Fourier Series User Guide

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Autumn 2017

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1 Requirements

For running this application on your desktop computer, you must install JRE (Java Runtime Environment) on your computer. For more information on installing Java, refer to our guide on installing java.

2 Brief Description Of Fourier Series[1]

Fourier series are infinite series that represent periodic functions in terms of cosines and sines. As such, Fourier series are of greatest importance to the engineer and applied mathematician. To define Fourier series, we first need some background material. A function $f(x)$ is called a periodic function if $f(x)$ is defined for all real x, except

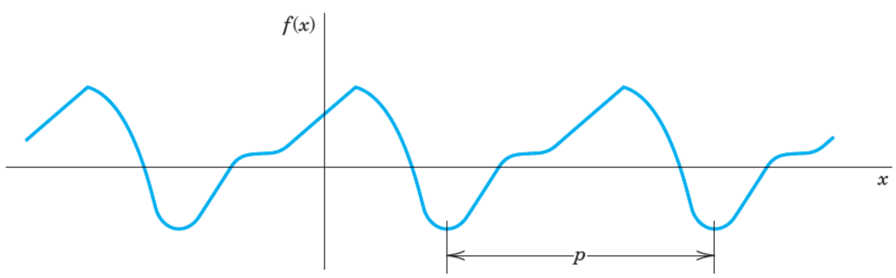

Figure 1: Periodic function of period p

possibly at some points, and if there is some positive number p, called a period of $f(x)$, such that

$$(1) \qquad f(x+p) = f(x) \qquad \text{for all } x$$

(The function $f(x) = tanx$ is a periodic function that is not defined for all real x but undefined for some points (more precisely, countably many points), that is $x = \pm\pi/2, \pm 3\pi/2, \ldots.$)

[1] Advanced Engineering Mathematics 10th Edition, Erwin Kreyszig

The graph of a periodic function has the characteristic that it can be obtained by periodic repetition of its graph in any interval of length p.

The smallest positive period is often called the fundamental period.

Familiar periodic functions are the cosine, sine, tangent, and cotangent. Examples of functions that are not periodic are $x, x^2, x^3, e^x, coshx, andlnx$, to mention just a few.

If$f(x)$ has period p, it also has the priod $2p$.; thus for any integer $n = 1, 2, 3, \ldots,$

$$(2) \qquad f(x + np) = f(x) \qquad \text{for all } x$$

Furthermore if $f(x)$ and $g(x)$ have period p, then $af(x) + bg(x)$ with any constants a and b also has the period p.

Our problem in the first few sections of this chapter will be the representation of various ***functions*** $f(x)$ ***of period*** 2π in terms of the simple functions

$$(3) \qquad 1, \qquad cosx, \qquad sonx \qquad cos2x, \qquad sin2x, \ldots, \qquad cosnx, \qquad sinnx, \ldots.$$

All these functions have the period 2π. They form the so-called trigonometric system. 2shows the first few of them (except for the constant 1, which is periodic with any period).

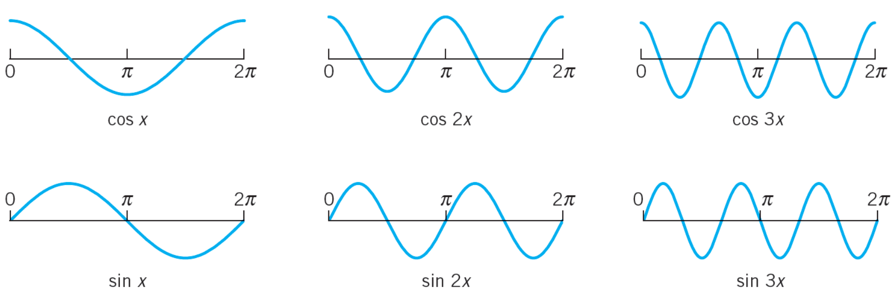

Figure 2: Cosine and sine functions having the period 2 (the first few members of the trigonometric system (3), except for the constant 1)

The series to be obtained will be a **trigonometric series** trigonometric series, that is, a series of the form

$$a_0 + a_1 cosx + b_1 sinx + a_2 cos2x + b_2 sin2x + \ldots$$

(4)

$$= a_0 + \sum_{n=1}^{\infty} (a_n cosnx + b_n sinnx).$$

$a_0, a_1, b_1, a_2, b_2, \ldots$ are constants, called the **coefficients** of the series. We see that each term has the period 2π. ence if the coefficients are such that the series converges, its sum will be a function of period 2π.

