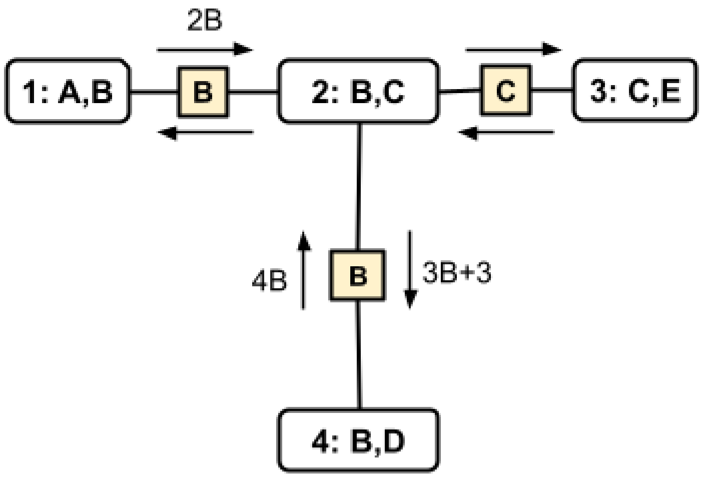
Probabilistic Graphical Models Final Exam - Spring 1401 (2022)	Instructor: B. Nasihatkon	K. N. TOOSI UNIVERSITY OF TECHNOLOGY
Name:	ID:	

Question 1- Junction tree, message passing (25 points)



Consider the following junction tree for performing max-sum message passing on an MRF with binary variables $A. B, C, D, E \in \{0, 1\}$, and the joint distribution

$$P(A, B, C, D, E) = \frac{1}{Z} exp(\theta_1(A, B) + \theta_2(B, C) + \theta_3(C, E) + \theta_4(B, D))$$

Assume that θ_i is assigned to the i-th cluster i = 1,2,3,4. Further, assume that

$\theta_1(A, B) = \alpha\, AB,$

$\theta_2(B, C) = B + \gamma\, C,$

$\theta_3(C, E) = -\ 1(C = E),$

$\theta_4(B, D) = \lambda\ 1(B = D = 1),$

$\delta_{1\to2}(B) = 2B$

$\delta_{4\to2}(B) = 4B$

$\delta_{2\to4}(B) = 3B + 3$

where 1(.) is the indicator function. Find $\delta_{2\to1}(B)$, $\delta_{2\to3}(C)$, $\delta_{3\to2}(C)$, as well as the values of α, γ, and λ.

Question 2-Graph Cuts (25 points)

Consider the following energy function over the *binary* variables A, B, C

$E(A, B, C) = E_{AB}(A, B) + E_{BC}(B, C) + E_A(A) + E_B(B) + E_C(C)$

where

$E_{AB}(0, 0) = E_{AB}(1, 1) = 0$

$E_{AB}(0, 1) = E_{AB}(1, 0) = 1$

$E_{BC}(0, 1) = 2 \quad E_{BC}(1, 0) = 4$

$E_{BC}(0, 0) = 4 \quad E_{BC}(1, 1) = 0$

$E_A(0) = -10 \quad E_A(1) = 1 \quad E_B(0) = 2 \quad E_B(1) = 0$

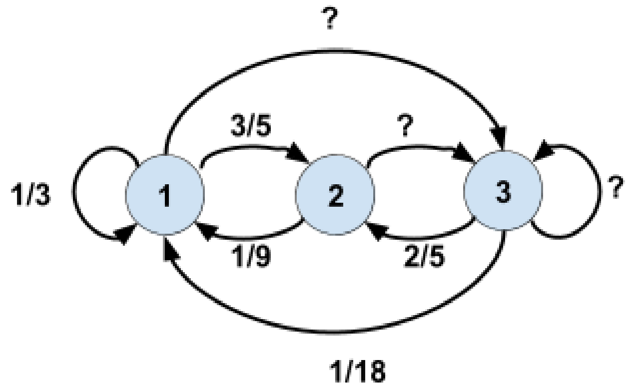
$E_C(0) = 4 \quad E_C(1) = -4$

A) Is $E(A, B, C)$ *submodular*? Why? **(5 points)**

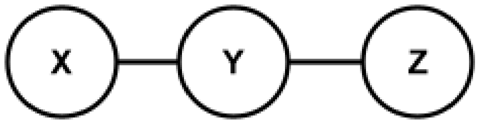
B) Construct a min-cut/max-flow graph for an equivalent energy function such that all edge weights are positive. **(20 points)**

Question 3- MCMC (25 points)

Consider a Markov chain with the following state transition graph for a 1D distribution, where the numbers by the edges are transition probabilities. Derive the corresponding stationary distribution $P^{\infty}(X = i) = \pi(i)$ for $i = 1, 2, 3$. Write down the complete derivations. Write down the complete solution.



Question 4 - Parameter learning (25 points)



Consider the following MRF on binary variables $X, Y, Z \in \{0, 1\}$, with the log-linear joint distribution $p(X, Y, Z) = \frac{1}{Z} exp(\alpha\, 1(X = Y) + \beta\, YZ)$

Your task is to obtain the optimal Maximum-Likelihood values of the parameters α and β given following the training data. To do this you have to follow the steps below

x^i	y^i	z^i
0	**0**	**0**
1	**0**	**1**
1	**1**	**0**
0	**1**	**1**
0	**0**	**1**

a) Find the partition function Z as a function of α and β. Simplify as much as possible. (9 points)

b) Write down the log-likelihood function using part (a) and the provided data. (6 points)

c) Find the optimal values of α and β by differentiating with respect to each of these parameters and setting equal to zero. (10 points)