Three-phase unbalance of distribution systems: Complementary analysis and experimental case study

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Abstract

Three-phase unbalance is a familiar issue for power system researchers and engineers. This can introduce additional power losses in distribution network in steady states due to both negative and zero sequence components. It could also limit the loading capability of distribution transformers, well below their nominal ratings. There are many voltage and current unbalance definitions (e.g., IEEE and NEMA) for three-phase three-wire systems, assuming zero sequence currents to be of negligible practical value, for they cannot flow in three-wire systems. However, the zero sequence unbalance has significant current magnitude in three-phase with four-wire distribution networks, particularly in developing countries. Hence, this paper concentrates on the distribution unbalance, completing the available definitions in order to maintain tangible relationships between the level of unbalance and the cited consequences in distribution networks. Furthermore, practical works were performed on 11 selected 20 kV/0.4 kV substations within Tehran North-West Distribution System (TNWDS), where data loggers have been installed for 7 days to measure and record operating conditions of substations. Then, detailed analysis and assessment are suggested on empirical data to substantiate the presented complementary definitions and relationships.

Keywords: Unbalance, Distribution systems, Substations, Over-capacity, Power losses

1. Introduction

Generating units supply three-phase sinusoidal positive sequence voltages, which are balanced in terms of their amplitudes and 120° phase differences at a single frequency. Any deviations from these two basic characteristics introduce an unbalanced condition. Hence, the terminology of unbalanced can be classified into three main parts; amplitude unbalance of the fundamental, phase difference unbalance of the fundamental, and unbalanced harmonic disturbances [1]. Occurrence of at least one of these features is enough for a distribution network to become unbalanced.

Unbalanced conditions are principally caused by both structural and operational factors [2]. Structural unbalance is usually negligible and can occur due to incomplete transposition of transmission lines and cables, asymmetrical distribution of wiring of transformers, open-A(V-V) or open-Y-open-A connected transformers, capacitor banks, aged fuses, etc. Nevertheless, operational unbalance can be considerable when single-phase and two-phase loads as well as any unbalanced three-phase loads such as arc-furnaces are being supplied by the distribution network [1,3].

The consequences of voltage unbalance are discussed in [1,3,6] on power system equipments such as induction motors, power converters, AC variable speed drive, and power transformers. Comprehensive studies have been carried out in [12,13], describing consequences of unbalance condition on induction motors. It is concluded that the obtained results are very sensitive to the unbalance condition. Moreover, unbalance condition could lower stability margin, increasing the power losses. It is noticeable that a small voltage unbalance might lead to a significant current unbalance because of low negative sequence impedance.

This paper takes a closer look into the unbalance issue of distribution systems, where available definitions are complemented for the four-wire distribution networks. In particular, eleven 20 kV/0.4 kV substations were studied in the TNWDS to assess the consequences of unbalance situations in terms of the developed formulas. This study, further aims at analyzing and relating the unbalance of the distribution networks with the emerged technical consequences (e.g. loading capability and power losses) using the experimental data compiled from the TNWDS. The analysis is performed for all unbalanced definitions and compared thereafter to show the suitability of the suggested complementation for the studied distribution network. In brief, Section 2 describes and analyzes unbalance formulation including both conventional and complementary definitions. Section 3 discusses the unbalance consequences in terms of unbalance definitions, where the theoretical...
conclusions are applied to an experimental case study in Section 4 for further analysis and verifications.

2. Unbalance formulation

Unbalance phenomenon is becoming more a vital issue for distribution systems as various customers with different priorities are appearing, demanding low-cost high-quality electricity. There is a lot of confusion, however, on the meaning of the term voltage/current unbalance, where different standards introduce different conventional definitions as follows (define phase-to-neutral voltages by \(V_{an}, V_{bn}, V_{cn}\), and line-to-line voltages by \(V_{ab}, V_{bc}, V_{ca}\)):

- **IEEE Std. 936 (1987): Phase voltage unbalance rate** [4].
  \[
  \text{IEEE}^\text{936} = \frac{\text{Maximum of } \{V_{an}, V_{bn}, V_{cn}\} - \text{Minimum of } \{V_{an}, V_{bn}, V_{cn}\}}{\text{Mean of } \{V_{an}, V_{bn}, V_{cn}\}} \times 100
  \]

