Electrical Power and Energy Systems 33 (2011) 817–826

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

Three-phase unbalance of distribution systems: Complementary analysis and experimental case study

M. Tavakoli Bina *, A. Kashefi

Faculty of Electrical Engineering, K.N. Toosi University of Technology, Seid Khandan, P.O. Box 16315-1355, Tehran 16314, Iran

ARTICLE INFO

Article history:
Received 12 June 2007
Accepted 7 December 2010
Available online 12 February 2011

Keywords:
Unbalance
Distribution systems
Substations
Over-capacity
Power losses

ABSTRACT

Three-phase unbalance is a familiar issue for power system researchers and engineers. This can introduce additional power losses in distribution network in steady states due to both negative and zero sequence components. It could also limit the loading capability of distribution transformers, well below their nominal ratings. There are many voltage and current unbalance definitions (e.g., *IEEE* and NEMA) for three-phase three-wire systems, assuming zero sequence currents to be of negligible practical value, for they cannot flow in three-wire systems. However, the zero sequence unbalance has significant current magnitude in three-phase with four-wire distribution networks, particularly in developing countries. Hence, this paper concentrates on the distribution unbalance, completing the available definitions in order to maintain tangible relationships between the level of unbalance and the cited consequences in distribution networks. Furthermore, practical works were performed on 11 selected 20 kV/0.4 kV substations within Tehran North-West Distribution System (TNWDS), where data loggers have been installed for 7 days to measure and record operating conditions of substations. Then, detailed analysis and assessment are suggested on empirical data to substantiate the presented complementary definitions and relationships.

© 2010 Elsevier Ltd. All rights reserved.

1. Introduction

Generating units supply three-phase sinusoidal positive sequence voltages, which are *balanced* in terms of their amplitudes and 120° phase differences at a single frequency. Any deviations from these two basic characteristics introduce an *unbalanced* condition. Hence, the terminology of *unbalanced* can be classified into three main parts; *amplitude unbalance* of the fundamental, *phase difference unbalance* of the fundamental, and *unbalanced harmonic disturbances* [1]. Occurrence of at least one of these features is enough for a distribution network to become unbalanced.

Unbalanced conditions are principally caused by both structural and operational factors [2]. Structural unbalance is usually negligible and can occur due to incomplete transposition of transmission lines and cables, asymmetrical distribution of wiring of transformers, open-Δ(*V*–*V*) or open-Y–open-Δ connected transformers, capacitor banks, aged fuses, etc. Nevertheless, operational unbalance can be considerable when single-phase and two-phase loads as well as any unbalanced three-phase loads such as arc-furnaces are being supplied by the distribution network [1,3].

The consequences of voltage unbalance are discussed in [1,3,6] on power system equipments such as induction motors, power converters, AC variable speed drive, and power transformers. Comprehensive studies have been carried out in [12,13], describing consequences of unbalance condition on induction motors. It is concluded that the obtained results are very sensitive to the unbalance condition. Moreover, unbalance condition could lower stability margin, increasing the power losses. It is noticeable that a small voltage unbalance might lead to a significant current unbalance because of low negative sequence impedance.

This paper takes a closer look into the unbalance issue of distribution systems, where available definitions are complemented for the four-wire distribution networks. In particular, eleven 20 kV/0.4 kV substations were studied in the TNWDS to assess the consequences of unbalance situations in terms of the developed formulas. This study, further aims at analyzing and relating the unbalance of the distribution networks with the emerged technical consequences (e.g. loading capability and power losses) using the experimental data compiled from the TNWDS. The analysis is performed for all unbalanced definitions and compared thereafter to show the suitability of the suggested complementation for the studied distribution network. In brief, Section 2 describes and analyzes unbalance formulation including both conventional and complementary definitions. Section 3 discusses the unbalance consequences in terms of unbalance definitions, where the theoretical

* Corresponding author. Tel.: +98 (21) 8406 2371; fax: +98 (21) 8846 2066.
E-mail address: tavakoli@kntu.ac.ir (M.T. Bina).

0142-0615/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijepes.2010.12.003

conclusions are applied to an experimental case study in Section 4 for further analysis and verifications.

2. Unbalance formulation

Unbalance phenomenon is becoming more a vital issue for distribution systems as various customers with different priorities are appearing, demanding low-cost high-quality electricity. There is a lot of confusion, however, on the meaning of the term *voltage/current unbalance*, where different standards introduce different conventional definitions as follows (define phase-to-neutral voltages by $\{V_{an}, V_{bn}, V_{cn}\}$, and line-to-line voltages by $\{V_{ab}, V_{bc}, V_{ca}\}$):

- *IEEE* Std. 936 (1987): Phase voltage unbalance rate [4].

$$IEEE\,v_{936} = \frac{\text{Maximum of } \{V_{an}, V_{bn}, V_{cn}\} - \text{Minimum of } \{V_{an}, V_{bn}, V_{cn}\}}{\text{Mean of } \{V_{an}, V_{bn}, V_{cn}\}} \times 100 \quad (1)$$

- *IEEE* Std. 112 (1991): Modified phase voltage unbalance rate [4].

$$IEEE\,v_{112} = \frac{\text{Maximum deviation from mean of } \{V_{an}, V_{bn}, V_{cn}\}}{\text{Mean of } \{V_{an}, V_{bn}, V_{cn}\}} \times 100 \quad (2)$$

- NEMA (National Electric Manufacturers Associations of the USA) Std. (1993) [4].

$$MDV = \frac{\text{Maximum deviation from mean of } \{V_{ab}, V_{bc}, V_{ca}\}}{\text{Mean of } \{V_{ab}, V_{bc}, V_{ca}\}} \times 100 \quad (3)$$

- *IEEE* true definition (1996) [4].

$$TDV = \frac{\text{Negative sequence voltage}}{\text{Positive sequence voltage}} \times 100 \quad (4)$$

Although, these four definitions could produce different values for a single case; note that current unbalance formulations are simply obtained by replacing voltage with current components ($IEEEi_{936}$, $IEEEi_{112}$, *MDI*, and *TDI*). The first two standards, $IEEE_{936}$ and $IEEE_{112}$, ignore the 120° phase difference unbalance and only take the amplitudes into account. The last two definitions, *MDV* and *TDV*, are sensitive to the phase difference unbalance and provide suitable measures of unbalance of three-phase *three-wire* networks.

Nevertheless, these definitions ignore zero sequence components that are practically unavoidable in three-phase *four-wire* distribution systems. Fig. 1a illustrates a reproduced unbalance case (reported in [2]), where negative and zero sequence voltages are 3% and 2% of positive sequence voltage respectively. When the relative phase angle of the negative sequence voltage vary within [0°, 360°], $IEEEv_{936}$, $IEEEv_{112}$ and *MDV* give different voltage unbalance percents even if no zero sequence voltage is present.

2.1. Complementary unbalance formulation

To establish complementary definitions for *three-phase four-wire* systems, the following points are carefully taken into account:

- The well-known symmetrical component [5] approach is used to compose complementary definitions.
- For three-wire systems, when no zero sequence current is present, the complementary definitions will give the same results as the *IEEE* true definition.
- Unbalance share of each phase will be identical on the concluding definitions.

