweeps region 1, its resultant vector with respect to all other 6 vectors is clarified using areas indicated by A1 to A6. After obtaining this vector's location, it should cross the reference hexagonal to generate switching pulses based on the location of the vector (inside or outside of the hexagonal). Fig. 3 indicates the status of current error vector in different parts of the hexagonal:

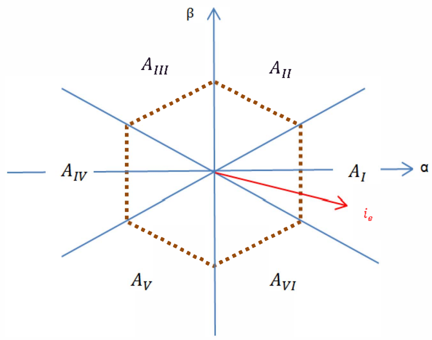

Fig. 3. Current error vector status

Every time the current error vector cuts one side of the indicated hexagonal in Fig. 3, the difference of reference voltage vector of the grid (V_{ref}) with all 8 possible vectors in output side of the Inverter should be calculated and the vector which could return the current error into the allowed region should be chosen. If in a specific instant there are more than one vector which could provide the same results in returning the current error, then in steady state conditions vector which could decrease the current error vector with a lower speed should be chosen and in dynamic conditions, like dramatic changes in load, an inverse of the above statement should be selected. Based on the above explanations, space vectors table according to the location of current error vector (for switching of tree legs related to the main three phases) is provided in Table I.

It should be considered that the following switching table is true only for steady state mode of operation and achieving appropriate dynamic operation requires some changes in Table I and considering the aforementioned conditions for dynamic mode. In dynamic mode neglecting the position of current error vector, only one switching state and one corresponding output would be considered for the inverter which is not this paper's concern.

TABLE I. Switching states of three main legs of the Inverter

sectors	A_I	A_{II}	A_{III}	A_{VI}	A_V	A_{IV}
A_1	V0	V0	V1	V1	V2	V2
A_2	V3	V7	V7	V2	V2	V3
A_3	V4	V4	V0	V0	V3	V3
A_4	V4	V5	V5	V7	V7	V4
A_5	V5	V5	V6	V6	V0	V0
A_6	V7	V6	V6	V1	V1	V7

To clarify the proposed switching technique to drive the Inverter switches, the following codes are used to accomplish desired results (How to implement Table I):

```
%%%% k : sector of Sampled Load Voltage
%%%% s : sector of Error Vector of Current
function [Sa,Sb,Sc,Sf] = fcn(k,s)
if k==1
    if s==1
        Sa=0;Sb=0;Sc=0;Sf=0;
    elseif s==2
        Sa=1;Sb=1;Sc=1;Sf=1;
    elseif s==3
        Sa=1;Sb=0;Sc=0;Sf=1;
    elseif s==4
        Sa=1;Sb=0;Sc=0;Sf=0;
    elseif s==5
        Sa=1;Sb=1;Sc=0;Sf=1;
    else
        Sa=1;Sb=1;Sc=0;Sf=0;
    end
elseif........
```

Exactly the same as above codes all 30 remained situations could be simulated.

PART TWO: SWITCHING STATES OF THE FOURTH LEG.

To describe switching states of the fourth leg of the Inverter, Hysteresis method in *ABC* frame for a three- phase four-wire system should be employed. In this part switching signals that were obtained by using Hysteresis method in *ABC* frame will provide pulses to drive switches of the fourth leg of the Inverter through the incoming equations. Using (2) the following equations could be derived for driving the switches in desired coordination (Indeed three variables of S_α, S_β, S_0 are three virtual variables which are defined below based on switching states of S_a, S_b, S_c, S_f):

$$S_\alpha = k_1(S_a + S_c - 2S_b) \quad (10)$$

$$S_\beta = k_2(S_c - S_a) \quad (11)$$

$$S_0 = k_3(3S_f - S_a - S_b - S_c) \quad (12)$$

Where k_1, k_2, k_3 are three constants based on DC link voltage and inductance of PCC. Eight space vectors for the output of a three phase and three leg Inverter could be defined as in Table II.