- **IEEE Std. 112 (1991): Modified phase voltage unbalance rate** [4].
  \[
  \text{IEEE}^\text{112} = \frac{\text{Maximum deviation from mean of } \{V_{an}, V_{bn}, V_{cn}\}}{\text{Mean of } \{V_{an}, V_{bn}, V_{cn}\}} \times 100
  \]

- **NEMA (National Electric Manufacturers Associations of the USA) Std. (1993)** [4].
  \[
  \text{MDV} = \frac{\text{Maximum deviation from mean of } \{V_{an}, V_{bc}, V_{ca}\}}{\text{Mean of } \{V_{an}, V_{bc}, V_{ca}\}} \times 100
  \]

- **IEEE true definition (1996)** [4].
  \[
  \text{TDV} = \frac{\text{Negative sequence voltage}}{\text{Positive sequence voltage}} \times 100
  \]

Although, these four definitions could produce different values for a single case; note that current unbalance formulations are simply obtained by replacing voltage with current components (\(\text{IEEE}^\text{936}, \text{IEEE}^\text{112}, \text{MDI}, \text{and } \text{TDI}\) ). The first two standards, \(\text{IEEE}^\text{936}\) and \(\text{IEEE}^\text{112}\), ignore the 120° phase difference unbalance and only take the amplitudes into account. The last two definitions, MDV and TDV, are sensitive to the phase difference unbalance and provide suitable measures of unbalance of three-phase three-wire networks.

Nevertheless, these definitions ignore zero sequence components that are practically unavoidable in three-phase four-wire distribution systems. Fig. 1a illustrates a reproduced unbalance case (reported in [2]), where negative and zero sequence voltages are 3% and 2% of positive sequence voltage respectively. When the relative phase angle of the negative sequence voltage vary within \([0°, 360°]\), \(\text{IEEE}^\text{936}, \text{IEEE}^\text{112}\) and \(MDV\) give different voltage unbalance percents even if no zero sequence voltage is present.

2.1. Complementary unbalance formulation

To establish complementary definitions for three-phase four-wire systems, the following points are carefully taken into account:

- The well-known symmetrical component [5] approach is used to compose complementary definitions.
- For three-wire systems, when no zero sequence current is present, the complementary definitions will give the same results as the IEEE true definition.
- Unbalance share of each phase will be identical on the concluding definitions.

Keeping these points in mind, we can now decompose an unbalanced system into two components; **completely balanced and completely unbalanced** as shown in Fig. 1b. It is also mathematically described as below:

\[
V = V_1 + \Delta V
\]

where

\[
\Delta V = \begin{bmatrix}
\Delta V_a \\
\Delta V_b \\
\Delta V_c
\end{bmatrix}
\]

\[
\begin{align*}
V_1 &= \begin{bmatrix}
V_{1a} \\
V_{1b} \\
V_{1c}
\end{bmatrix} = \begin{bmatrix}
V_{1a} = V_1 \\
V_{1b} = a^2 V_1 \\
V_{1c} = V_1
\end{bmatrix}, \\
\Delta V &= \begin{bmatrix}
\Delta V_a \\
\Delta V_b \\
\Delta V_c
\end{bmatrix} = \begin{bmatrix}
V_{1a} + V_{2a} = V_0 + V_2 \\
V_{1b} + V_{2b} = V_0 + aV_2 \\
V_{1c} + V_{2c} = V_0 + a^2 V_2
\end{bmatrix}
\]

where \(V_1\) is the positive sequence voltage as the **completely balanced** component, \(a = e^{j120°}\), and \(\Delta V\) consists of both negative and zero sequence as the **completely unbalanced** component. Further, the unbalance ratio \(r\) is defined (as a phasor vector):

