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Employing the Spectral Methods for Solving Ordinary and Partial Differential Equations by the Mapped Basic Functions

A Thesis Presented for the Degree of
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Abstract

Ordinary and partial differential equations have an important role in modelling different phenomena. But obtaining exact solution is not possible in most of the cases. Consequently, it is necessary to use efficient numerical methods to get a suitable approximate solution. Among all of numerical methods we can refer to finite difference, finite element and finite volume methods.

In the past four decades, spectral methods have been developed rapidly and paid attention because of their high accuracy. Several spectral methods are employed to problems on finite intervals. On the other hand, many problems arising in fluid dynamics, astrophysics, financial mathematics and other fields are set in unbounded domains. Many spectral methods have been developed for solving such problems.

This thesis is organized as follows: In the first chapter we present some definitions and preliminaries about spectral methods. In the second chapter, which is extracted from [54] as the main reference, some results on the Jacobi rational approximation are established that play important roles in analyzing the Jacobi rational spectral methods. Also in this chapter, from [54], Galerkin method based on Jacobi rational functions is presented for the following model problem which includes important equations in financial mathematics:

$$\begin{cases} \partial_t V(x,t) + \partial_x(a(x,t)\partial_x V(x,t)) + b(x,t)\partial_x V(x,t) - c(x,t)V(x,t) = F(x,t), \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad x \in \Lambda = (0,+\infty), t \in [0,T), \\ V(x,T) = V_0(x), \qquad\qquad\qquad\qquad\quad x \in \overline{\Lambda}. \end{cases}$$

The convergence of the proposed method is proved and numerical results are discussed. In the last chapter, we employ collocation method in order to solve semi-linear heat equation with Neumann boundary conditions and discuss about the convergence and stability of the method. Some numerical examples are solved in order to illustrate the performance of the presented method.

Keywords: *spectral method, Galerkin method, collocation, Jacobi rational function.*