K.N. Toosi University of Technology

Faculty of Mathematics
Department of Applied Mathematics

Ph.D. Thesis in Applied Mathematics

Title

A Generalization of Lagrange Interpolation and Its Applications

By

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Abstract

This thesis deals mainly with several topics related to interpolation and how they are employed in numerical analysis. It begins with an overview of the general formalism and the aspects of interpolation problems that are required throughout the thesis. This includes history, applications and generalization of interpolation problems.

The first new result is a functional generalization of Lagrange interpolation which is motivated by the definition of the Lagrange polynomials. Existence and uniqueness of the interpolating function in addition to remainder formula are considered for the introduced interpolation class. This class of interpolation formulae captures the main features of interpolation and covers some well known classes such as polynomial, cosine polynomial, Müntz polynomial and exponential interpolation as well as interpolation by rational functions which all can be observed in explained particular cases. Also, as an application we introduce a new type of weighted interpolatory quadrature rules. Based on this new interpolation formula under some assumptions and by introducing generalized divided difference operator we construct the so-called generalized Newton functions to derive an extension of Newton interpolation formula. We also consider the interpolation problem at coincident points which leads to an extension of Taylor interpolation. This generalization also allows the construction of an extension of Taylor series expansion. Specifically, by using Bell polynomials we demonstrate how some special cases of this generalization lead to many new series and identities. We also apply the extended expansion for generating functions of some famous sequences of numbers. In another point of view, by using a variant of the classical integration by parts we obtain a new type of Taylor series expansion. In this direction, as an application we include some closed forms for integrals containing Jacobi and Laguerre polynomials, which cannot be directly obtained by usual symbolic computation programs, i.e. only some very specific values can be computed by the mentioned programs. An error analysis is given in the sequel for the introduced expansion.

Finally, we mention two numerical methods which are derived from the interpolation scheme. In the first method, a new application of Hermite interpolation is studied to solve ordinary differential equations numerically. A recent representation of Hermite interpolation enables us to derive a modification of multistep methods based on Hermite quadrature rule for solving initial value problems. A special focus lies on modification of Adams-Bashforth methods. This procedure yields a method that enjoys higher order and wider stability region in comparison to standard Adams-Bashforth method when the number of steps is considered equal in both methods. In a similar manner, weighted type of Hermite quadrature rules are employed to develop modified Adams-Bashforth methods with the purpose to solve a special case of singular Cauchy problem. The second discussed numerical method concerns a class of weighted quadratures which can be directly connected to the Gaussian quadratures and thus determined by its corresponding nodes and weights. We include several particular examples and finally some comments on the error analysis are considered.

Keywords: polynomial and nonpolynomial interpolations, Hermite interpolation, generalized Lagrange and Newton interpolations, generalized divided differences, Taylor series expansion, Adams-Bashforth method, weighted quadrature rule.