Fig. 1. (a) Variations of voltage unbalance percent evaluated by \(\text{IEEE}^\text{936}, \text{IEEE}^\text{112}\) and \(MDV\) due to zero sequence voltage when phase of negative sequence vary within \([0°, 360°]\) and (b) decomposition of a general unbalanced system into **completely balanced** and **completely unbalanced** systems.
where \( r_a, r_b, r_c \) introduce the contribution of each phase from the completely unbalanced component. But, these need to be combined together to get an unbalance representation for the three-phase four-wire networks. Hence, considering (5) and (6), the following possible combinations can be examined:

- **Choice 1**: Mean of \( \{r_a, r_b, r_c\} \).
  \[
  MV = \frac{r_a + r_b + r_c}{3}
  \]  
  (7)

- **Choice 2**: Geometric mean of \( \{r_a, r_b, r_c\} \).
  \[
  GMV = \sqrt[3]{r_a \times r_b \times r_c} = \frac{1}{V_1} \sqrt{V_0^2 + V_2^2}
  \]  
  (8)

- **Choice 3**: Geometric mean of \( \{|r_a|, |r_b|, |r_c|\} \).

- **Choice 4**: Geometric root mean squares of \( \{|r_a|, |r_b|, |r_c|\} \).

- **Choice 5**: Mean of \( \{|r_a|, |r_b|, |r_c|\} \).
  \[
  AMV = \frac{|r_a| + |r_b| + |r_c|}{3} = \frac{|V_0 + V_2| + |V_0 + a \times V_2| + |V_0 + a^2 \times V_2|}{3|V_1|}
  \]  
  (9)

- **Choice 6**: Root mean squares of \( \{|r_a|, |r_b|, |r_c|\} \).
  \[
  RMSV = \sqrt{\frac{|r_a|^2 + |r_b|^2 + |r_c|^2}{3}} = \sqrt{\frac{|V_0|^2 + |V_2|^2}{|V_1|^2}}
  \]  
  (10)

Complemented current unbalance can be obtained by replacing voltage with current components (ML, GMI, AMI, RMSI), as well. Considering (6), the first choice cannot represent a real unbalance value because the right hand side of (7) is always equal to zero. Also, when \( |V_0 + V_2| \) is about zero, the resulting unbalance value of the second choice in (8) will be about zero despite the presence of system unbalance. Choices three and four lead to relationships that are equivalent to the second choice, where any unbalance ratios about zero would result in unreal unbalance value. The last two choices provide reasonable definitions especially choice 6 that present a simple relationship for analysis and calculations. Therefore, examination and assessment of these choices ([9] and [10]) will be performed using the practical compiled data obtained from the studied distribution substations.

3. Unbalance consequences

Three-phase unbalance imposes certain consequences on the distribution networks, including power losses of substation transformers and lines along with the viable capacity of transformers out of their nominal balanced ratings. Power system losses refer to the amount of power lost in various processes and steps of delivery from generation to customer since the devices that transfer energy have physical resistance to the energy flow. Thus, the unbalance of power systems contributes to the intensification of physical process of losing energy. Moreover, when a distribution transformer is operating under unbalance condition, then the transformer will be unable to serve up to its nominal ratings because of unequal current magnitudes. Most distribution companies purchase wholesale electricity at transmission voltage levels. The companies then step the voltage down to primary distribution levels through their own substation transformers. This section intends to analyze and express the share of unbalance of distribution net-
where $K_1 = \frac{V_a I_a}{I_1}$, $K_2 = \frac{V_b I_b}{I_2}$, and $K_3 = \frac{V_c I_c}{I_3}$. It can be shown that 20 kV/0.4 kV transformers of Δ-Y type produce no zero sequence voltage neither at the low-voltage Y-end nor at 20 kV three-wire line-end [7,8]. Hence, considering $V_0 \approx 0$ and neglecting $\left|\frac{V_0}{V}\right|^2$ that is practically very small (the average of $\left|\frac{V_0}{V}\right|$ during a week was found to be about 0.0077 for 11 studied substations), the balanced system losses using unbalance compensation is approximated to $BLoss = 3R|I|^2$ (experimental case study verifies the stated approximation). Therefore, the share of unbalance losses out of total transformer and line losses is

$$\Delta Loss = ULoss - BLoss = 3R(|I_1|^2 + |I_2|^2 + |I_3|^2) + 9R_0|I_0|^2$$  \hspace{1cm} (16)$$

