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Controlling the Deflection of Steel Cantilever Beam Using Pre-tensioning Cable

*** Nader Fanaie [1], Fatemeh Partovi [2]**

Abstract

Despite appropriate design of beams under bending and shear, the deflection of long steel beams usually exceeds the allowable range, and therefore the structural designers encounter challenges in this regard. Considering significant features of the cables, namely, low weight, small cross section, and high tensile strength, they are used in this research so as to control the deflection of beams, rather than increasing their heights, and obtain acceptable responses. In this study, for the first time, theoretical relation is developed to calculate the increase in pre-tensioning force of steel cables under external loading based on the method of least work as well as the deflection of steel cantilever beam with cable based on the virtual work method. Moreover, required cross-sectional area of steel cable has been calculated to reach allowable deflection in steel cantilever beam with cable. To verify the theoretical relations, the steel cantilever beam is modeled in the finite element ABAQUS software without cable and with cable. The obtained results show that the theoretical relations can appropriately predict the deflection of cantilever beam with cable. Furthermore, in bending moment diagrams of cantilever beam without cable and with cable, if the cable leads to neutral axis at free end of the cantilever beam, the bending moment of cantilever beam with cable will not increase compared to that of cantilever beam without cable.

Keywords:

Deflection, Steel cantilever beam, Cable, Pre-tensioning

1. Introduction

Cables as important components of structure are materials which can tolerate the tensile force and generally increase the stiffness and bearing capacity of the structure (Razavi and Sheidaii 2012). Nowadays, the cables are increasingly used in the structures. Hou and Tagawa (2009) applied cable-cylinder bracing in the

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*[1] . Associate Professor, Department of Civil Engineering, K. N. Toosi University of Technology, Tehran, Iran, Email: fanaie@kntu.ac.ir (responsible author)

[2]. Master of Science, Department of Civil Engineering, K. N. Toosi University of Technology, Tehran, Iran, Email: he.partovi@gmail.com

seismic retrofitting of steel flexural frames. From their view point, through this retrofitting method, the lateral strength of the storey augments without decreasing the ductility of flexural frame. Fanaie et al. (2016a) presented the theoretical relations for cable-cylinder bracing system using rigid cylinder like steel cylinder. They verified the results by finite element ABAQUS software. Fanaie et al. (2016b) also studied seismic behavior of steel flexural frames strengthened with cable-cylinder bracing, and obtained reasonable results. Pre-tensioning the steel beams through high strength cables is one of the most efficient methods to decrease the required steel, and to increase their bearing capacity. Pre-tensioning technique has been primarily used in reinforced concrete structures; however, for the first time, it was utilized by Dischinger and Magnel in steel beams. Pre-tensioned steel structures are constructed all over the world, especially in America, Russia, and Germany. This fact shows the structural and economic merits of pre-stressed steel beams compared to non-prestressed ones. Pre-tensioning technique is appropriate for constructing new structures as well as strengthening the existing ones (Troitsky 1990).

Some researchers have studied pre-stressed composite beams using steel cable. Ayyub et al. (1990, 1992) assessed pre-stressed steel-concrete composite beams experimentally as well as analytically using steel cable in the regions of positive and negative bending moments. They concluded that pre-tensioning increases the ultimate strength. Nie et al. (2007) presented theoretical relations to calculate the deflection as well as yield and ultimate moments of simply supported pre-stressed steel-concrete composite beam considering the slip effect. They verified the suggested formulas with the experimental results. Pre- stressed steel beams equipped with steel cables have been investigated by some researchers. Troitsky (1990) evaluated the behavior of pre-stressed steel beam using cables, and observed the increase in the stiffness and decrease in the deformation of the beam. Belletti and Gasperi (2010) studied the behavior of pre-stressed simply supported steel I-shaped beams by tendons with focusing on two parameters, namely, the number of deviators and the value of prestressing force. Park et al. (2010) analytically and experimentally evaluated the flexural behavior of steel I-beam pre-stressed with externally unbonded tendons. They figured out considerable increase in the yielding and ultimate bearing capacity of steel I-beam.