Keeping these points in mind, we can now decompose an unbalanced system into two components; *completely balanced* and *completely unbalanced* as shown in Fig. 1b. It is also mathematically described as below:

$$\mathbf{V} = \mathbf{V}_1 + \Delta\mathbf{V} \quad (5)$$

where

$$\mathbf{V} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}, \quad \mathbf{V}_1 = \begin{bmatrix} V_{1a} = V_1 \\ V_{1b} = a^2 V_1 \\ V_{1c} = aV_1 \end{bmatrix},$$

$$\Delta\mathbf{V} = \begin{bmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \end{bmatrix} = \begin{bmatrix} V_{0a} + V_{2a} = V_0 + V_2 \\ V_{0b} + V_{2b} = V_0 + aV_2 \\ V_{0c} + V_{2c} = V_0 + a^2 V_2 \end{bmatrix}$$

where V_1 is the positive sequence voltage as the *completely balanced* component, $a = e^{j120°}$, and $\Delta\mathbf{V}$ consists of both negative and zero sequence voltages as the *completely unbalanced* component. Further, the unbalance ratio $\mathbf{r}$ is defined (as a phasor vector):

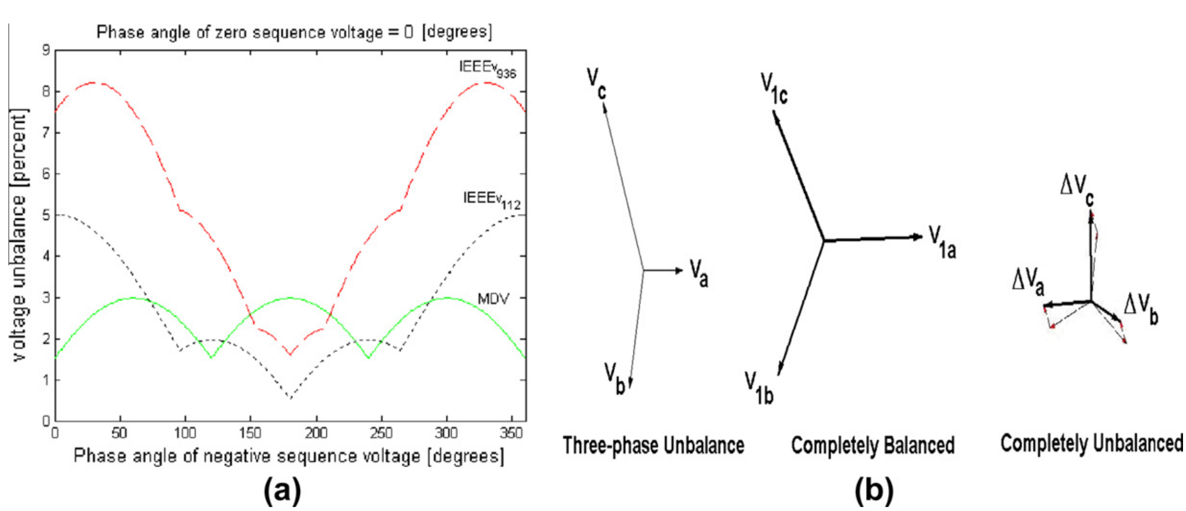

Fig. 1. (a) Variations of voltage unbalance percent evaluated by $IEEEv_{936}$, $IEEEv_{112}$ and *MDV* due to zero sequence voltage when phase of negative sequence vary within [0°, 360°] and (b) decomposition of a general unbalanced system into *completely balanced* and *completely unbalanced* systems.

$$\mathbf{r} \stackrel{\text{def}}{=} \begin{bmatrix} r_a \\ r_b \\ r_c \end{bmatrix} = \begin{cases} \frac{\Delta V_a}{V_{1a}} = \frac{V_0+V_2}{V_1} \\ \frac{\Delta V_b}{V_{1b}} = \frac{V_0+a^2V_2}{a^2V_1} \\ \frac{\Delta V_c}{V_{1c}} = \frac{V_0+aV_2}{aV_1} \end{cases} \tag{6}$$

where r_a, r_b, r_c introduce the contribution of each phase from the *completely unbalanced* component. But, these need to be combined together to get an unbalance representation for the *three-phase four-wire* networks. Hence, considering (5) and (6), the following possible combinations can be examined:

- *Choice 1*: Mean of $\{r_a, r_b, r_c\}$.

$$MV = \frac{r_a + r_b + r_c}{3} \tag{7}$$

- *Choice 2*: Geometric mean of $\{r_a, r_b, r_c\}$.

$$GMV = \sqrt[3]{r_a \times r_b \times r_c} = \frac{\sqrt[3]{V_0^3 + V_2^3}}{V_1} \tag{8}$$

- *Choice 3*: Geometric mean of $\{|r_a|, |r_b|, |r_c|\}$.
- *Choice 4*: Geometric root mean squares of $\{|r_a|, |r_b|, |r_c|\}$.
- *Choice 5*: Mean of $\{|r_a|, |r_b|, |r_c|\}$.

$$AMV = \frac{|r_a| + |r_b| + |r_c|}{3} = \frac{|V_0 + V_2| + |V_0 + a \times V_2| + |V_0 + a^2 \times V_2|}{3|V_1|} \tag{9}$$

- *Choice 6*: Root mean squares of $\{|r_a|, |r_b|, |r_c|\}$.

$$RMSV = \sqrt{\frac{|r_a|^2 + |r_b|^2 + |r_c|^2}{3}} = \frac{\sqrt{|V_0|^2 + |V_2|^2}}{|V_1|} \tag{10}$$

Complemented current unbalance can be obtained by replacing voltage with current components (*MI*, *GMI*, *AMI*, *RMSI*), as well. Considering (6), the first choice cannot represent a real unbalance value because the right hand side of (7) is always equal to zero. Also, when $|V_0 + V_2|$ is about zero, the resulting unbalance value of the second choice in (8) will be about zero despite the presence of system unbalance. Choices three and four lead to relationships that are equivalent to the second choice, where any unbalance ratios about zero would result in unreal unbalance value. The last two choices provide reasonable definitions especially *choice 6* that present a simple relationship for analysis and calculations. Therefore, examination and assessment of these choices ((9) and (10)) will be performed using the practical compiled data obtained from the studied distribution substations.

3. Unbalance consequences

Three-phase unbalance imposes certain consequences on the distribution networks, including power losses of substation transformers and lines along with the viable capacity of transformers out of their nominal balanced ratings. Power system losses refer to the amount of power lost in various processes and steps of delivery from generation to customer since the devices that transfer energy have physical resistance to the energy flow. Thus, the unbalance of power systems contributes to the intensification of physical process of losing energy. Moreover, when a distribution transformer is operating under unbalance condition, then the transformer will be unable to serve up to its nominal ratings because of unequal current magnitudes. Most distribution companies purchase wholesale electricity at transmission voltage levels. The companies then step the voltage down to primary distribution levels through their own substation transformers. This section intends to analyze and express the share of unbalance of distribution networks on the power losses and loading capacity of transformers, which will further be assessed by experimental work and empirical data gathered from the TNWDS selected substations.

3.1. Substation transformers

With the existence of power losses in three forms of *copper losses*, *core losses* and *auxiliary losses*, there are no perfect transformers as such. A large part of distribution losses appears in distribution transformers, while maximum efficiency of a transformer is achieved when copper losses are equal to iron losses [9]. Meanwhile, if various transformer losses were analyzed exclusively in terms of unbalance issue, copper losses have the largest share which will vary with the unbalanced load currents. For the studied transformers within the TNWDS, copper losses are about twice bigger than iron losses in average power; eight times at the rated power.