where $\Delta Loss$ can be compared with both $BLoss (ULoss_{1\%})$ and $ULoss (ULoss_{2\%})$ as follows:

$$ULoss_{1\%} = \frac{ULoss_{1\%}}{BLoss} = \frac{3R(|I_1|^2 + |I_2|^2 + |I_3|^2) + 9R_0|I_0|^2}{3R|I|^2} \times 100$$

$$ULoss_{2\%} = \frac{ULoss_{2\%}}{BLoss} = \frac{ULoss_{2\%}}{3R|I|^2} \times 100$$  \hspace{1cm} (17)$$

It should be noted that the defined $R_0$ is practically very small compared to $R$, leading to approximate relationships like

$$ULoss_{1\%} = RMSI_1^2\%$$

$$ULoss_{2\%} = RMSI_2^2\%$$  \hspace{1cm} (18)$$

Moreover, by expansion of Taylor series for $ULoss_{2\%}$ in (18), it can be shown that when $RMSI$ is small, then $ULoss_{1\%}$ and $ULoss_{2\%}$ are approximately equal.

### 3.3. Loading capability of unbalanced transformers

Unbalance condition reduces the actual loading capability of a distribution transformer, where the maximum current of the three phases determines the remaining capacity of transformer. In this situation, the substation transformer may not be able to supply the load as much as it is designed to and as high as the intended rate, which is called over-capacity of unbalanced transformer (OCUT) in this paper. In practice, when the maximum current reaches the overload zone, even if the other two phases operate well below their rated currents the protective alarm could turn on. Also, as the OCUT tends towards zero in a balanced operation, the OCUT could be considered as a measure of unbalance of distribution feeders at every steady state operation point. The lower the OCUT, the more balanced operates the distribution substation. Additionally, lowering the high OCUT of substation transformers postpones their proper resizing and emplacement, which is a necessary task in the process of loss reduction and economic operation of distribution systems.

A solution to formulate the OCUT is to calculate the unbalance loading of transformer (ULT) on the assumption that the network is operating under three-phase balanced currents $I_{in}$, $\omega(S_{max})$ such that $ULT = 3S_{max}$. Then, the equivalent balanced loading of the transformer (BLT) is calculated when the load unbalance is compensated for, such that the source currents become balanced ($BLT = S_a + S_b + S_c$, see Fig. 2b). The OCUT can be expressed by:

$$OCUT_{1\%} = \frac{ULT - BLT}{BLT} \times 100$$  \hspace{1cm} (19)$$

Alternatively, $OCUT_{2\%}$ can be evaluated with respect to ULT as:

$$OCUT_{2\%} = \frac{ULT - BLT}{ULT} \times 100$$  \hspace{1cm} (20)$$

### 4. Experimental case study

Fig. 3a shows the outline of part of the TNWDS. The 63 kV/20 kV substation numbered as #2 feeds eleven 20 kV/0.4 kV substations, having a possible route from another 63 kV/20 kV substation #1 for urgent situations. Parameters of the 11 substations are listed by Table 1. Also, Fig. 3b illustrates the single-line diagram of 11 chosen distribution substations along with their locations, rated powers, types of transformers ($T_1, T_2, T_3, T_4$) and cables ($A, B, C$).

Load loggers were installed in the primary of distribution substations, connecting monitoring devices directly to the low-voltage conductors. They have been employed for 7 days in September 2004, started on Sunday the fifth and ended on Saturday the 11th. The collected data were measured 96 times a day (four times an hour), 672 samples per distribution substation. (This procedure is quite expensive as it takes much time to install, and many resources are required.) The collected data include phase voltages, line currents, three power factors, total harmonic distortion (THD) both for voltages and currents, and odd harmonics up to 13th. Then, the empirical load trend data were downloaded to the computer for further study and research.
4.1. Practical unbalance consequences

Considering (17), (19), and (20), the consequences of unbalance issue can now be analytically related to the unbalance definitions expressed by (3), (4), (9), and (10). It can be seen that, on average, the calculated voltage unbalance of all studied substations is very small; less than 1%, that is in compliance with the IEC standard 61000-2-12, while current unbalance is considerably high. Also, the analysis performed on empirical data related to transformers and line losses together with over-capacity of distribution transformers, discussed in Section 3, suggests relating the unbalance consequences to the current unbalance.