III. SIMULATION RESULTS

Simulated system is a distribution grid whose parameters are illustrated in Table III. The schematic of simulated system is illustrated in Fig. 4 which was implemented in Matlab /Simulink. The load in the simulations is a nonlinear unsymmetrical load which is obtained by putting a resistive-capacitive load in series with a rectifier where the load neutral point is available, also the amplitudes of the capacitors are different which would affect the positive and negative peak values of the currents. In simulation process it was assumed that the system is supplying a nonlinear load with specific active and reactive power. Table III contains nominal values of the implemented circuit in Matlab.

TABLE II. Relationship between Switching States in ABC and αβ Frame

Vectors	S	S	S	S	S	S	S
1	1	0	0	0	0	1	1
						0	0
2	1	1	0	0	0	1	1
						0	0
3	0	1	0	1	0	1	1
						0	0
4	0	1	1	1	0	1	0
						0	1
5	0	0	1	1	1	1	1
						0	0
6	1	0	1	0	1	1	0
						0	1
7	1	1	1	0	1	1	0
						0	0
0	0	0	0	0	1	1	1
						0	0

TABLE III. Three phase system parameters

DC link voltage	400 Volts	
Inductance of PCC	1 mH	
Resistance of PCC	0.1Ω	
Three level Hysteresis band width	h1=2	h2=1
Grid nominal voltage	380 v	
Load active power	8 kw	
Load reactive power	4.8 kvar	

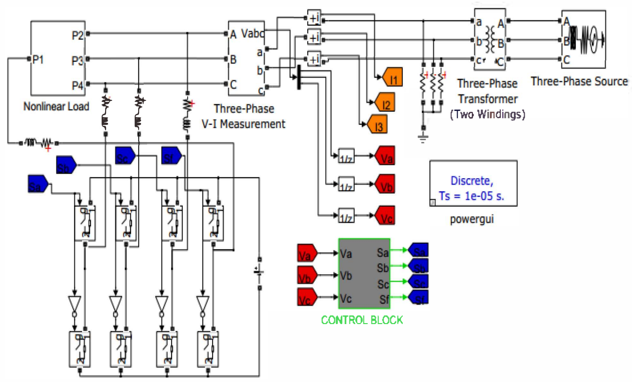

Fig. 4. Implemented circuit in Matlab/ Simulink

Fig. 5 indicates different parts of control block in Fig. 4.

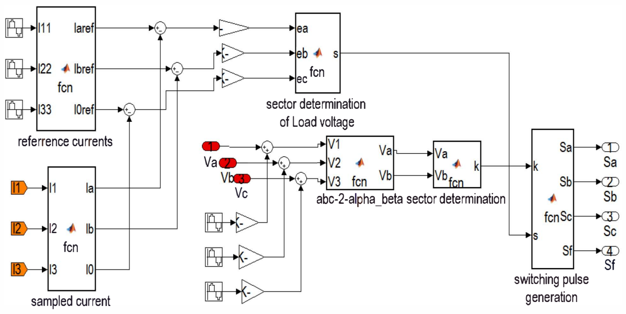

Fig. 5. Contol block to drive 4-leg Inverter

Now if changes in current i_α is plotted with respect to changes in current i_β for a three leg Inverter it would be a half circle [16] and for a four-leg Inverter it will be a complete circle which is the validity to confirm that both currents i_α , i_β are in allowed region of Hysteresis band.

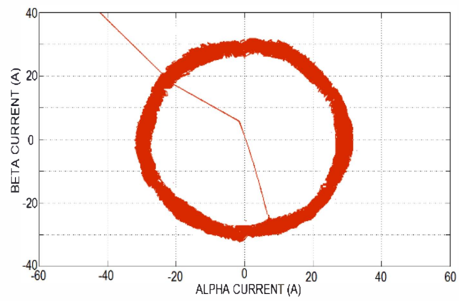

Fig. 6. Changes in i_β with respect to changes in i_α

If β current error is plotted with respect to α current error, a hexagonal will appear which is the allowed region plotted in Fig. 3. This is illustrated in Fig. 7. It can be modified using changes in Hysteresis band.