Since controlling the deflection of beam, especially long beams, confronts the structural designers with challenge, this research focuses on the deflection of steel cantilever beam with cable. The increase of pre-tensioning force of steel cable subjected to external loading is determined using method of least work. Then, method of virtual work is applied to develop the deflection relation of steel cantilever beam equipped with cable. In order to validate the obtained deflection relation, the results of theoretical relation is compared with its of finite element model of the beam.

2. Pre-tensioning symmetric I-shaped steel cantilever beam with steel cable

As shown in Fig. 1, pre-stressed cables have been used in both sides of beam web, and subjected to external loading.

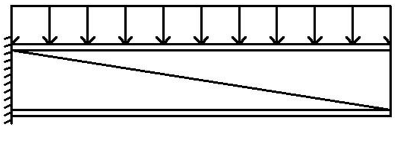

Figure 1. Pre-stressed symmetric I-shaped steel cantilever beam with steel cable under external loading

The following assumptions are taken into account to analyze pre-stressed symmetric I-shaped steel cantilever beam with steel cable:

1- The materials of steel beam and cable are linearly elastic;

2-The deformations are small;

3- Shear deformation is not considered;

4- The friction loss in the region of cable deformation and the relaxation of steel cable are ignored;

5- Steel beam section is rolled; therefore, it is compact;

3. Increase of pre-tensioning force of the steel cable in a beam under external loading

The cable length increased by ΔL , and its pre-tensioning force, F_{pt}, increased by ΔF , under uniform distributed loading. As the structure is statically indeterminate, the static equilibrium equations are not enough to calculate ΔF . The increase of the force in the cable can be calculated using the method of least work.

The total strain energy of the beam with cable caused by its bending moment and axial force as well as the strain energy of the cable owing to its axial force are determined to calculate the increase in pre-tensioning force of the cable through the method of least work. Then, the relation of total strain energy is differentiated with respect to ΔF and the result is equated to zero to obtain the relation to increase the pre-tensioning force of the cable (ΔF).

3.1. Calculating the increase in pre-tensioning force of steel cable in the cantilever beam along with cable

As the slope of steel cable is constant in the cantilever beam with cable (as shown in Fig. 2), the increase in pre-tensioning force of steel cable is equal to ΔF . Therefore, the axial force of the beam is equal to $\Delta F \cos\theta$. Moreover, the bending moment of cantilever beam along with cable subjected to uniform distributed loading (Fig. 2) is obtained to calculate the strain energy caused by bending.

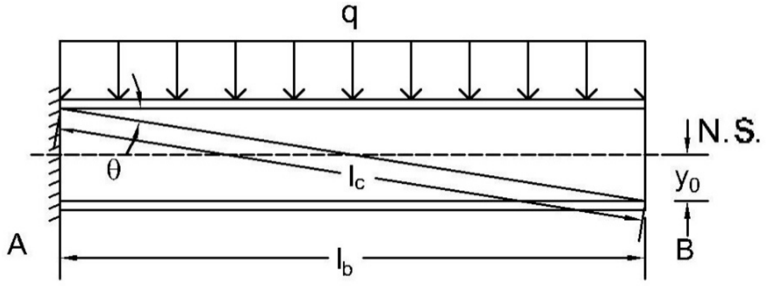

Figure 2. Cantilever beam along with cable

For $0 \le x \le l_b$ range:

$$M(x)=-\Delta F\sin\theta x+\Delta F\cos\theta y_0-\frac{qx^2}{2}+ql_bx-\frac{ql_b^{\,2}}{2} \tag{1}$$

