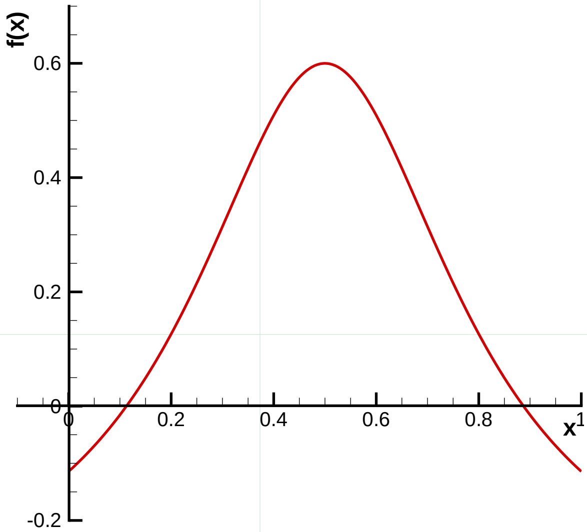
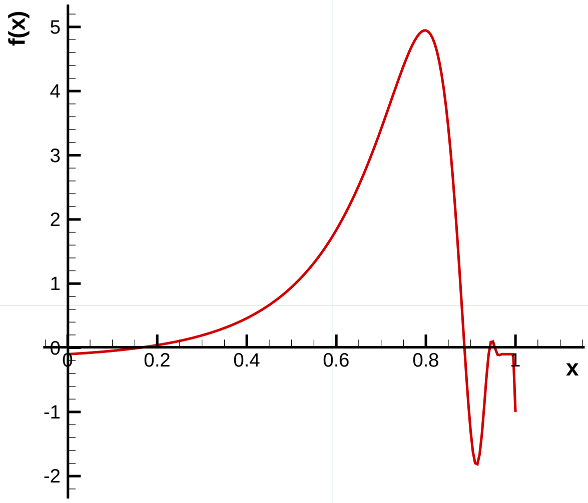
Advanced Numerical Method

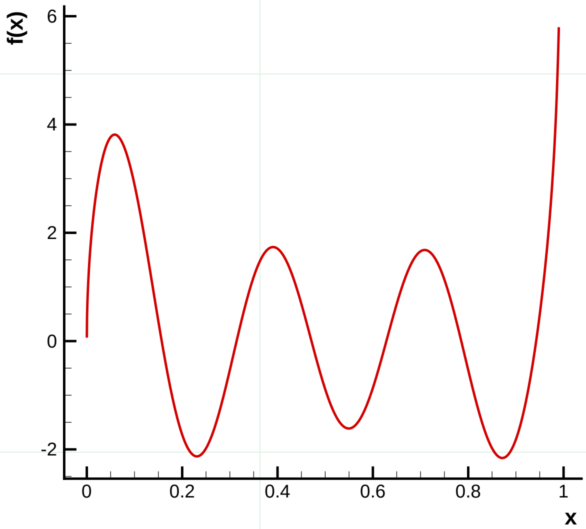
Dr. Farschad Torabi **Homework #4**



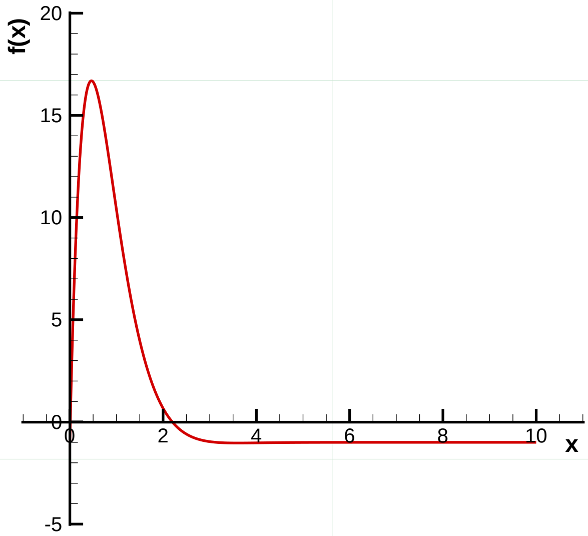
(a) $\dfrac{1}{10(0.5-x)^2+1}-0.4$



(b) $-\dfrac{\exp(0.4(1-\frac{1}{1-x}))\sin(0.4(1-\frac{1}{1-x}))}{(1-x)^2}-0.1$



(c) $\dfrac{\sin(20x)}{\sqrt{x-x^3}}$



(d) $100\exp(-2x)\sin(x)-1$

Consider four different functions illustrated in Figure 1. We are to integrate these functions as described below with theaccuracy of 10^4. Before integrating, it is good to plot each function and check the output using TECPLOT software, to ensure that you have coded the functions properly.

1. For figure 1(a)

   (a) Find the roots of the function, i.e. x_1 and x_2 in the range of $x \in [0 \cdots 1]$. The best method for root finding is Newton–Raphson rule since it is not a very complicated function and its derivative can be found analytically.

   (b) Integrate this function from x_1 to x_2 using Trapezoidal rule, Simpson rule and Mote–

Carlo method.

2. For figure 1(b)

    (a) Find the smallest and largest roots of the function, i.e. x_1 and x_2 in the range of $x \in [0 \cdots 1]$. Note that this function has 4 distinct roots in this range two of which are very close near the point $x = 0.95$. Make sure that you have found the largest root. Since this function is a little bit complicated, it is better to find the roots using bisection method.

    (b) Integrate this function from x_1 to x_4 using Trapezoidal rule, Simpson rule and Mote–Carlo method.

    (c) When using Trapezoidal rule, show that by means of adaptive method the solution can be found faster than the normal method.

3. For figure 1(c)

    (a) Integrate this function from $x_1 = 0$ to $x_2 = 1$ using Gauss–Legendre and Gauss–Chebyshev methods.

    (b) How many Gaussian points are required to achieve the required accuracy, foreach method.

4. For figure 1(d)

    (a) Integrate this function from $x_1 = 0$ to $x_2 = 10$ using Trapezoidal rule and Gauss–Legendre methods.

    (b) When using Trapezoidal rule, show that by means of adaptive method the solution can be found faster than the normal method.

    (c) How many Gaussian points are required to achieve the required accuracy, for Gauss–Legendre method.