3.2. Transmission lines

Line losses follow the RI^2 law, and thus any steps taken to reduce the flow of amperes in lines will produce loss reduction advantages. This will also introduces dependence of the line losses on its operating currents. Nevertheless, we are interested in analyzing the share of unbalance operation of a distribution system, in particular, from the transformer and line losses. Hence, the unbalanced currents (I_a, I_b, I_c) are decomposed into the symmetrical components (I_0, I_1, I_2), and the power losses are obtained as below:

$$\begin{cases} ULoss \stackrel{\text{def}}{=} Loss_0 + Loss_1 + Loss_2 \\ Loss_0 = 3(R + 3R_n)|I_0|^2 \\ Loss_1 = 3R|I_1|^2 \\ Loss_2 = 3R|I_2|^2 \end{cases} \tag{11}$$

where R is the resistance seen from the three-phase low-voltage feeders including the resistance of transformer windings, R_n is the equivalent resistance of the neutral-wire situated between the substation transformer and the low-voltage feeders (see Fig. 2a), and *ULoss* is the total system losses under unbalanced condition that is decomposed into zero ($Loss_0$), positive ($Loss_1$) and negative ($Loss_2$) sequence components. The unbalanced load power (S_{Load}) before connecting a compensator can be expressed by [11]:

$$S_{Load} = 3(V_0I_0^* + V_1I_1^* + V_2I_2^*) \tag{12}$$

Then, assume the compensator of Fig. 2b ideally injects the *unbalance component* of the load such that the system feeders become *completely balanced* in terms of both load terminal voltages and source currents. It should be noted that, by using the compensator, the load unbalanced active and reactive powers are supplied by the source in the form of balanced currents (I_{1comp}); thus, S_{Load} can alternatively be introduced by:

$$S_{Load} = 3V_1I_{1comp}^* \tag{13}$$

Comparing (12) with (13) results in:

$$I_{1comp}^* = \frac{V_0I_0^* + V_1I_1^* + V_2I_2^*}{V_1} \tag{14}$$

In practice, I_{1comp} could be very close to I_1; hence, the source balanced power losses after compensation (*BLoss*) can be worked out as below:

$$BLoss = 3R\left[|I_1|^2 + \left|\frac{V_0}{V_1}\right|^2|I_0|^2 + \left|\frac{V_2}{V_1}\right|^2|I_2|^2 + 2Re(K_1 + K_2 + K_3)\right] \tag{15}$$

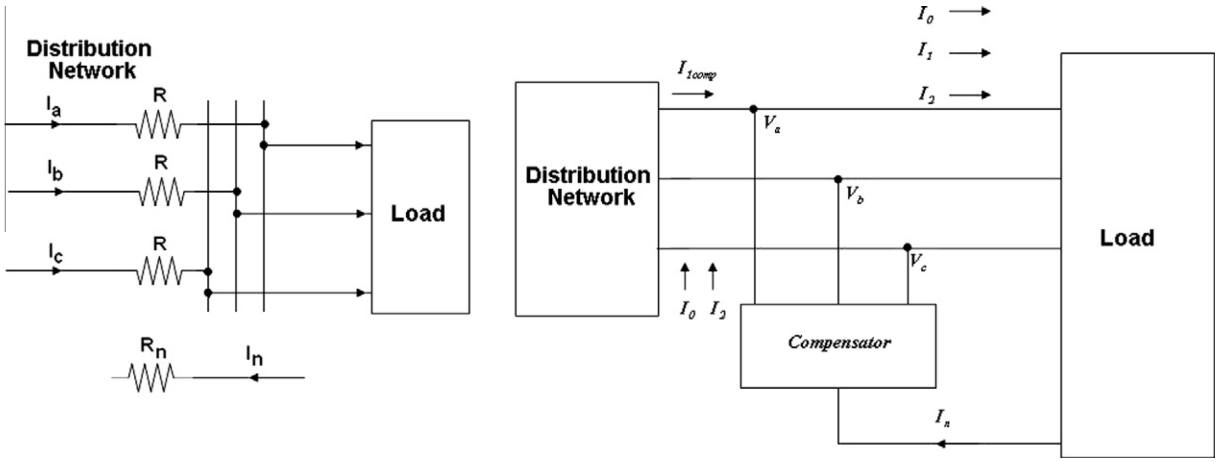

Fig. 2. (a) Representation of distribution equivalent resistances of the three-phases R and the neutral-wire R_n and (b) compensation of the unbalance of the substation feeder using an unbalance compensator.

where $K_1 = \frac{V_0}{V_1} I_0^* I_1$, $K_2 = \frac{V_0 V_2^*}{|V_1|^2} I_0^* I_2$ and $K_3 = \frac{V_2}{V_1} I_2^* I_1$. It can be shown that 20 kV/0.4 kV transformers of Δ–Y type produce no *zero sequence voltage* neither at the low-voltage Y-end nor at 20 kV three-wire line-end [7,8]. Hence, considering $V_0 \approx 0$ and neglecting $\left|\frac{V_2}{V_1}\right|^2$ that is practically very small (the average of $\left|\frac{V_2}{V_1}\right|$ during a week was found to be about 0.0077 for 11 studied substations), the balanced system losses using unbalance compensation is approximated to $BLoss \approx 3R|I_1|^2$ (experimental case study verifies the stated approximation). Therefore, the share of unbalance losses out of total transformer and line losses is

$$\Delta Loss = ULoss - BLoss = 3R(|I_0|^2 + |I_2|^2) + 9R_n|I_0|^2 \tag{16}$$

where $\Delta Loss$ can be compared with both $BLoss$ ($ULoss_1\%$) and $ULoss$ ($ULoss_2\%$) as follows:

$$\begin{cases} ULoss_1\% = \frac{\Delta Loss}{BLoss}\% = \frac{3R(|I_0|^2+|I_2|^2)+9R_n|I_0|^2}{3R|I_1|^2} \times 100 \\ \qquad\qquad = ((1 + 3\frac{R_n}{R})RMSI^2 - 3\frac{R_n}{R}TDI^2) \times 100 \\ ULoss_2\% = \frac{\Delta Loss}{ULoss}\% = \frac{ULoss_1}{1+ULoss_1} \times 100 \end{cases} \tag{17}$$

It should be noted that the defined R_n is practically very small compared to R, leading to approximate relationships like

$$\begin{cases} ULoss_1\% = RMSI^2\% \\ ULoss_2\% = \frac{RMSI^2}{1+RMSI^2}\% \end{cases} \tag{18}$$

Moreover, by expansion of Taylor series for $ULoss_2\%$ in (18), it can be shown that when $RMSI$ is small, then $ULoss_1\%$ and $ULoss_2\%$ are approximately equal.