The total Strain energy equation is written as follows:

$$\begin{aligned}U&=\frac{1}{2(EI)_b}\int_0^{l_b}\left(-\Delta F\sin\theta x+\Delta F\cos\theta y_0-\frac{qx^2}{2}+ql_bx-\frac{ql_b^{\,2}}{2}\right)^2dx+\frac{\Delta F^2l_c}{2(AE)_c}\\&+\frac{(\Delta F\cos\theta)^2l_b}{2(AE)_b}=\frac{1}{2(EI)_b}\left\{\begin{matrix}\dfrac{q^2l_b^{\,5}}{20}+\dfrac{\Delta F^2l_b^{\,3}\sin^2\theta}{3}+\Delta F^2l_by_0^{\,2}\cos^2\theta\\-\Delta F^2l_b^{\,2}y_0\sin\theta\cos\theta+\dfrac{q\Delta Fl_b^{\,4}\sin\theta}{12}-\dfrac{q\Delta Fl_b^{\,3}y_0\cos\theta}{3}\end{matrix}\right\}\\&+\frac{\Delta F^2l_c}{2(AE)_c}+\frac{\Delta F^2l_b\cos^2\theta}{2(AE)_b}\end{aligned} \tag{2}$$

In order to calculate the increase in pre-tensioning force of the cable (ΔF) through the method of least work, the relation of whole strain energy is differentiated with respect to ΔF and the obtained result is equated to zero:

$$\frac{\partial U}{\partial(\Delta F)}=0 \tag{3}$$

The relation for calculating the increase of pre-tensioning force of the cable (ΔF) is obtained as follows:

$$\Delta F=\frac{-ql_b^{\,4}\sin\theta+4ql_b^{\,3}y_0\cos\theta}{8\left(l_b^{\,3}\sin^2\theta+3l_by_0^{\,2}\cos^2\theta-3l_b^{\,2}y_0\sin\theta\cos\theta+\dfrac{3(EI)_bl_c}{(AE)_c}+\dfrac{3I_bl_b\cos^2\theta}{A_b}\right)} \tag{4}$$

Where, q is the intensity of uniform distributed load; l_b and l_c are the lengths of beam and inclined cable, respectively; A_b and A_c are the cross sections of the beam and cable at both sides of the web, respectively; E_b and E_c are modulus of elasticity of the beam and cable, respectively; I_b is the moment of inertia of steel section; y_0 is the distance of neutral surface to the connection point of steel cable to the beam flanges (half of the height of beam web); and θ is the angle between the inclined cable and horizontal axis.

4. Deflection

Virtual work method can be used to calculate the deflection of cantilever beam without cable and with cable, ignoring the effects of shear and axial forces.

4.1. Maximum deflection of cantilever beam without cable under uniform distributed loading

If the length and flexural rigidity of the beam are l_b and $(EI)_b$, respectively, maximum deflection of cantilever beam without cable under uniform distributed loading q is calculated as follows:

The deflection of the end of cantilever beam without cable:

$$\Delta_{end}=\frac{ql_b^{\ 4}}{8\left(EI\right)_b} \tag{5}$$

4.2. Calculating maximum deflection of cantilever beam along with cable

Assuming the force of cable to be equal to F, the bending moment of cantilever beam along with cable is obtained under real loading (presented in Fig. 2) as follows:

For $0 \le x \le l_b$ range:

$$M\left(x\right)=-F\sin\theta x+F\cos\theta y_0-\frac{qx^2}{2}+ql_bx-\frac{ql_b^{\ 2}}{2} \tag{6}$$

Where, $F=F_{pt}+\Delta F$ is the total force of the cable; F_{pt} is the pre-tensioning force of the cable; and ΔF is the increase of pre-tensioning force of the cable.

In analyzing the structure under virtual loading, if the structure is indeterminate, its constraints can be eliminated up to being converted to a stable determinate structure. In the beam and cable system, the cable is a redundancy and can be omitted in analyzing the structure under virtual loading. The bending moment of cantilever beam is obtained under unit virtual load (Fig. 3) as follows:

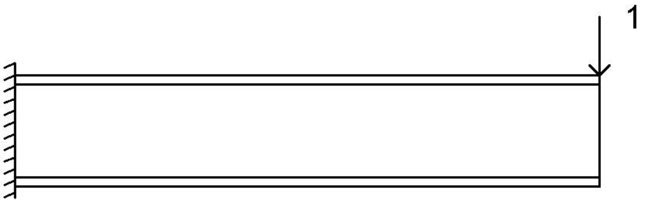

Figure 3. Cantilever beam under unit virtual load

For $0 \le x \le l_b$ range:

$$m\left(x\right)=x-l_b \tag{7}$$

The deflection at the end of cantilever beam along with cable is calculated as follows:

$$\begin{aligned}1\times\Delta&=\int\frac{M\left(x\right)m\left(x\right)}{EI}dx=\frac{1}{\left(EI\right)_b}\int_0^{l_b}\left(-F\sin\theta x+F\cos\theta y_0-\frac{qx^2}{2}+ql_bx-\frac{ql_b^{\ 2}}{2}\right)\left(x-l_b\right)dx\\&=\frac{1}{\left(EI\right)_b}\left[\frac{ql_b^{\ 4}}{8}+\frac{Fl_b^{\ 3}\sin\theta}{6}-\frac{Fl_b^{\ 2}y_0\cos\theta}{2}\right]\end{aligned} \tag{8}$$

5. Calculating the required cross-sectional area of steel cable to reach allowable deflection

In this section, the required cross-sectional area of steel cable with specified pre-tensioning force F_{pt} is calculated for steel cantilever beam in which the maximum deflection is not within the allowable range so that by adding cable, the maximum deflection of the beam along with cable reach allowable amount.

5.1. Calculating the required cross-sectional area of steel cable to reach the allowable deflection in the cantilever beam along with cable

Calculating the required cross-sectional area of steel cable with specified pre-tensioning force F_{pt} to reach the allowable deflection, the maximum deflection of cantilever beam along with cable, according to Eq. (8) by replacing Eq. (5), must be equal to the allowable amount Δ_a as follows:

$$\Delta_a = \Delta_{end} + \frac{1}{(EI)_b}\left[\frac{Fl_b^3 \sin\theta}{6} - \frac{Fl_b^2 y_0 \cos\theta}{2}\right] \tag{9}$$

From Eq. (9), the total cable force is obtained as follows:

$$F = \frac{12(EI)_b(\Delta_{end} - \Delta_a)}{-2l_b^3 \sin\theta + 6l_b^2 y_0 \cos\theta} = \alpha(\Delta_{end} - \Delta_a) \tag{10}$$

In Eq. (10), α is as follows:

$$\alpha = \frac{12(EI)_b}{-2l_b^3 \sin\theta + 6l_b^2 y_0 \cos\theta} \tag{11}$$

The increase in pre-tensioning force of cable ΔF according to Eq. (4) is obtained as follows:

$$\Delta F = \frac{-ql_b^4 \sin\theta + 4ql_b^3 y_0 \cos\theta}{8\left(l_b^3 \sin^2\theta + 3l_b y_0^2 \cos^2\theta - 3l_b^2 y_0 \sin\theta\cos\theta + \frac{3(EI)_b l_c}{(AE)_c} + \frac{3I_b l_b \cos^2\theta}{A_b}\right)} = \frac{\beta}{\gamma + \frac{\mu}{A_c}} \tag{12}$$

In Eq. (12), β , γ and μ are as follows:

$$\beta = -ql_b^4 \sin\theta + 4ql_b^3 y_0 \cos\theta \tag{13}$$

$$\gamma = 8\left(l_b^3 \sin^2\theta + 3 l_b y_0^2 \cos^2\theta - 3 l_b^2 y_0 \sin\theta\cos\theta + \frac{3 I_b l_b \cos^2\theta}{A_b} \right) \tag{14}$$

$$\mu = 8\left(\frac{3(EI)_b l_c}{E_c} \right) \tag{15}$$

From Eq. (12), the required cross-sectional area of cable A_c is obtained as follows:

$$A_c = \frac{\mu \Delta F}{\beta - \gamma \Delta F} \tag{16}$$

The increase in pre-tensioning force of cable ΔF with regards to total cable force is obtained in accordance with Eq. (10) as follows:

$$\Delta F = F - F_{pt} = \alpha(\Delta_{end} - \Delta_a) - F_{pt} \tag{17}$$

By replacing Eq. (17) in Eq. (16), the required cross-sectional area of cable with specified pre-tensioning force F_{pt} to reach the allowable deflection in the cantilever beam along with cable is obtained as follows:

$$A_c = \frac{\mu(\alpha(\Delta_{end} - \Delta_a) - F_{pt})}{\beta - \gamma(\alpha(\Delta_{end} - \Delta_a) - F_{pt})} \tag{18}$$

6. Finite element modeling of steel cantilever beam pre-stressed with steel cable

Cantilever beam has been designed based on Load and Resistance Factor Design (LRFD) method using AISC360-10 code (2010). Cantilever beam has been designed in such a way that the maximum deflection under dead and live loads is greater than the allowable deflection ($\frac{1}{240}$ of the beam length). Table 1 presents the property of the cantilever beam as well as related allowable and maximum deflection under service load. It should be noted that the length of loading span is 1.5 m for the beam; dead and live loads are 450 and 200 kg/m^2 respectively.

Table 1. Property and deflection of cantilever beam

Type of beam	Beam span length(m)	Cross-section of beam	Maximum deflection (cm)	Allowable deflection (cm)
Cantilever beam	2	IPB120	1.128	0.833

The cantilever beam without cables and with cables were modeled in ABAQUS finite element software. Fig. 4 presents finite element model of the cantilever beam along with cable. The beam and cables have been modeled in 3-dimensional space with shell and truss elements (as wire) respectively. The weld's connector is used to connect the cable to the top flange of the beam at two ends providing a perfect connection between two nodes. Moreover, coupling constraint is used to connect the cable to the bottom flange of the beam. Uniform distributed load is applied as a surface traction type on the top flange. Predefined field tool is used to create the initial pre-tensioning stress in the cable as well. The initial pre-tensioning stress of the cable decreases prior to applying a uniform distributed load due to not being the rigid beam. The amount of the initial pre-tensioning stress is considered to be much greater in ABAQUS software to reach the desired pre-stressing value after its loss. Fig. 5 presents the locations of cables in the cantilever beam.

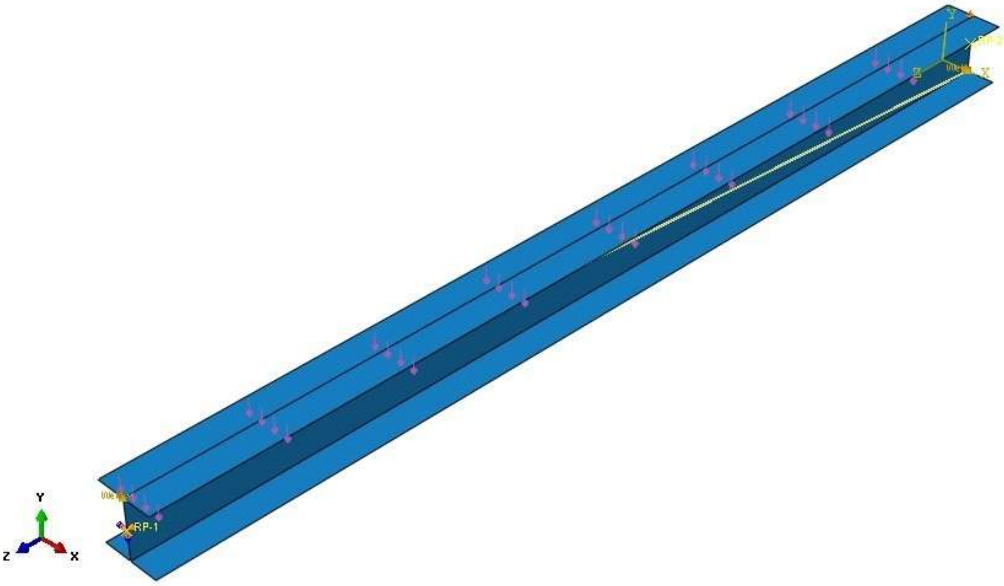

Figure 4. Finite element model of the cantilever beam along with cable

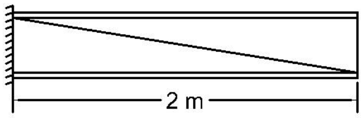

Figure 5. The locations of cables in the cantilever beam along with cable

For better presenting the behavior of cantilever beam with cable, first it has been modeled in the software without cable, and then with cable; and the obtained results have been compared with each other.

