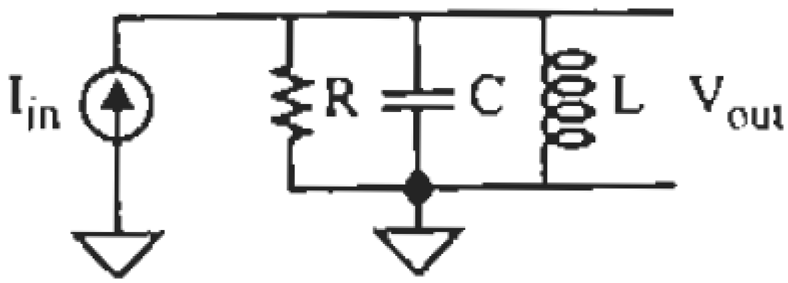
Passive RLC Networks

Parallel RLC Tank



$$Y = G + j\omega C + \frac{1}{j\omega L} = G + j\left(\omega C - \frac{1}{\omega L}\right)$$

Resonant Frequency: $\left(\omega_0 C - \frac{1}{\omega_0 L}\right) = 0 \implies \omega_0 = \frac{1}{\sqrt{LC}}.$

L=1 nH, C=1 pF ➔ f=5 GHz

Q (Quality Factor)

$$Q \equiv \omega \frac{\text{energy stored}}{\text{average power dissipated}}.$$

$$E_{\text{tot}} = \tfrac{1}{2} C (I_{\text{pk}} R)^2.$$

$$P_{\text{avg}} = \tfrac{1}{2} I_{\text{pk}}^2 R$$

$$Q = \omega_0 \frac{E_{\text{tot}}}{P_{\text{avg}}} = \frac{1}{\sqrt{LC}} \frac{\frac{1}{2} C (I_{\text{pk}} R)^2}{\frac{1}{2} I_{\text{pk}}^2 R} = \frac{R}{\sqrt{L/C}}$$

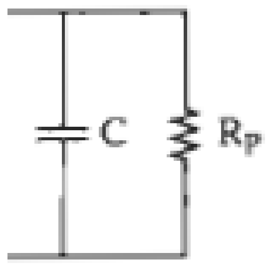
The quantity $\sqrt{L/C}$ has the dimensions of resistance and is sometimes called the *characteristic impedance* of the network.