3.3. Loading capability of unbalanced transformers

Unbalance condition reduces the actual loading capability of a distribution transformer, where the maximum current of the three phases determines the remaining capacity of transformer. In this situation, the substation transformer may not be able to supply the load as much as it is designed to and as high as the intended rate, which is called *over-capacity of unbalanced transformer (OCUT)* in this paper. In practice, when the maximum current reaches the overload zone, even if the other two phases operate well below their rated currents the protective alarm could turn on. Also, as the *OCUT* tends towards zero in a balanced operation, the *OCUT* could be considered as a measure of unbalance of distribution feeders at every steady state operation point. The lower the *OCUT*, the more balanced operates the distribution substation. Additionally, lowering the high *OCUT* of substation transformers postpones their proper resizing and emplacement, which is a necessary task in the process of loss reduction and economic operation of distribution systems.

A solution to formulate the *OCUT* is to calculate the unbalance loading of transformer (*ULT*) on the assumption that the network is operating under three-phase balanced currents $I_{max}(S_{max})$ such that $ULT = 3S_{max}$. Then, the equivalent balanced loading of the transformer (*BLT*) is calculated when the load unbalance is compensated for, such that the source currents become balanced ($BLT = S_a + S_b + S_c$, see Fig. 2b). The $OCUT_1\%$ can be expressed by:

$$OCUT_1\% = \frac{ULT - BLT}{BLT} \times 100 \tag{19}$$

Alternatively, $OCUT_2\%$ can be evaluated with respect to ULT as below:

$$OCUT_2\% = \frac{ULT - BLT}{ULT} \times 100 \tag{20}$$

4. Experimental case study

Fig. 3a shows the outline of part of the TNWDS. The 63 kV/20 kV substation numbered as #2 feeds eleven 20 kV/0.4 kV substations, having a possible route from another 63 kV/20 kV substation #1 for urgent situations. Parameters of the 11 substations are listed by Table 1. Also, Fig. 3b illustrates the single-line diagram of 11 chosen distribution substations along with their locations, rated powers, types of transformers (T_1, T_2, T_3, T_4) and cables (A, B, and C).

Load loggers were installed in the primary of distribution substations, connecting monitoring devices directly to the low-voltage conductors. They have been employed for 7 days in September 2004, started on Sunday the fifth and ended on Saturday the 11th. The collected data were measured 96 times a day (four times an hour), 672 samples per distribution substation. (This procedure is quite expensive as it takes much time to install, and many resources are required.) The collected data include phase voltages, line currents, three power factors, total harmonic distortion (THD) both for voltages and currents, and odd harmonics up to 13th. Then, the empirical load trend data were downloaded to the computer for further study and research.

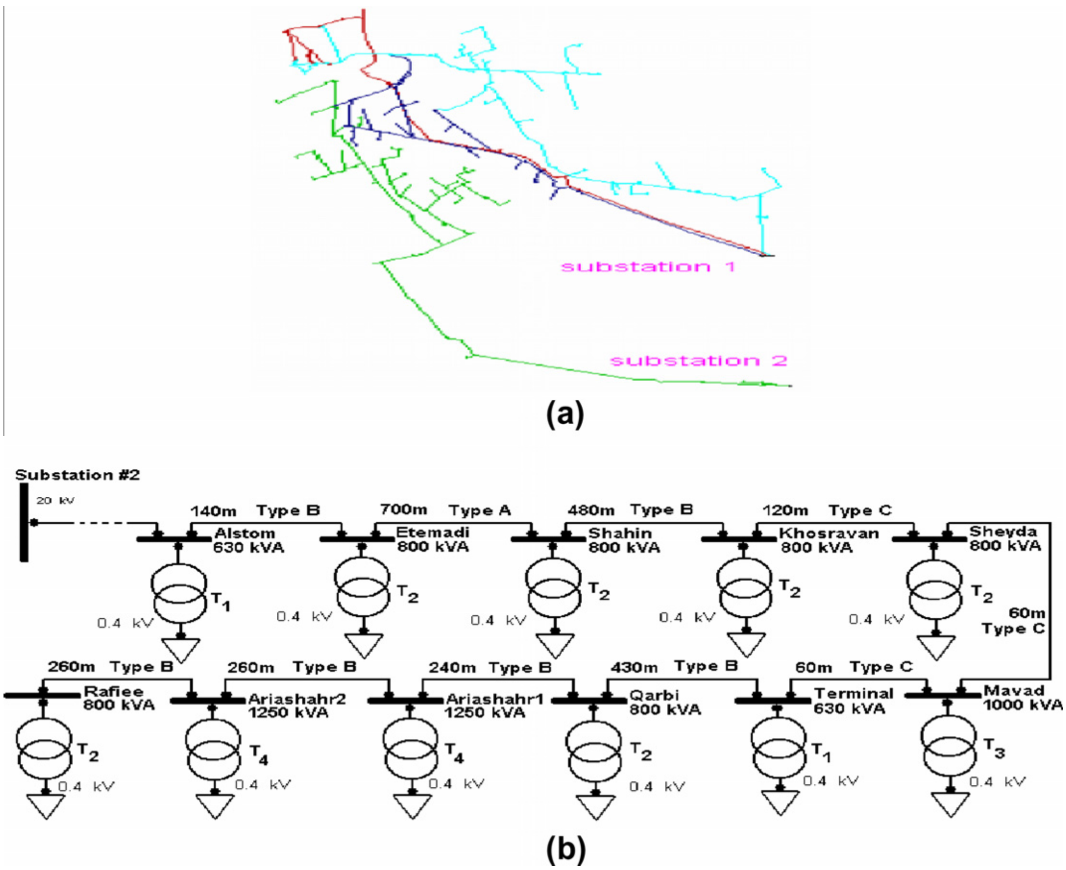

Fig. 3. (a) Outline of 20 kV distribution section within the TNWDS supplied by 63 kV/20 kV substation #2, (b) the voltage profile of the studied 20 kV distribution system, and (c) single-line diagram of the 11 studied 20 kV/0.4 kV substations.

Table 1
Cable and transformer resistances for analysis of their power losses.

Resistances of cables, 30 °C (Ω/m)	A: 7.647×10^{-5}	B: 9.686×10^{-5}	C: 18.862×10^{-5}	
HV-side resistance of transformers (Ω/phase)	T_1: 16.89	T_2: 11.788	T_3: 6.945	T_4: 6.81
LV-side resistance of transformers (Ω/phase)	T_1: 1.876×10^{-3}	T_2: 1.374×10^{-3}	T_3: 1.080×10^{-3}	T_4: 0.840×10^{-3}
LV-side iron-core resistance of transformers (Ω/phase)	T_1: 1.0×10^{6}	T_2: 0.827×10^{6}	T_3: 0.635×10^{6}	T_4: 0.635×10^{6}

4.1. Practical unbalance consequences

Considering (17), (19), and (20), the consequences of unbalance issue can now be analytically related to the unbalance definitions expressed by (3), (4), (9), and (10). It can be seen that, on average, the calculated voltage unbalance of all studied substations is very small; less than 1%, that is in compliance with the IEC standard 61000-2-12, while current unbalance is considerably high. Also, the analysis performed on empirical data related to transformers and line losses together with over-capacity of distribution transformers, discussed in Section 3, suggests relating the unbalance consequences to the current unbalance.