For example, this is illustrated by Fig. 4, where $U_{\text{Loss},1}$ and $U_{\text{Loss},2}$ are calculated for the collected data from substation Shahin and plotted against $MDV$, $TDV$, and $RMSI$. Analytical results confirm that $U_{\text{Loss},1}$ and $U_{\text{Loss},2}$ correlate to $RMSI$ (Fig. 4c) much more appropriately compared to $MDV$ and $TDV$ (Fig. 4a and b). Examining the exact logged data (e.g. Fig. 4c) also suggests that unbalance consequences can be approximated to a power function of current unbalance like [10]:

$$UI = \alpha \times EU^\beta$$  \hspace{1cm} (21)

where $EU$ is any of the current unbalance definitions, and $UI$ could be each of the unbalance consequences under study $U_{\text{Loss},1}$, $U_{\text{Loss},2}$ or $OCUT$% that are calculated using the collected data from substations.

Calculation of $\alpha$ and $\beta$ can be performed using the well-known least mean squares (LMS) method. However, this case study leads to solving a non-linear LMS problem. Thus, here an algorithm is suggested to seek parameters $\alpha$ and $\beta$ as illustrated by Fig. 5, aiming at looking for the best possible relationship between $UI$ and $EU$. Meanwhile, the algorithm is arranged in a way that the solutions adapt those methods employed for finding LMS.

First, the three described $UI$ for the 672 sets of logged data are calculated in each distribution substation. This can be managed using (14)-(17). Then, an auxiliary parameter $\beta'$ can be changed from zero to a big number (e.g. 40) by a small step (e.g. $\Delta \beta' = 0.05$). Thus, for every chosen $\beta'$, there are 672 values for $EU^\beta'$, and 672 values for the ratio $r_{\beta'} = \frac{UI}{EU}$. We can now find the standard deviation for the computed data set $r_{\beta'}$ for each $\beta'$, providing a set of standard deviations ($\sigma_{\beta'}$) corresponding to the swept parameter $\beta' \in [0, 40]$.

The calculated $\sigma_{\beta'}$ can be normalized by their corresponding mean values of $r_{\beta'}$ to achieve a proper measure to compare them. Hence, the lowest normalized standard deviation presents the
seeking optimal parameter $\beta$. Furthermore, using the resultant $\beta$, the parameter $a$ can be obtained from the mean values of $UI$ ($UI^b$) and $EU^b$ ($EU^b$). Resultant parameters $a$ and $b$ can be expressed by

$$\begin{align*}
\sigma_{r_b} &= \min\{\sigma_r\} \beta^{-0.5} 4^{1/2} \\
\alpha &= \frac{b}{EU^b}
\end{align*}$$

(22)

This algorithm was simulated by MATLAB to fit the best functions of type (18) for the unbalance consequences as follows:

4.1.1. Power losses of substation transformers and transmission lines

Transformer and line losses due to the unbalance operation can be worked out by (11)–(17) for the 11 distribution substations under the study, using the empirical data. Also, different current unbalance definitions are calculated by (3), (4), (9), and (10). Then, the resultant losses are associated to current unbalance definitions in the form of (18), using the fitting algorithm of Fig. 5.

This has been performed for 11 substations, where Fig. 6a–d typically present both the empirical and fitted outcomes of substation Shahin for the four current unbalance definitions TDI, MDI, AMI, and RMSI, respectively. Table 2 shows the calculated parameters $a$ and $b$, and correlation factor $\gamma$ of substation Shahin for the four unbalance definitions.