It is Proven that: $Q = \dfrac{\omega_o}{\text{BW}}$

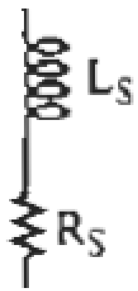
Series RLC Network

$$Q = \frac{\sqrt{L/C}}{R}$$

Q of the Capacitor and Inductor

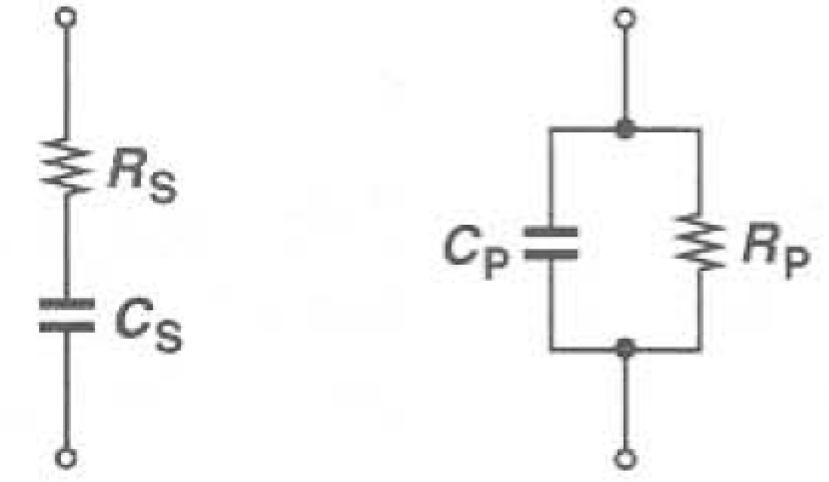


$$Q = R_p C \omega_o$$



$$Q = \frac{L\omega_o}{R_s}$$

Equivalent Series and Parallel Circuits



$$\frac{R_P}{R_P C_P s + 1} = \frac{R_S C_S s + 1}{C_S s}$$

$$[1 - R_s C_s R_p C_p \omega^2] + j\omega[R_s C_s + R_p C_p - R_p C_s] = 0$$

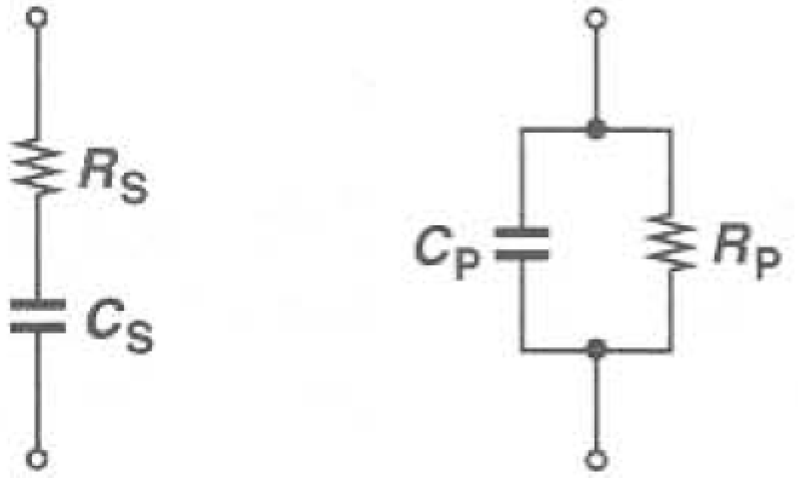
$$[1 - R_s C_s R_p C_p \omega^2] = 0 ==> Q_s = Q_p = Q = \frac{1}{R_s C_s \omega} = R_p C_p \omega$$

$$j\omega\left[\frac{1}{Q\omega} + \frac{Q}{\omega} - R_p C_s\right] = 0$$

$$R_p = \frac{1}{C_s \omega}\left[\frac{1 + Q^2}{Q}\right]$$

$$R_p = R_s(1 + Q^2)$$

Equivalent Series and Parallel Circuits



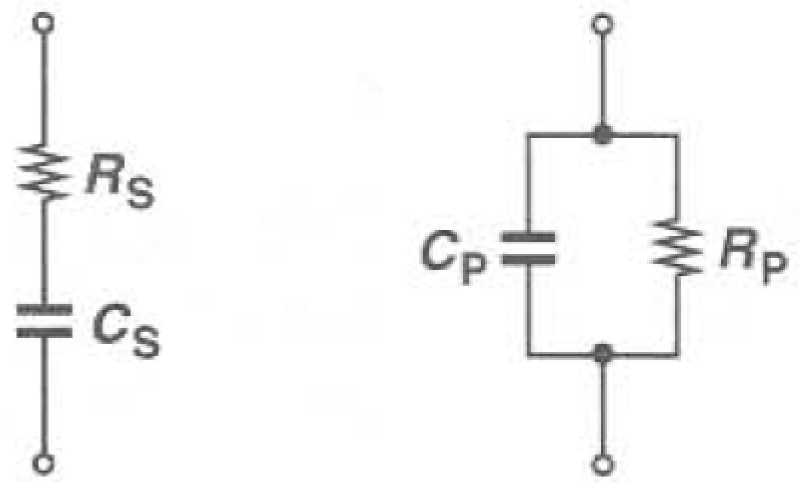
$$\frac{R_P}{R_P C_P s + 1} = \frac{R_S C_S s + 1}{C_S s}$$

$$R_p = \frac{1}{C_s\omega}\left[\frac{1+Q^2}{Q}\right]$$

$$Q = R_p C_p \omega = \frac{1}{C_s\omega}\left[\frac{1+Q^2}{Q}\right] C_p \omega$$

$$C_p = \left[\frac{Q^2}{1+Q^2}\right] C_s$$

Equivalent Series and Parallel Circuits



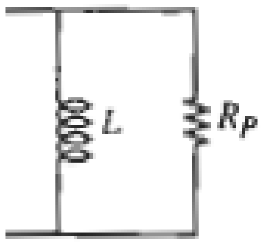
$$\frac{R_P}{R_P C_P s + 1} = \frac{R_S C_S s + 1}{C_S s}$$