For example, this is illustrated by Fig. 4, where *ULoss*$_1$% and *ULoss*$_2$% are calculated for the collected data from substation *Shahin* and plotted against *MDV*, *TDV*, and *RMSI*. Analytical results confirm that *ULoss*$_1$% and *ULoss*$_2$% correlate to *RMSI* (Fig. 4c) much more appropriately compared to *MDV* and *TDV* (Fig. 4a and b). Examining the exact logged data (e.g. Fig. 4c) also suggests that unbalance consequences can be approximated to a power function of current unbalance like [10]:

$$UI = \alpha \times EU^{\beta} \quad (21)$$

where *EU* is any of the current unbalance definitions, and *UI* could be each of the unbalance consequences under study *ULoss*$_1$%, *ULoss*$_2$% or *OCUT*% that are calculated using the collected data from substations.

Calculation of α and β can be performed using the well-known least mean squares (LMS) method. However, this case study leads to solving a non-linear LMS problem. Thus, here an algorithm is suggested to seek parameters α and β as illustrated by Fig. 5, aiming at looking for the best possible relationship between *UI* and *EU*. Meanwhile, the algorithm is arranged in a way that the solutions adapt those methods employed for finding LMS.

First, the three described *UI* for the 672 sets of logged data are calculated in each distribution substation. This can be managed using (14)–(17). Then, an auxiliary parameter β' is changed from zero to a big number (e.g. 40) by a small step (e.g. $\Delta\beta' = 0.05$). Thus, for every chosen β', there are 672 values for $\mathbf{EU}^{\beta'}$, and 672 values for the ratio $\mathbf{r}_{\beta'} = \frac{UI}{EU^{\beta'}}$. We can now find the standard deviation for the computed data set $\mathbf{r}^{\beta'}$ for each β', providing a set of standard deviations $\{\boldsymbol{\sigma}_{\mathbf{r}_{\beta'}}\}$ corresponding to the swept parameter $\beta' \in [0, 40]$.

The calculated $\{\boldsymbol{\sigma}_{\mathbf{r}_{\beta'}}\}$ can be normalized by their corresponding mean values of $r_{\beta'}$ to achieve a proper measure to compare them. Hence, the lowest normalized standard deviation presents the

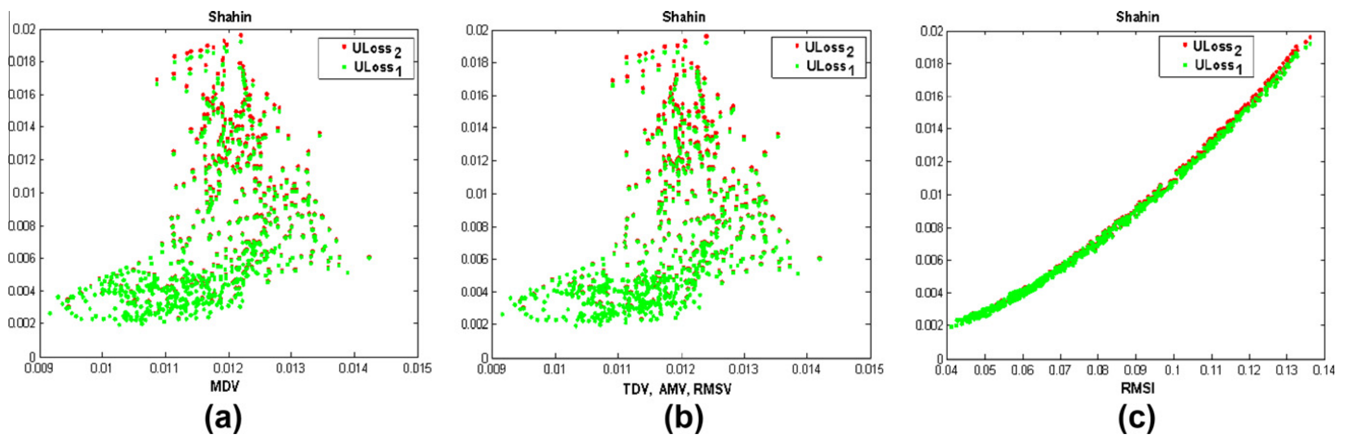

Fig. 4. The calculated $ULoss_1\%$ and $ULoss_2\%$ using 672 collected data from substation *Shahin* to show their correlations with voltage unbalance definitions (a) *MDV* and (b) *TDV*, compared to (c) the current unbalance definition *RMSI*.

seeking optimal parameter β. Furthermore, using the resultant β, the parameter α can be obtained from the mean values of **UI** ($\overline{\mathbf{UI}}$) and $\mathbf{EU}^{\beta}$ ($\overline{\mathbf{EU}^{\beta}}$). Resultant parameters α and β can be expressed by

$$\begin{cases} \sigma_{\mathbf{r}_{\beta}} = \min\{\sigma_{\mathbf{r}_{\beta'}}\}_{\beta'=0:\Delta\beta':40} \mapsto \beta \\ \alpha = \frac{\overline{\mathbf{UI}}}{\overline{\mathbf{EU}^{\beta}}} \end{cases} \tag{22}$$

This algorithm was simulated by MATLAB to fit the best functions of type (18) for the unbalance consequences as follows:

4.1.1. Power losses of substation transformers and transmission lines

Transformer and line losses due to the unbalance operation can be worked out by (11)–(17) for the 11 distribution substations under the study, using the empirical data. Also, different current unbalance definitions are calculated by (3), (4), (9), and (10). Then, the resultant losses are associated to current unbalance definitions in the form of (18), using the fitting algorithm of Fig. 5.

This has been performed for 11 substations, where Fig. 6a–d typically present both the empirical and fitted outcomes of substation *Shahin* for the four current unbalance definitions *TDI*, *MDI*, *AMI*, and *RMSI*, respectively. Table 2 shows the calculated parameters α and β, and correlation factor γ of substation *Shahin* for the four unbalance definitions.

Examining the experimentally analyzed results taken from distribution substations show that:

(1) Transformer and line losses relate to current unbalance much more relevantly than voltage unbalance definitions (Figs. 4 and 6).
(2) The *RMSI* is the best relevant unbalance definition to the power losses amongst the four definitions for all the 11 substations, presenting a current unbalance definition that is properly applicable to both *three-wire* and *four-wire* systems.
(3) The *AMI* is the second best relevant unbalance definition, having complicated formulation and computation compared to the *RMSI*, though.
(4) The NEMA definition (*MDI*) introduces better results compared to the *IEEE TDI* in four-wire systems, showing more suitability for three-wire systems than four-wire distribution networks though.
(5) Since the *IEEE TDI* ignores the presence of zero sequence current, it should be applied to the three-wire systems rather than to the four-wire distribution networks.
(6) Analyzed results of 11 substations show that power losses are as expected, approximated by the *RMSI* squared ($\beta \approx 2$), while α is close to one (see Table 3). This confirms $ULoss_1\%$ given by (18). Other current unbalance definitions introduce more or less dissimilar outcomes though.
(7) Table 3 provides average and standard deviation (σ) of the three parameters α, β and correlation factor γ obtained from the 11 substations, confirming the stated conclusions.