Expressions such as (4) will occur frequently in Fourier analysis. To compare the expression on the right with that on the left, simply write the terms in the summation. Convergence of one side implies convergence of the other and the sums will be the same. Now suppose that $f(x)$ is a given function of period 2π and is such that it can be **represented** by a series (4), that is, (4) converges and, moreover, has the sum $f(x)$. Then, using the equality sign, we write

$$(5) \qquad f(x) = \sum_{n=1}^{\infty}(a_n cosnx + b_n sinnx)$$

and call (5) the **Fourier series** of $f(x)$. We shall prove that in this case the coefficients of (5) are the so-called **Fourier coefficients** of $f(x)$, given by the **Euler formulas**

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$(6) \qquad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cosnxdx \qquad n = 1, 2, \ldots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sinnxdx \qquad n = 1, 2, \ldots$$

The name "Fourier series" is sometimes also used in the exceptional case that (5) with coefficients (6) does not converge or does not have the sum $f(x)$ —this may happen but is merely of theoretical interest.

3 How To Use The Application

For using this application you must download it from the followed link: https://wp.kntu.ac.ir/aliakbarian/pde/visualizations/fourier-transform/FourierFX.jar

When you run the application something like figure 3 will be shown to you on the screen.

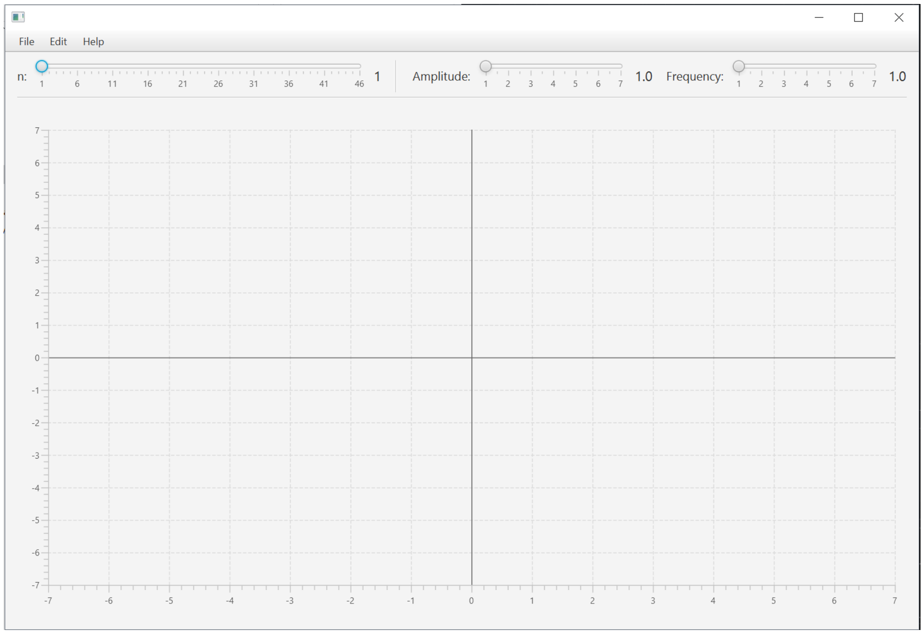

Figure 3: First Perspective

There are some buttons on top of the application, in File menu, you can save the result of your test as, a picture, by next button you can choose your function arbitrarily, and by the last one you can see some information about the application.

For starting you must choose one function, for example, I choose Half Wave, this is shown in 4.

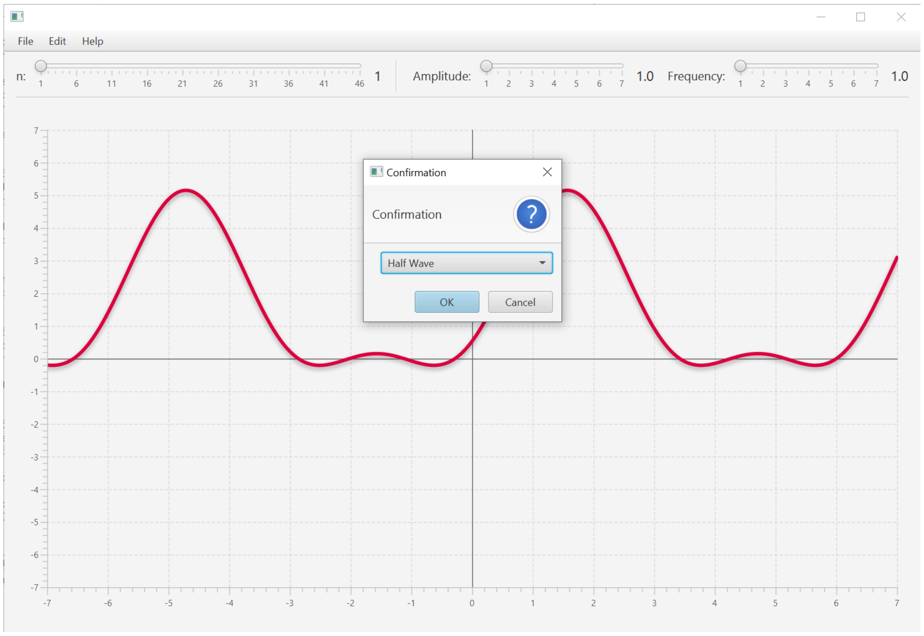

Figure 4: Half Wave Is Chosen

finally you have three sliders in application, n, Amplitude, and Frequency like 5.