If $Q^2 >> 1$ ⟹

$$R_p \cong R_s Q^2$$

$$C_p \cong C_s$$

Equivalent Series and Parallel Circuits



$$R_P = R_S(1 + Q^2)$$

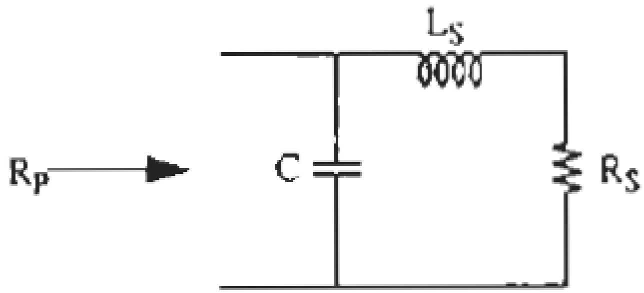
$$L_P = L_S(\frac{1 + Q^2}{Q^2})$$

If: $Q^2 \gg 1$

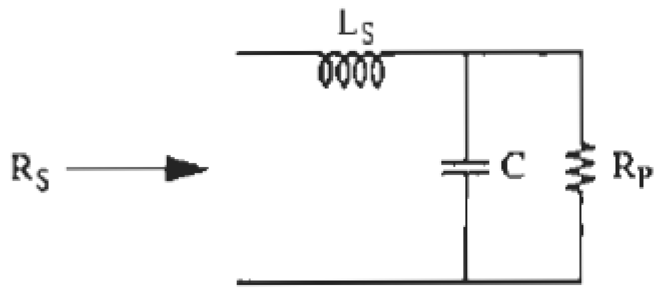
$$R_P \approx R_S Q^2 = \frac{{L_S}^2\omega^2}{R_S}$$

$$L_P \approx L_S$$

The L-Match



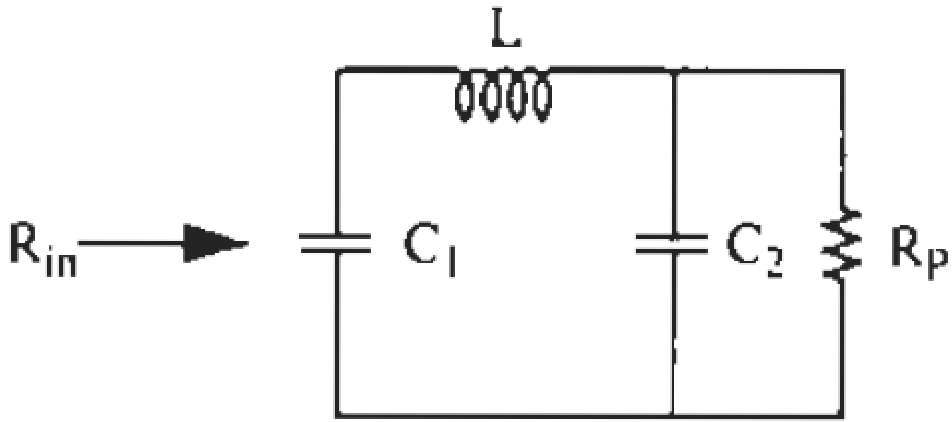
Upward Impedance Transformer

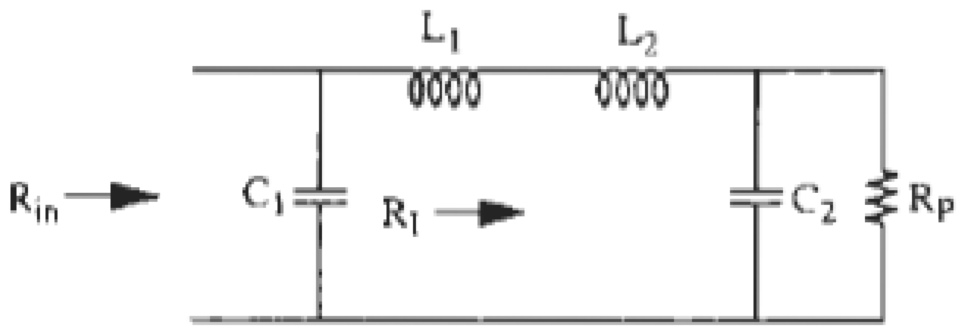


Downward Impedance Transformer

there are only two degrees of freedom (one can choose only L and C).