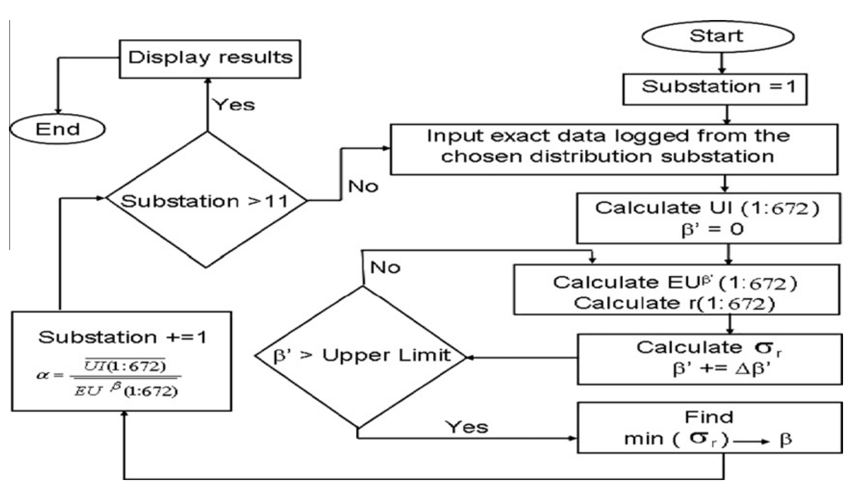

Fig. 5. Flow chart of the fitting algorithm to seek for suitable α and β in (18) for the unbalance consequences.

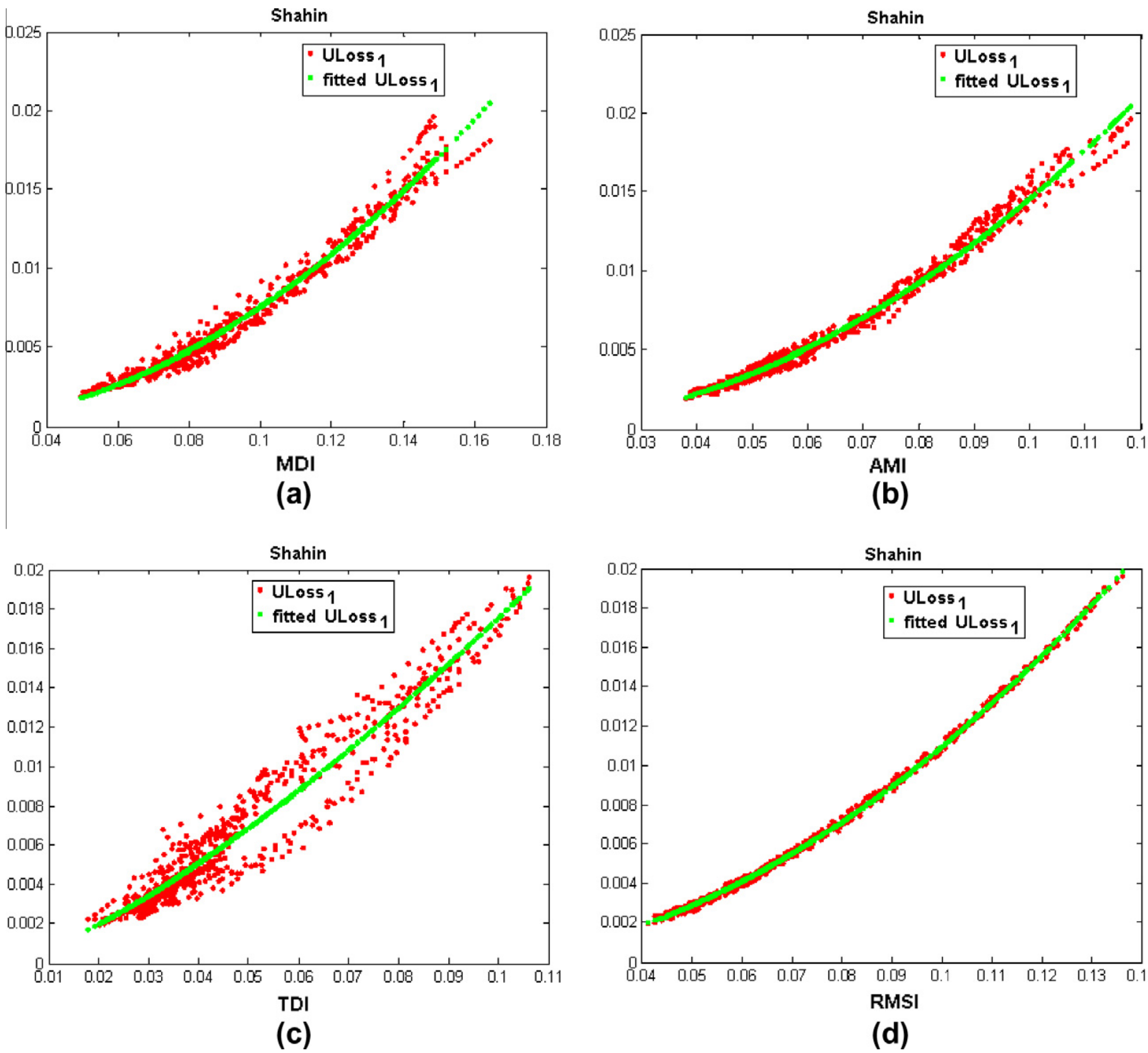

Fig. 6. Experimental and fitted unbalanced losses $ULoss_1$ of substation *Shahin* for the four current unbalance definitions: (a) *MDI*, (b) *TDI*, (c) *AMI*, and (d) *RMSI*.

4.1.2. Over-capacity of unbalanced transformers (OCUT)

Experimental logged data correspond to various unbalanced operating conditions, which in turn relates to different *OCUT*. It is experimentally examined once again that the *OCUT*, like *ULoss*, cannot be fitted properly to *voltage* unbalance of distribution substations.

Thus, the $OCUT_1$, $OCUT_2$ have been evaluated using current unbalance definitions for all 11 substations using (17), (19), and (20). As a typical example, Fig. 7 introduces the calculated over-capacity formulas for the extremely unbalanced substation *Terminal* that are worked out against the four current unbalance definitions *MDI*, *TDI*, *AMI*, and *RMSI*. Every picture includes $OCUT_1$ and $OCUT_2$.

Fig. 7a–d illustrate the calculated $OCUT_1$ and $OCUT_2$ based on (19) and (20), while experimentally analyzed $OCUT_1$ provides better possibility to fit proper function compared to $OCUT_2$. Hence, the algorithm of Fig. 5 was applied to fit $OCUT_1$ of substation *Terminal* to the current unbalance definitions *MDI*, *TDI*, *AMI* and *RMSI*. Fig. 8a–d illustrate both the exact $OCUT_1$ and the fitted $OCUT_1$ of substation *Terminal* versus the four current unbalance definitions. Table 4 also summarizes the correlation factors γ of substation *Terminal* for the two over-capacity formulations.

It can also be seen from Table 4 that $OCUT_1$ provides better fitting parameters compared to $OCUT_2$. Assessing the experimentally analyzed results show that:

Table 2
Fitting parameters of substation *Shahin* for transformer and line losses against the four current unbalance definitions.

Unbalance category	α	β	Correlation factor (γ)
MDI	0.7917	2.0200	0.9887
TDI	0.3939	1.3500	0.9689
AMI	1.5978	2.0400	0.9934
RMSI	0.9151	1.9200	0.9994

Table 3
Average and standard deviation of the three parameters α, β and correlation factor γ calculated for power losses.