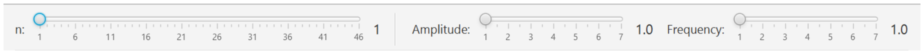

Figure 5: Sliders

The first slider indicates numbers of terms you want to add for calculating Fourier series and drawing plot and as more terms you add, the plot delivered to you is much like your function like 6.

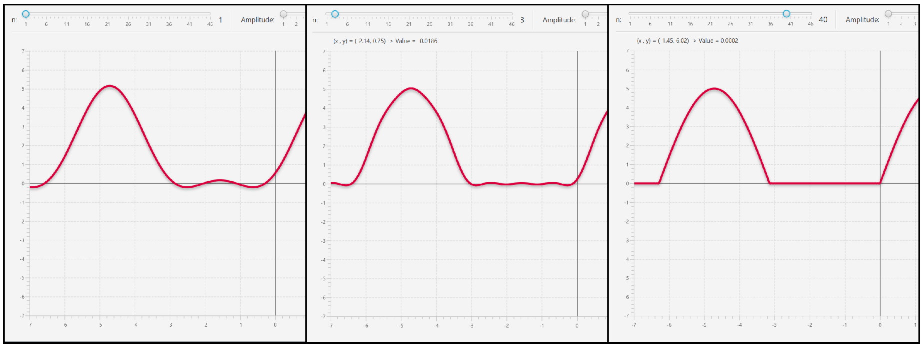

Figure 6: More Terms Give You A Better Plot

Amplitude slider can modify the amplitude of Fourier series plot like 7.

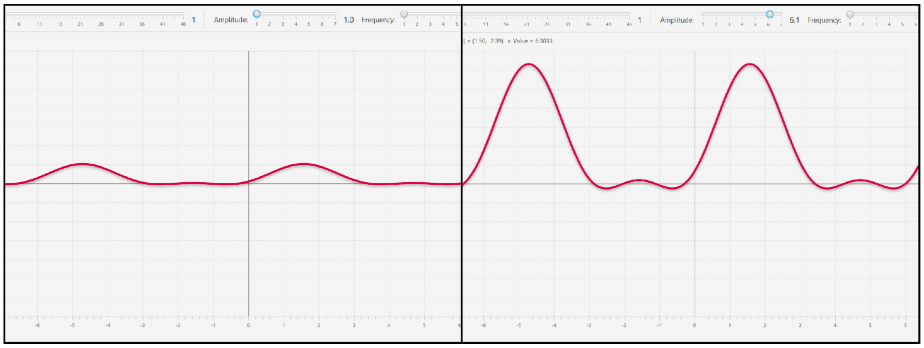

Figure 7: You Can Change Amplitude Of Plot

And by frequency sliders, you be able to change frequency of plot like 8.

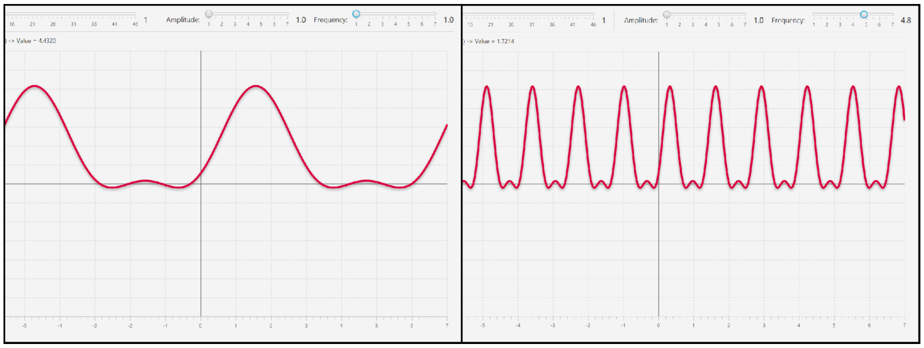

Figure 8: You Can Change Frequency Of Plot

4 About Us

User guide is written by Mahdi Kafi on 11/16/2017
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