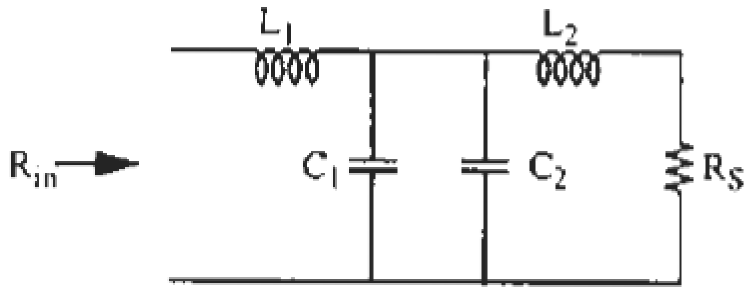
The π-Match



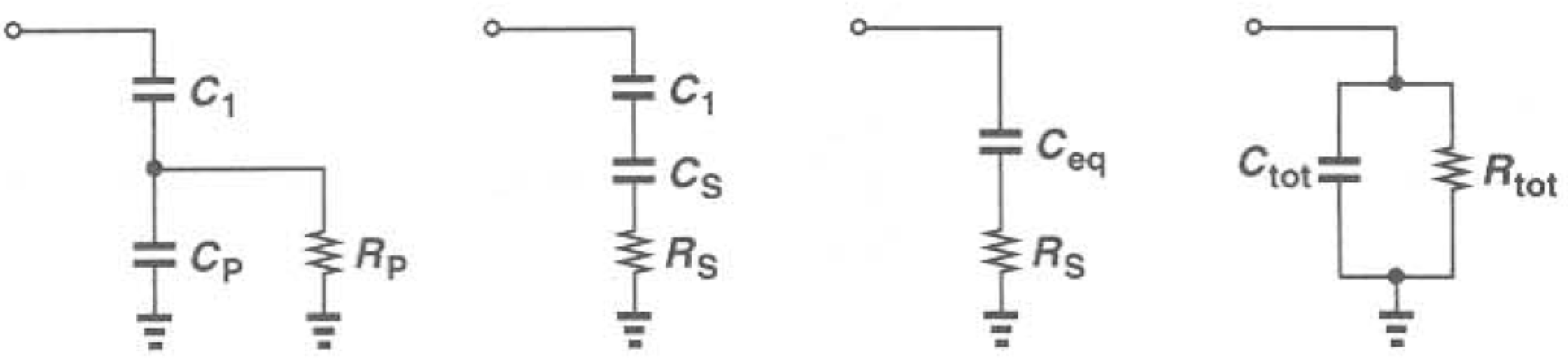


Π-match as cascade of L-matches

The T-Match



Impedance Transformation by Means of a Capacitor Divider

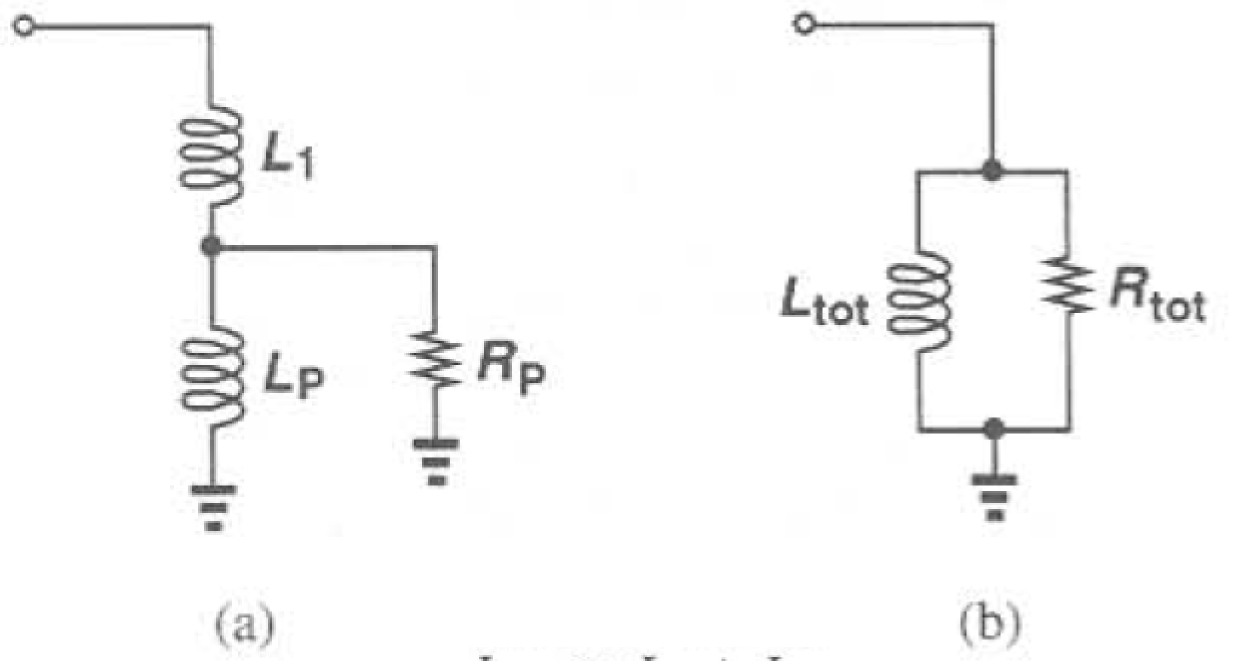


(a) (b) (c) (d)

$$C_{tot} \approx C_1 C_P/(C_1+C_P)$$