The steel material of beam considered in this research is ST-37; yield stress is 240MPa; modulus of elasticity of steel is 200 GPa; and Poisson's ratio is 0.3. The material of steel cable is in accordance with the ASTM A416M standard. 7-wire strand (grade 270 (1860)) is considered for steel cable with low relaxation, minimum ultimate strength (f_{pu}) of 270 ksi (1860 MPa), minimum yield strength at 1% extension of 52.74 kip (234.6 kN), elasticity module of 28.5×10^6psi (196501.8 MPa) and Poisson's ratio of 0.3.

Adding cable to the beam converts it to beam-column because the horizontal component of the cable force creates an axial force in the beam. To prevent the beam buckling about the longitudinal axis, the cantilever beam with 1 lateral brace at the end of the beam. The beams are designed according to AISC360-10 so as to take the simultaneous effects of axial force and bending moment into account.

7. Verification of theoretical relations of deflection with results of ABAQUS models

Static general analysis of ABAQUS software has been used to analyze the catilever beam (Table 1), without cable and with cable. The cross-sectional area of steel cable is considered as 7-wire strand with low relaxation for cantilever beam with equal numbers of cables at both sides of the webs with the cross section area of 74.19 mm^2, for each cable. The total cross-section area of steel cables is 297 mm^2.These values are considered according to ASTM A416 standard. Pre-tensioning of the steel cable is considered as 600 MPa. Controlling the accuracy of theoretical relation, maximum deflection obtained from modeling are compared to those of of Eqs. 5 and 8 for the cantilever beam without cable and with cable.

Table 2. Maximum deflection values obtained from modeling and theoretical equations for the cantilever beam without cable and with cable

Type of beam		Maximum deflection of beam obtained from modeling(cm)	Maximum deflection of beam obtained from theoretical equations(cm)	Allowable deflection (cm)
Cantilever beam	Without cable	1.151	1.128	0.833
	With cable	0.685	0.783	

As presented in Table 2, maximum deflection of the beam without cable obtained from modeling is slightly more than that of theoretical relation. The reason is that the beam has been modeled in ABAQUS software in the form of shell; and therefore beam haunch cannot be modeled. By decreasing the moment of inertia of beam section in the software, maximum deflection of the beam without cable, obtained from modeling, becomes slightly more than that of theoretical relation. Maximum deflection of the cantilever beam along with cable, obtained from modeling, is very close to that of theoretical equation. Considering the theoretical relation of increasing the pre-tensioning of the cable, reducing the moment of inertia in ABAQUS software due to not modeling the beam haunch results in obtaining more increase in the pre-tensioning force in modeling than that of theoretical equation. Consequently, the steel cantilever beam deflection related to the increase in pre-tensioning force of the cable, obtained from modeling, is slightly more than that of theoretical relation. Therefore, in calculating the deflection of cantilever beam along with cable, the errors arise from different deflection of the beam along with cable, related to the status of increasing in pre-tensioning force of the cable, obtained from modeling and those of theoretical equation operate as opposed to that of the beam without cable related to the uniform distributed loading. Consequently, they cancel out the effects of each other.

Bending moment caused by cable force is in the opposite direction of bending moment due to uniform distributed loading. As presented in Table 2, maximum deflection of the cantilever beam along with pre-stressed cable is less than that of the beam without cable. Moreover, maximum deflection is less than allowable limit in cantilever beam. Therefore, using the cable satisfies the deflection criterion under service load.