Unbalance category	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\gamma}$	σ_α	σ_β	σ_γ
MDI	0.4996	1.7855	0.9770	0.1315	0.1652	0.0300
TDI	1.2970	1.6427	0.9292	0.8090	0.4321	0.1140
AMI	1.2469	1.9755	0.9811	0.2526	0.1555	0.0412
RMSI	1.0330	1.9982	0.9989	0.1030	0.0632	0.0022

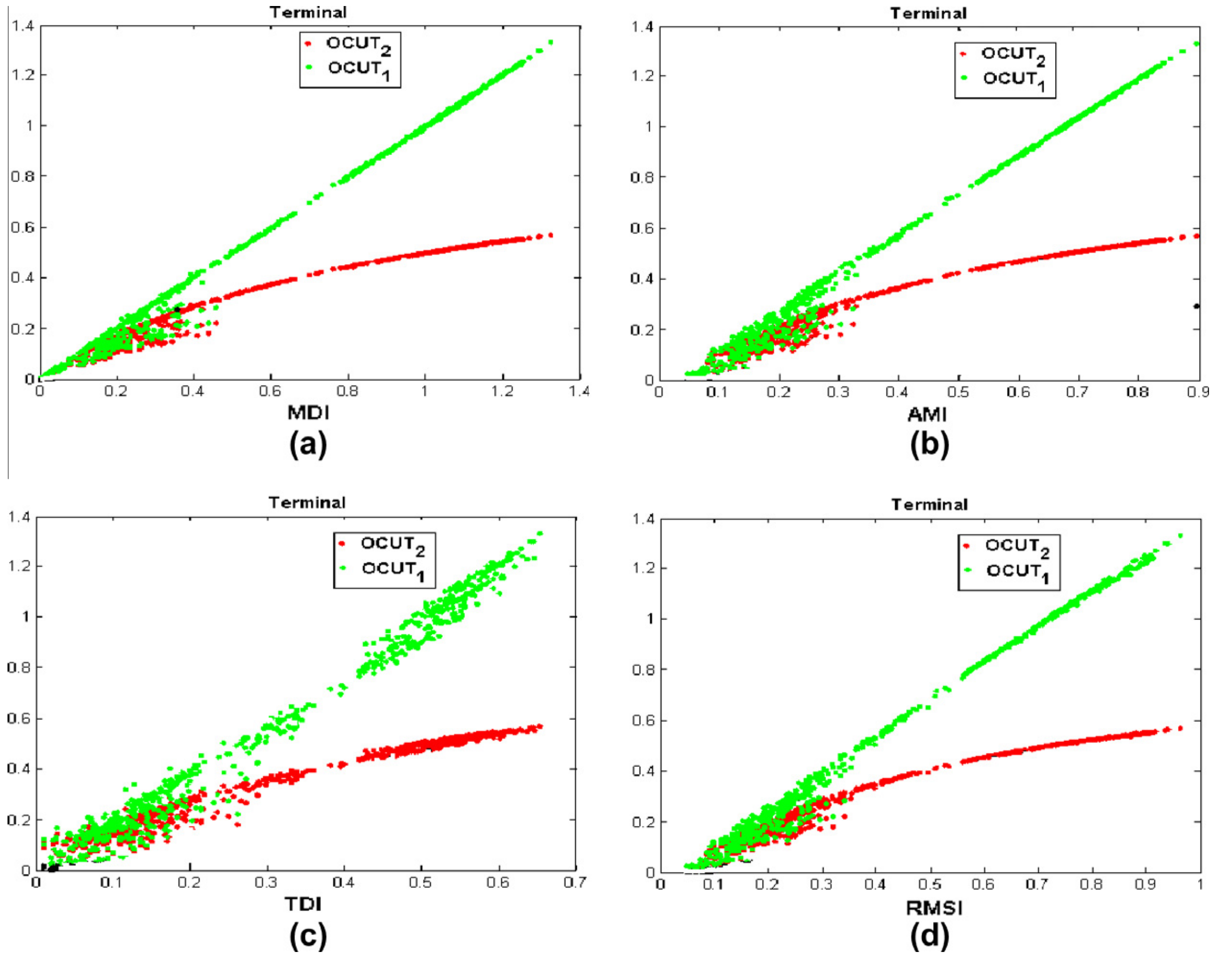

Fig. 7. Experimental exact $OCUT_1$ and $OCUT_2$ of substation *Terminal* for the four current unbalance definitions: (a) *MDI*, (b) *TDI*, (c) *AMI*, and (d) *RMSI*.

(1) Over-capacity of substation transformers are related to current unbalance definitions much more appropriate than voltage unbalance definitions. Meanwhile, experimental results show lower correlation factors for the *OCUT* compared to those of the unbalance power losses *ULoss*.
(2) Once again, the *RMSI* is the best fitted variable amongst the four definitions of all the 11 substations, applicable to both *three-wire* and *four-wire* systems.
(3) The *IEEE TDI* is the second best fitted unbalance definition, suitable for three-wire systems though.
(4) The NEMA definition (*MDI*) introduces close fitting correlation factor to *AMI*.
(5) Among the studied substations, substation *Mavad* introduces an exceptional case in which γ is too poor to fit a proper curve. Hence, excluding this substation from analysis would lead to a bigger $\bar{\gamma}$ than those of Table 5.
(6) Table 5 lists average and standard deviation of the three parameters α, β and correlation factor γ obtained by $OCUT_1$ for the 11 substations.
(7) Table 5 indicates that the $OCUT_1$ can be approximated as a linear function of unbalance definitions ($\bar{\beta} \approx 1$).

4.1.3. Impacts of unbalance compensation on feeders losses

The foregoing discussions explain the contribution of unbalance condition on transmission line losses, transformer losses and over-capacity of transformers. Experimentally analyzed results also showed that it is often possible to find a straight relationship between the current unbalance percentage and the mentioned unbalance consequences (analytically like (18), or statistically like Tables 3 and 4).

Nevertheless, it should be emphasized that unbalance compensation might lead to even a slight increase in *feeder's losses*. This is examined by studying several substations shown by Fig. 3 as follows:

- First, unbalance compensation usually contributes in loss reduction; for example, feeder's losses by unbalance compensation at two heavily loaded substations, *Shahin* and *Mavad*, have been lowered about 15%.
- Second, elimination of unbalance components at the low-voltage bus of substation *Rafiee* causes a slight increase in feeder's losses.

As a result, unbalance compensation of distribution systems have to be studied to seek the optimal locations for compensators to achieve loss reduction objectives.