Examining the experimentally analyzed results taken from distribution substations show that:

1. Transformer and line losses relate to current unbalance much more relevantly than voltage unbalance definitions (Figs. 4 and 6).
2. The RMSI is the best relevant unbalance definition to the power losses amongst the four definitions for all the 11 substations, presenting a current unbalance definition that is properly applicable to both three-wire and four-wire systems.
3. The AMI is the second best relevant unbalance definition, having complicated formulation and computation compared to the RMSI, though.
4. The NEMA definition (MDI) introduces better results compared to the IEEE TDI in four-wire systems, showing more suitability for three-wire systems than four-wire distribution networks though.
5. Since the IEEE TDI ignores the presence of zero sequence current, it should be applied to the three-wire systems rather than to the four-wire distribution networks.
6. Analyzed results of 11 substations show that power losses are as expected, approximated by the RMSI squared ($b^2/C^2$), while $a$ is close to one (see Table 3). This confirms $ULoss_1$ given by (18). Other current unbalance definitions introduce more or less dissimilar outcomes though.
7. Table 3 provides average and standard deviation ($\sigma$) of the three parameters $a$, $b$, and correlation factor $\gamma$ obtained from the 11 substations, confirming the stated conclusions.

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Fig. 4. The calculated $ULoss_1$ and $ULoss_2$% using 672 collected data from substation Shahin to show their correlations with voltage unbalance definitions (a) MDV and (b) TDV, compared to (c) the current unbalance definition RMSI.

Fig. 5. Flow chart of the fitting algorithm to seek for suitable $a$ and $b$ in (18) for the unbalance consequences.
4.1.2. Over-capacity of unbalanced transformers (OCUT)

Experimental logged data correspond to various unbalanced operating conditions, which in turn relates to different OCUT. It is experimentally examined once again that the OCUT, like ULoss, cannot be fitted properly to voltage unbalance of distribution substations.

Thus, the OCUT₁, OCUT₂ have been evaluated using current unbalance definitions for all 11 substations using (17), (19), and (20). As a typical example, Fig. 7 introduces the calculated over-capacity formulas for the extremely unbalanced substation Terminal that are worked out against the four current unbalance definitions MDI, TDI, AMI, and RMSI. Every picture includes OCUT₁ and OCUT₂.

Fig. 7a–d illustrate the calculated OCUT₁ and OCUT₂ based on (19) and (20), while experimentally analyzed OCUT₁ provides better possibility to fit proper function compared to OCUT₂. Hence, the algorithm of Fig. 5 was applied to fit OCUT₁ of substation Terminal to the current unbalance definitions MDI, TDI, AMI and RMSI. Fig. 8a–d illustrate both the exact OCUT₁ and the fitted OCUT₁ of substation Terminal versus the four current unbalance definitions. Table 4 also summarizes the correlation factors γ of substation Terminal for the two over-capacity formulations.

It can also be seen from Table 4 that OCUT₁ provides better fitting parameters compared to OCUT₂. Assessing the experimentally analyzed results show that:

<table>
<thead>
<tr>
<th>Unbalance category</th>
<th>a</th>
<th>b</th>
<th>Correlation factor (γ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDI</td>
<td>0.7917</td>
<td>2.0200</td>
<td>0.9887</td>
</tr>
<tr>
<td>TDI</td>
<td>0.3939</td>
<td>1.3500</td>
<td>0.9689</td>
</tr>
<tr>
<td>AMI</td>
<td>1.5978</td>
<td>2.0400</td>
<td>0.9934</td>
</tr>
<tr>
<td>RMSI</td>
<td>0.9151</td>
<td>1.9200</td>
<td>0.9994</td>
</tr>
</tbody>
</table>

Table 3

Average and standard deviation of the three parameters a, b and correlation factor γ calculated for power losses.

<table>
<thead>
<tr>
<th>Unbalance category</th>
<th>a</th>
<th>b</th>
<th>γ</th>
<th>σa</th>
<th>σb</th>
<th>σγ</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDI</td>
<td>0.4996</td>
<td>1.7855</td>
<td>0.9770</td>
<td>0.1315</td>
<td>0.1652</td>
<td>0.0300</td>
</tr>
<tr>
<td>TDI</td>
<td>1.2570</td>
<td>1.6427</td>
<td>0.9292</td>
<td>0.8090</td>
<td>0.4321</td>
<td>0.1140</td>
</tr>
<tr>
<td>AMI</td>
<td>1.2469</td>
<td>1.9755</td>
<td>0.9811</td>
<td>0.2526</td>
<td>0.1555</td>
<td>0.0412</td>
</tr>
<tr>
<td>RMSI</td>
<td>1.0330</td>
<td>1.9982</td>
<td>0.9989</td>
<td>0.1030</td>
<td>0.0632</td>
<td>0.0022</td>
</tr>
</tbody>
</table>
(1) Over-capacity of substation transformers are related to current unbalance definitions much more appropriate than voltage unbalance definitions. Meanwhile, experimental results show lower correlation factors for the OCUT compared to those of the unbalance power losses ULoss.