8. Comparison of bending moment diagrams of cantilever beam without cable and with cable

The bending moment diagrams of cantilever beam without cable and with cable are plotted based on Eq. (6) and are shown in Fig. 6. The total cross-section area of steel cables is 297 mm^2 and pre-tensioning stress of steel cables is assumed 600 MPa.

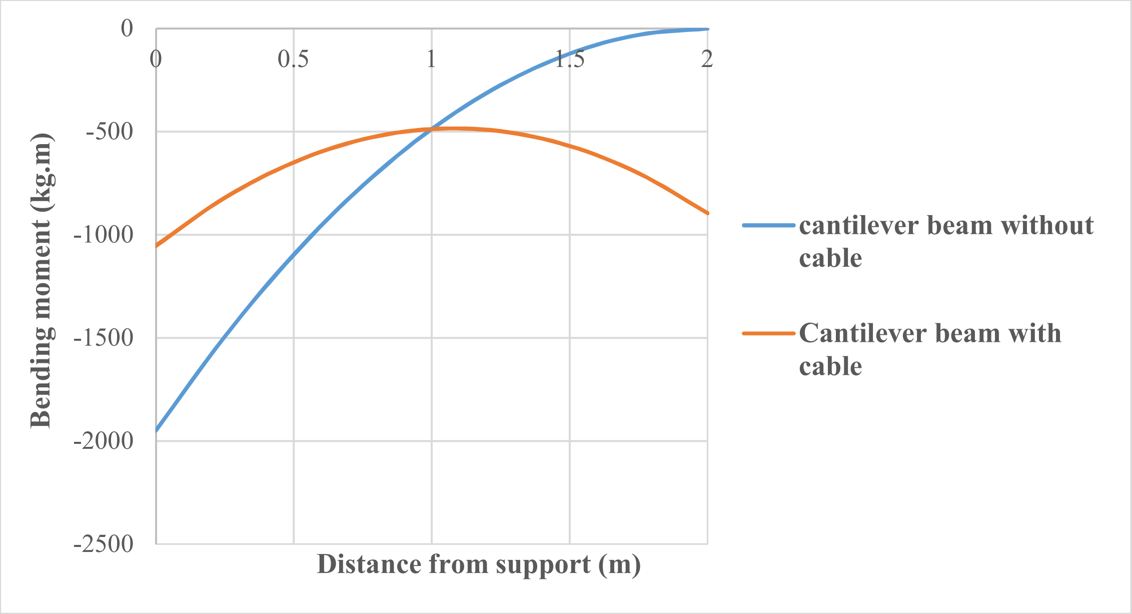

Fig. 6. Bending moment diagrams of cantilever beam without cable and with cable

As shown in Fig. 6, the cable passes through the neutral axis at a distance of 1 m from the support in the cantilever beam with cable resulting in the bending moment of cantilever beam with cable is to be equal to that of cantilever beam without cable at this distance. Moreover, the bending moment of cantilever beam with cable is decreased compared to that of cantilever beam without cable from support to the place of passing cable through the neutral axis, thereafter, the bending moment of cantilever beam with cable is increased comparing to that of cantilever beam without cable. It is concluded that if the cable leads to neutral axis at free end of the cantilever beam, the bending moment of cantilever beam with cable will not increase compared to that of cantilever beam without cable.

9. Sensitivity analysis on the cross-section of steel cable

In the sensitivity analysis on the cross-section of steel cable, the cross-section of steel cable is considered as 7-wire strand with low relaxation for cantilever beam along with equal numbers of cables at both sides of the webs with the cross section areas of 74.19 mm^2 for each cable. These values are considered according to ASTM A416 standard. Constant pre-tensioning has been considered as 600 MPa. Tables 3 present the maximum deflections in the cantilever beam with cable modeled in ABAQUS software for different cross-sections of steel cable.