5. Conclusion

This paper concentrates on the unbalance issue of the three-phase four-wire distribution systems, where zero sequence current is unavoidable. Considering the *IEEE* and NEMA unbalance definitions for three-wire systems, they are complemented to get applicable definitions for distribution networks. A comprehensive practical case study is considered in North-West of Tehran power distribution system, where data loggers were installed at 11

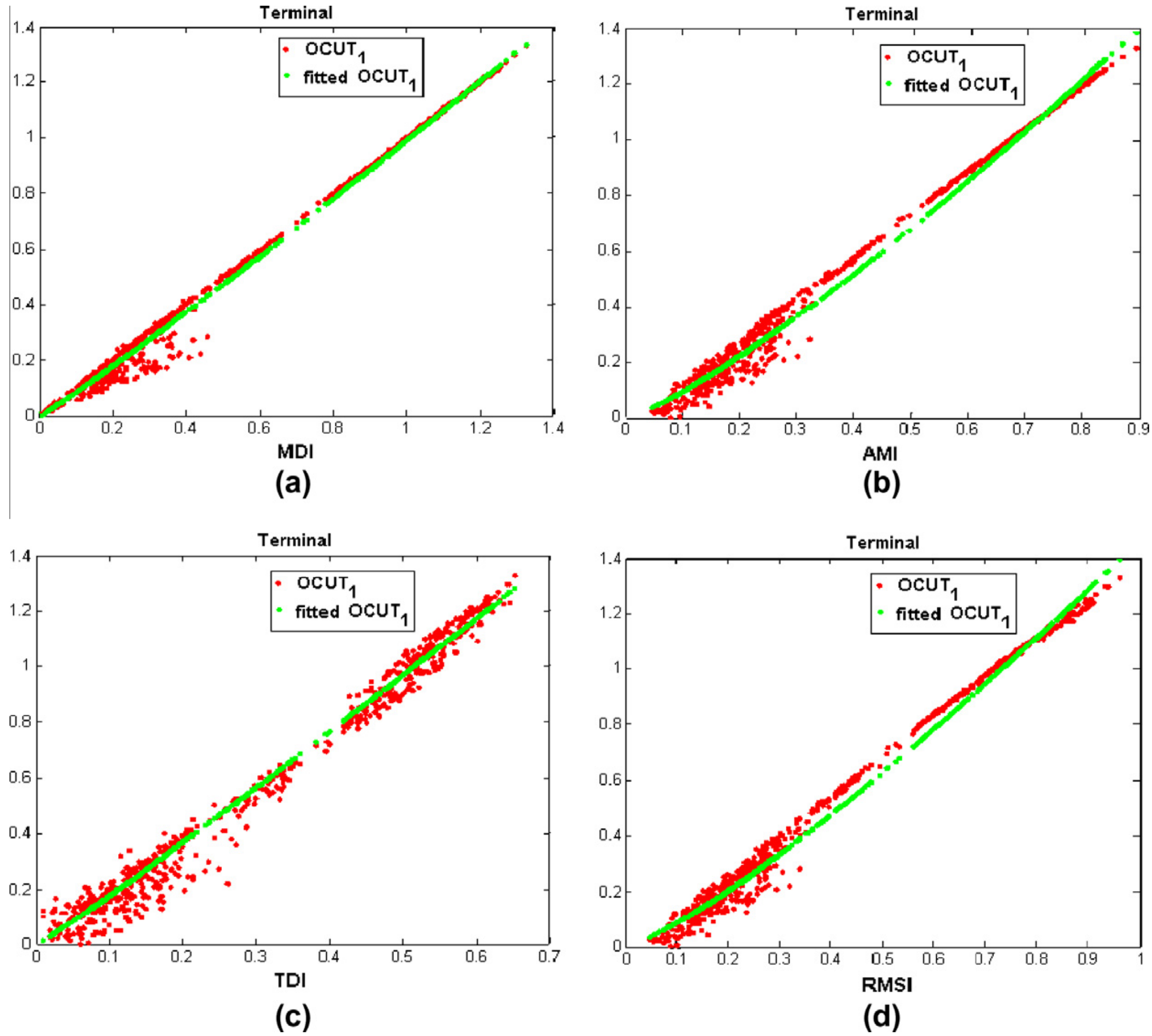

Fig. 8. The exact $OCUT_1$ along with the fitted $OCUT_1$ of substation *Terminal* for the four current unbalance definitions: (a) *MDI*, (b) *TDI*, (c) *AMI*, and (d) *RMSI*.

Table 4
Fitting correlation factors of the extremely unbalanced substation *Terminal* for various over-capacity of its transformer against the four current unbalance definitions.

Unbalance category	Correlation factor of $OCUT_1$	Correlation factor of $OCUT_2$
MDI	0.9969	0.9895
TDI	0.9919	0.9842
AMI	0.9951	0.9829
RMSI	0.9950	0.9832

Table 5
Average and standard deviation of the three parameters α, β and γ calculated for $OCUT_1$.

Unbalance category	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\gamma}$	σ_α	σ_β	σ_γ
MDI	0.9553	0.9809	0.8435	0.0703	0.1158	0.1218
TDI	1.7106	1.0027	0.8752	0.3703	0.0293	0.0253
AMI	1.6322	1.0727	0.8382	0.3339	0.1358	0.1348
RMSI	1.4427	1.1091	0.8829	0.1831	0.0444	0.0463

distribution substations for 1 week in September 2004. Then, consequences of unbalance operating conditions were formulated, analyzing the share of the unbalance issue on transformer and transmission lines losses, as well as the over-capacity of substation transformers. The collected data was used to work out both unbalance consequences and unbalance definitions. Further, these results are related using a suggested optimization algorithm to fit unbalance consequence functions to the available and complemented unbalance definitions. Analysis of the experimentally logged data suggest the root mean squared current unbalance definition as the best fitted variable for unbalance consequence functions, hence applicable to both three-wire and four-wire networks.

Acknowledgments

The support of Tehran North-West Distribution Company is gratefully acknowledged. This work has involved enormous effort by many individuals, impossible to mention all who have contributed. The authors would also like to thank the support of the Reactive Power Research Laboratory of K.N. Toosi University of Technology as well as Mr. Haery who has supported throughout the case study practical management.

References

[1] Bollen MHJ. Definitions of voltage unbalance. IEEE Power Eng Rev 2002;1(November):49–50.

[2] Pillay P, Manyage M. Definitions of voltage unbalance. IEEE Power Eng Rev 2001:50–1.
[3] Jouanne Annette von, Banerjee Basudeb (Ben). Assessment of voltage unbalance. IEEE Trans Power Del 2001;16(4):782–90.
[4] Wang Yaw-Juen, Yang Ming-Jer. Probabilistic modeling of three-phase voltage unbalance caused by load fluctuations. IEEE Power Eng Soc Winter Meet 2000;4(January):2588–93.
[5] Fortescue CL. Method of symmetrical coordinates applied to the solution of polyphase networks. In: 34th Annual convention of the American institute of electrical engineers, vol. 36; June 1918. p. 1027–140.
[6] Office of industrial technologies (energy efficiency and renewable energy). Eliminate voltage unbalance. The US Department of Energy. Motor tip sheet #2; January 2000.
[7] Jeong Seung-Gi. Representing line voltage unbalance. In: IEEE 37th IAS annual meeting, vol. 3; October 2002. p. 1724–32.
[8] Ghijselen Jozef AL, Van den Bossche Alex PM. Exact voltage unbalance assessment without phase measurements. IEEE Trans Power Syst 2005;20(1):519–20.
[9] Bimbhra PS. Electrical machinery (theory, performance and applications). 5th ed. Delhi: Khanna Publishers; 1997.
[10] Kirkpatrick Elwood G. Introductory statistics and probability for engineering, science and technology. Prentice-Hall: New Jersey; 1974.
[11] Duncan Glover J, Sarma Mulukuta. Power system analysis and design. 2nd ed. Boston: PWS Publishing Company; 1994.
[12] Faiz J, Ebrahimpour H. Influence of unbalanced voltage supply on efficiency of three phase squirrel cage induction motor and economic analysis. IEEE Trans Energy Convers Manage 2006;47(March):289–302.
[13] Faiz J, Ebrahimpour H. Precise derating of three-phase induction motors with unbalanced voltages. Energ Convers Manage 2007;48:2579–86.