(2) Once again, the RMSI is the best fitted variable amongst the four definitions of all the 11 substations, applicable to both three-wire and four-wire systems.

(3) The IEEE TDI is the second best fitted unbalance definition, suitable for three-wire systems though.

(4) The NEMA definition (MDI) introduces close fitting correlation factor to AMI.

(5) Among the studied substations, substation Mavad introduces an exceptional case in which γ is too poor to fit a proper curve. Hence, excluding this substation from analysis would lead to a bigger γ than those of Table 5.

(6) Table 5 lists average and standard deviation of the three parameters α, β and correlation factor γ obtained by OCUT for the 11 substations.

(7) Table 5 indicates that the OCUT can be approximated as a linear function of unbalance definitions (β ≈ 1).

4.1.3. Impacts of unbalance compensation on feeders losses

The foregoing discussions explain the contribution of unbalance condition on transmission line losses, transformer losses and over-capacity of transformers. Experimentally analyzed results also showed that it is often possible to find a straight relationship between the current unbalance percentage and the mentioned unbalance consequences (analytically like (18), or statistically like Tables 3 and 4).

Nevertheless, it should be emphasized that unbalance compensation might lead to even a slight increase in feeder’s losses. This is examined by studying several substations shown by Fig. 3 as follows:

- First, unbalance compensation usually contributes in loss reduction; for example, feeder’s losses by unbalance compensation at two heavily loaded substations, Shahin and Mavad, have been lowered about 15%.
- Second, elimination of unbalance components at the low-voltage bus of substation Rafiee causes a slight increase in feeder’s losses.

As a result, unbalance compensation of distribution systems have to be studied to seek the optimal locations for compensators to achieve loss reduction objectives.

5. Conclusion

This paper concentrates on the unbalance issue of the three-phase four-wire distribution systems, where zero sequence current is unavoidable. Considering the IEEE and NEMA unbalance definitions for three-wire systems, they are complemented to get applicable definitions for distribution networks. A comprehensive practical case study is considered in North-West of Tehran power distribution system, where data loggers were installed at 11...
distribution substations for 1 week in September 2004. Then, consequences of unbalance operating conditions were formulated, analyzing the share of the unbalance issue on transformer and transmission lines losses, as well as the over-capacity of substation transformers. The collected data was used to work out both unbalance consequences and unbalance definitions. Further, these results are related using a suggested optimization algorithm to fit unbalance consequence functions to the available and complemented unbalance definitions. Analysis of the experimentally logged data suggest the root mean squared current unbalance definition as the best fitted variable for unbalance consequence functions, hence applicable to both three-wire and four-wire networks.

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**References**


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**Table 4**

<table>
<thead>
<tr>
<th>Unbalance category</th>
<th>Correlation factor of $OCUT_1$</th>
<th>Correlation factor of $OCUT_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MDI</td>
<td>0.9969</td>
<td>0.9895</td>
</tr>
<tr>
<td>TDI</td>
<td>0.9919</td>
<td>0.9842</td>
</tr>
<tr>
<td>AMI</td>
<td>0.9951</td>
<td>0.9829</td>
</tr>
<tr>
<td>RMSI</td>
<td>0.9950</td>
<td>0.9832</td>
</tr>
</tbody>
</table>

**Table 5**

Average and standard deviation of the three parameters $\alpha$, $\beta$ and $\gamma$ calculated for $OCUT_1$.

<table>
<thead>
<tr>
<th>Unbalance category</th>
<th>$\alpha$</th>
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