تأسیس ۱۳۰۷
دانشگاه صنعتی خواجه نصیرالدین طوسی

Adaptive Filters **Lecturer: Ali Bastami**

Homework 4 **Due date: Azar 3, 1398**

1. Consider a simple AWGN channel where the transmitted signal is corrupted by an additive white noise process as

$$u(n) = s(n) + v(n)$$

where $s(n)$ is the transmitted signal, $u(n)$ is the output signal of the channel and $v(n)$ is the additive white noise process. Assume that $v(n)$ is independent of $s(n)$ and all the processes are real. Also assume that $E\{v(n)\} = 0$, $\sigma_v^2 = 0.1$, $r_s(k) = E\{s(n)s(n-k)\} = 2^{-|k|}$, $E\{s(n)\} = 0$. At the receiver, the signal $u(n)$ is passed through a finite impulse response filter such that the output SNR is maximized. As you know, the eigen filter maximizes the output SNR.

(a) Determine the impulse response of the eigen filter for length $M = 2$, $M = 4$, $M = 8$, $M = 10$. Also determine the corresponding output SNR.

(b) Generate $N = 10000$ samples of the signal and noise with the statistics given above. Evaluate the output SNR by passing the signal and noise through the eigen filters found in part (a).

2. Consider a Wiener filtering problem characterized by the following values for the correlation matrix R of the tap-input vector $\underline{\mathrm{u}}(n)$ and the cross-correlation vector $\underline{\mathrm{p}}$ between $\underline{\mathrm{u}}(n)$ and the desired response $d(n)$:

$$\mathrm{R} = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$$

$$\underline{\mathrm{p}} = [0.5 \ \ 0.25]^T$$

(a) Suggest a suitable range of values for the step-size parameter μ that would ensure convergence of the method of steepest descent, based on the given value for matrix R.

(b) For $\mu = 1$, determine the recursions for computing the tap-weight vector $\underline{\mathrm{w}}(n)$. Assume that the initial condition is equal to zero, i.e. $\underline{\mathrm{w}}(0) = \underline{0}$. For sufficiently large values of n compare $\underline{\mathrm{w}}(n)$ with the optimum coefficients of Wiener filter, i.e. $\underline{\mathrm{w}}_{opt}$.

(c) Let $\sigma_d^2 = 1$. Plot the learning curve for different values of the step-size parameter μ and compare the rate of convergence. By inspecting these curves, in each case, determine the value of $J(n)$ as $n \to \infty$. Compare these values with $J_{min} = \sigma_d^2 - \underline{\mathrm{p}}^H \mathrm{R}^{-1} \underline{\mathrm{p}}$.

3. The steepest descent algorithm becomes unstable when the step size parameter μ is assigned a negative value. Justify the validity of this statement.

4. In the method of steepest descent, show that the correction applied to the tap-weight vector after $n + 1$ iterations may be expressed as

$$\delta \underline{\mathrm{w}}(n+1) = \mu E\{\underline{\mathrm{u}}(n) e^*(n)\}$$

where $\underline{\mathrm{u}}(n)$ is the tap-input vector and $e(n)$ is the estimation error. What happens to this adjustment at the minimum point of the error performance surface? Discuss your answer in light of the principle of orthogonality.