$$R_{tot} \approx 1/[R_S(C_{eq}\omega)^2] = (1+C_P/C_1)^2 R_P$$

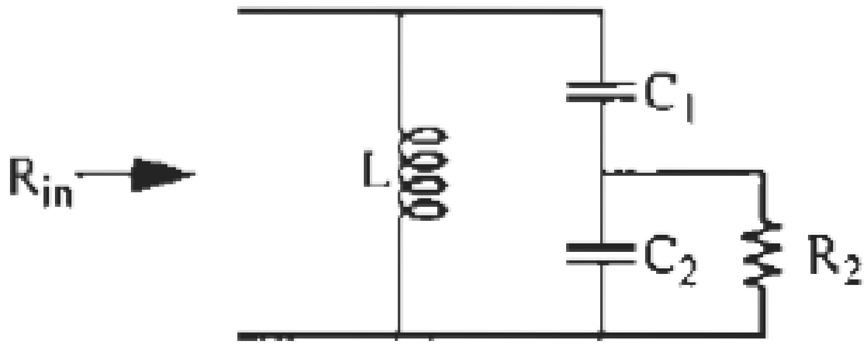
Impedance Transformation by Means of an Inductor Divider



$$L_{\text{tot}} \approx L_1 + L_P$$

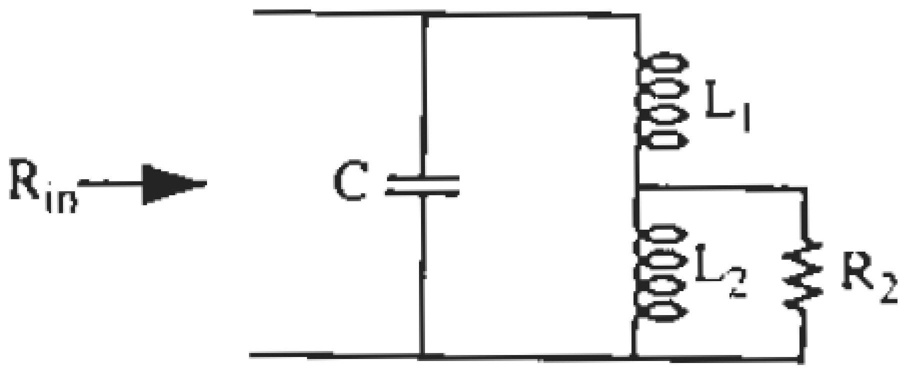
$$R_{\text{tot}} \approx (1 + L_1/L_P)^2 R_P$$