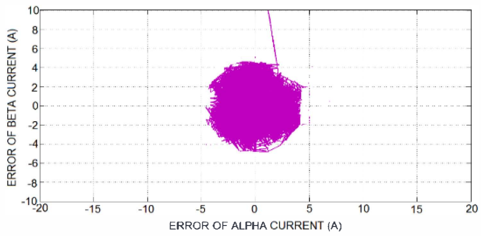

Fig. 7. Changes in β current error with respect to changes in α current error.

Fig. 8 illustrates voltage and current of nonlinear loads at PCC which includes odd harmonics.

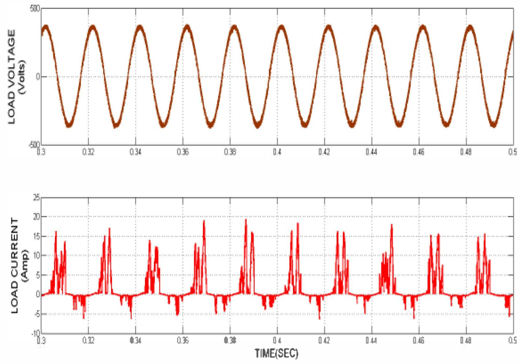

Fig. 8. Load voltage and current.

After employing the proposed Four-leg Inverter to eliminate harmonic effects of the load, the grid currents will be modified to the ones indicated in Fig. 9. Fig. 9 indicates elimination of unwanted effects of the loads on the grid with respect to Hysteresis allowed band for three phases.

Current THD decreases to less than 5% after employing the proposed control technique where it was about 85% before. This statement is obvious in Fig. 8 and Fig. 9.

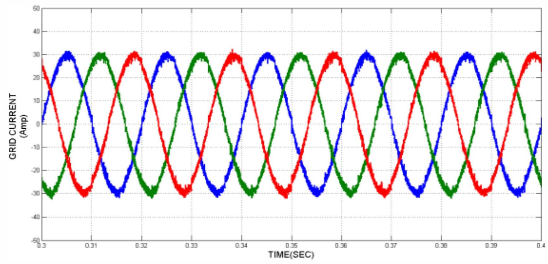

Fig. 9. Grid currents after employing the four-leg Inverter, reduction in Current THD to less than 5%.

To specify how the switching has been done, Fig. 10 indicates the relationship between grid voltage sector and the Inverter output region (As Fig. 10 illustrates, for every sector of the grid voltage, there are six different sectors for Inverter output according to simulation codes).

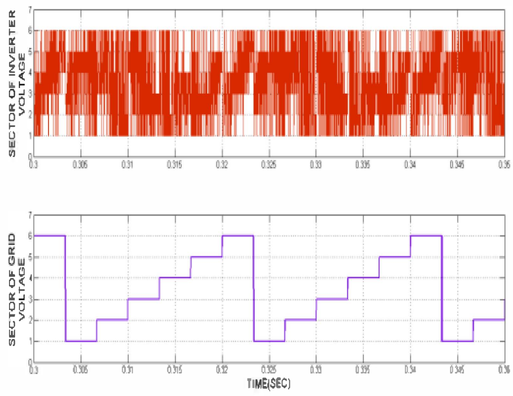

Fig. 10. Grid voltage sector and Inverter output voltage.

In order to calculate Inverter losses and compare it with previous works, traditional hysteresis controller, space vector based hysteresis controller [9], and proposed method have been studied. Table IV indicates that the proposed method reduces losses in comparison with other methods. The reason in reduction of losses is reduction in switching frequency. As it is obvious in Table IV, losses of the proposed method is half of traditional hysteresis method.

Table IV. Comparison of losses in traditional hysteresis controller, space vector based hysteresis controller, and proposed method.

Control Method	Traditional Hysteresis	Space Vector Based Hysteresis Control	Proposed Method
Losses (Watt)	1060	820	680
THD (%)	28.5	10.5	5.8

IV. Conclusion

In this paper a vector control in αβo system was proposed and finally simulated using Matlab/Simulink. According to the simulation results it was observed that the proposed method for switching a Four-leg Inverter could help steady state performance of the Inverter in addition to solving Hysteresis problems in *ABC* frame. In the presented simulations current THD at the PCC has been decreased from 85 percent to 5 percent and consequently the Inverter losses have been also decreased. Also switching of the fourth leg is done without existing problems in ABC frame and without switching states of the main three legs of the Inverter the fourth leg's switching state could be obtained. However transferring parameters between two coordination systems are a little bit difficult, with development of nonlinear loads in distribution systems the need to use such methods in order to eliminate low order harmonics is increasing daily.