Table 3. Maximum deflection results of cantilever beam along with cable in sensitivity analysis on the cross-section area of steel cable

Total cross-section area of steel cable (mm^2)	Maximum deflection of cantilever beam along with cable (cm)	Allowable deflection (cm)
148	0.918	0.833
297	0.685	0.833
445	0.454	0.833

According to the Table 3, maximum deflection is reduced in the cantilever beam with cable with the increase in steel cable cross-section area due to the increase in stiffness in the beam along with cable.

According to the Table 3, the steel cable with 148 mm^2cross-section area cannot satisfy the deflection criterion. Considering the steel cables with the cross-section areas of 297 mm^2 and 445 mm^2, maximum deflection is less than the allowable value in the cantilever beam along with cable. Therefore, the steel cables with these two cross section areas can satisfy the deflection criterion.

10. Sensitivity analysis on the pre-tensioning stress of the steel cable

In sensitivity analysis on pre-tensioning of steel cable, the cross-section of steel cable is considered as 7-wire strand with low relaxation for cantilever beam along with equal number of cables at both sides of the webs with the cross section areas of 74.19 mm^2 for each cable. These values are considered according to ASTM A416 standard and the total cross-section area of steel cables is 297 mm^2. Table 4 presents the values of maximum deflection of the cantilever beam with cable modeled in ABAQUS software, for different values of pre-tensioning of steel cable.

Table 4. Maximum deflection results of cantilever beam with cable in sensitivity analysis on the cable pre-tensioning stress

Type of beam	cable pre-tensioning stress (MPa)			Allowable deflection (cm)
	400	600	800	
Cantilever beam along with cable	0.835	0.685	0.535	0.833

According to Table 4, maximum deflection is reduced in the cantilever beam with cable, with increasing in the pre-tensioning of steel cable due to the increase in bending moment caused by cable force, compared to that resulted from uniform distributed loading.

If pre-tensioning of steel cable increases by 200 MPa, maximum deflection is reduced by 0.1cm in the cantilever beam along with cable. According to Table 4, for 400 MPa pre-tensioning of steel cable, maximum deflection of cantilever beam along with cable is slightly more than the allowable deflection. Therefore, the cable with 400 MPa pre-tensioning cannot satisfy the deflection criterion. When the values of pre-tensioning of steel cable are 600 MPa and 800 MPa, maximum deflection of cantilever beam along with cable is less than the allowable deflection. Therefore, these two pre-tensioning of steel cable satisfy the deflection criterion.

11. Conclusion

Cables, due to their low weights, small cross sections, and high tensile strengths, are reckoned as proper alternatives for pre-tensioning long steel beams subjected to uniform distributed loads. In this research, cables are employed to pre-stress the cantilever beam in which the deflection are not within the allowable range, despite appropriate design under bending and shear. Theoretical equations have been derived to calculate the increase in pre-tensioning force of the cable, the deflection of cantilever beam with and without cable and required cross-sectional area of steel cable to reach the allowable deflection in steel cantilever beam with cable. The results are obtained from finite element model and theoretical equation of cantilever beam along with cable under uniform distributed loads. They are briefly summarized as follows:

1. Comparing the results obtained from theoretical equation and its of finite element model demonstrates that the theoretical equation developed in this article can properly predict the deflection of cantilever beam without cable and along with cable;
2. Adding cable to the beam results in reducing the deflection of cantilever beam with cable;
3. The bending moment diagrams show that in cantilever beam without cable and with cable, if the cable leads to neutral axis at free end of the cantilever beam, the bending moment of cantilever beam with cable will not increase compared to that of cantilever beam without cable.
4. In cantilever beam with cable, the deflection is reduced by increasing in the cross section of steel cable, considering equal pre-tensioning. Moreover, proper values of steel cable cross sections are obtained, as per which the deflection criterion under service load of cantilever beam with cable is satisfied.
5. By increasing in pre-tensioning in the steel cables of equal cross-sections, the deflection is reduced in the cantilever beam with cable. Moreover, proper values of pre-tensioning are obtained for steel cables, as per which the deflection criterion under service load of cantilever beam with cable is satisfied.

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