Tapped Capacitor Match

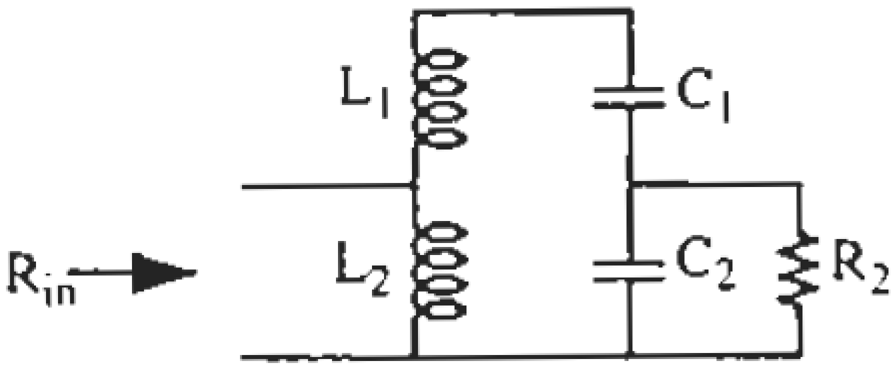


Tap: an intermediate point in an electric circuit where a connection may be made.

Tapped Inductor Match

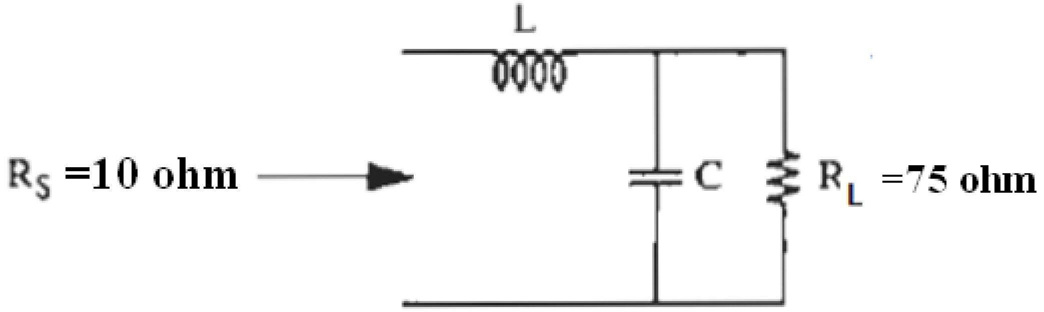


Double-Tapped Match

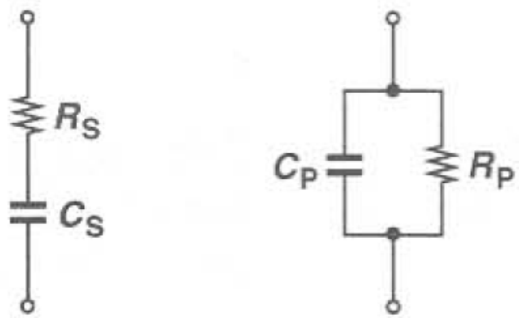


Example#1

Determine L, C in order to have good matching between R_L and Rs. BW=?



f=150 MHz



$$R_p = R_s(1 + Q^2)$$
$$C_p \cong C_s$$

$$\frac{R_L}{R_s} = 1 + Q_1^2 \xrightarrow{yields} 7.5 = 1 + Q_1^2 \xrightarrow{yields} Q_1 = 2.55$$