References

[1] J. Rodríguez, J. I. Leon, S. Kouro, R. Portillo, and M. A. M. Prats, "The Age of Multilevel Converters Arrives," Industrial Electronics Magazine, IEEE ,vol. 2, no. June, pp. 28–39, 2008.

[2] M. P. Kazmierkowski and L. Malesani, "Current control techniques for three-phase voltage-source PWM converters: a survey," *IEEE Trans. Ind. Electron.*, vol. 45, no. 5, pp. 691–703, 1998.

[3] P. Karuppanan and S. K. Ram, "Three Level Hysteresis Current Controller based Active Power Filter for Harmonic Compensation," International Conference on Emerging Trends in Electrical and Computer Technology (ICETECT) ,pp. 1–6, 2011.

[4] A. Nachiappan, K. Sundararajan, and V. Malarselvam, "Current controlled voltage source inverter using Hysteresis controller and PI controller," *2012 Int. Conf. Power, Signals, Control. Comput.*, pp. 1–6, Jan. 2012.

[5] M. Ucar and E. Ozdemir, "Control of a 3-phase 4-leg active power filter under non-ideal mains voltage condition," Electr. Power Syst. Res. 78 (2008) 58–73.

[6] R. Vyas, "Hysteresis Band Current Control For Harmonic Compensation of Unbalanced & Nonlinear load," vol. 1, no. 5, pp. 1–10, 2014.

[7] L. Bin and T. Minyong, "Control Method of the Three-Phase Four-Leg Shunt Active," Energy Procedia, vol. 14, no. 2011, p. 1, Jan. 2012.

[8] M. P. Kazmierkowski and W. Sulkowski, "Novel space vector based current controllers for PWM-inverters - Power Electronics, IEEE Transactions on," vol. 6, no. I, 1991.

[9] Q. Yin, Z. Bai, "Space Vector Based Hysteresis Control Strategy with Fixed Switching Frequency for UPQC," International Journal of Automation and Power Engineering (IJAPE) vol. 2, no. 4, pp. 151–156, 2013.

[10] M. R. Amer, O. A. Mahgoub, and S. A. Zaid, "New Hysteresis Control Method for Three Phase Shunt Active Power Filter ," Proceeding of the International MultiConference of Engineers and Computer Scientists, vol. II, 2011.

[11] N. Belhaouchet, S. Begag, and L. Rahmani, "A Digital Hysteresis current controller for three phase shunt active power filter," 4th International Conference on Computer Integrated Manufacturing CIP, no.3 November, 2007.

[12] F. Zare and G. Ledwich, "A New Hysteresis Current Control for Three-phase Inverters Based on Adjacent Voltage Vectors and Time Error," *2007 IEEE Power Electron. Spec. Conf.*, pp. 431–436, 2007.

[13] L. C. De Freitas and J. B. Vieira, "The Bang-Bang Hysteresis Current Waveshaping Control Technique Used to Implement a High Power Factor Power Supply,"IEEE Transactions on Power Electronic, vol. 19, no. 1, pp. 160–168, 2004.

[14] M. J. Ryan, R. W. De Doncker, and R. D. Lorenz, "Decoupled Control of a Four-Leg Inverter via a New 4 4 Transformation Matrix," IEEE Transactions on Power Electronics, vol. 16, no. 5, pp. 694–701, 2001.

[15] P. Rodríguez, J. I. Candela, R. Pindado, P. Filters, and P. Conditioning, "State-Space Averaged Model of Three-Phase Four-Wire Shunt Active Power Filter based on Current-Controlled VSI Key words," IEEE Transactions on Power Electronics, vol. 2.

[16] M. A. Dzieniakowslu, Marian P. Kaunierkowski, "Review of Current Regulation Techniques For Three-phase PWM Inverters ," 20th International Conference on Industrial Electronics, Control and Instrumentation, IECON '94., pp. 567–575, 1994.