$$Q_1 = R_L C W$$

2.55=75×C×2×π×150× (10^6) $\xrightarrow{yields}$ C=36.075PF

$$LC\omega^2 = 1$$

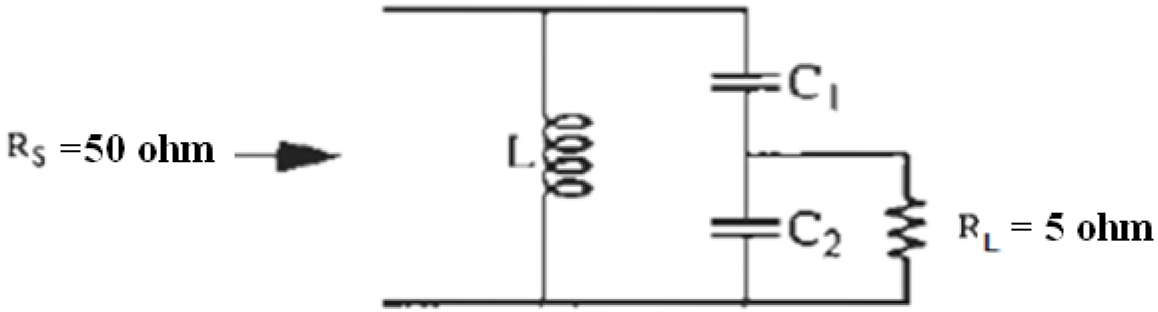
$$L \times 36p \times (2\pi \times 150MHz)^2 = 1$$

$$L = 31nH$$

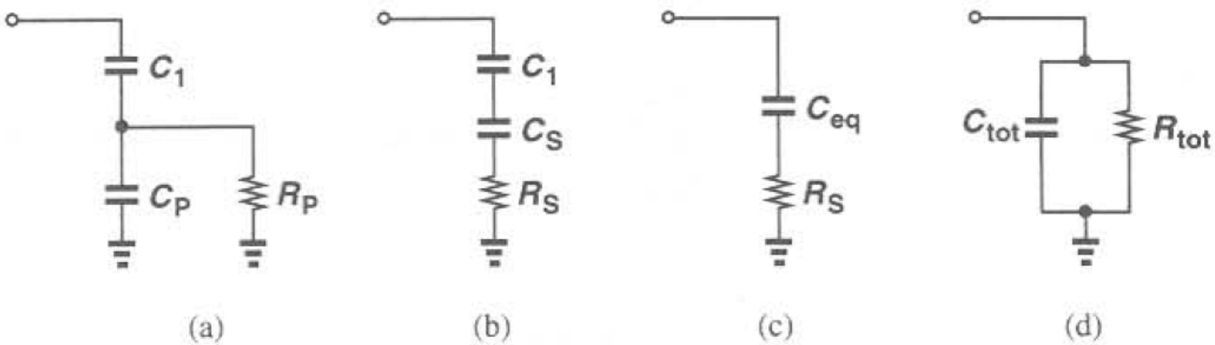
$$Q_1 = \frac{W}{BW} \xrightarrow{yields} 2.55 = \frac{150MHz}{BW} \xrightarrow{yields} BW = 58.8MHz$$

Example#2

Determine L, C1, C2 in order to have good matching between R_L and Rs over the 15 MHz bandwidth.



f=150 MHz



$$C_{tot} \approx C_1 C_P/(C_1 + C_P)$$

$$R_{tot} \approx 1/[R_S(C_{eq}\omega)^2] = (1 + C_P/C_1)^2 R_P$$

$$Q = \frac{\omega}{BW} = \frac{2\pi \times 150MHz}{2\pi \times 15MHz} = 10$$

$$Parallel \quad RLC: \quad Q = \frac{Rs}{\sqrt{\frac{L}{C_{tot}}}} \qquad 10 = \frac{50}{\sqrt{\frac{L}{C_{tot}}}}$$

$$\rightarrow \sqrt{\frac{L}{C_{tot}}} = 5$$

$$LC_{tot}\omega^2 = 1$$

$$\omega = 2\pi \times 150MHz$$

$$\Rightarrow \quad L = \mathbf{5.3\ nH} \qquad C_{tot} = \mathbf{212\ pF}$$

$$R_{tot} = \left(1 + {}^{C_2}/_{C_1}\right)^2 \times R_L \longrightarrow 10 = \left(1 + {}^{C_2}/_{C_1}\right)^2 \xrightarrow{yields} {}^{C_2}/_{C_1} = 2.16$$

$$C_{tot} \approx C_1 \times \frac{C_2}{C_1 + C_2} = 212PF \xrightarrow{yields} 212PF = \frac{C_2}{1 + \frac{C_2}{C_1}} \xrightarrow{yields} C_2 = 670PF$$

$$C_1 = 310PF$$