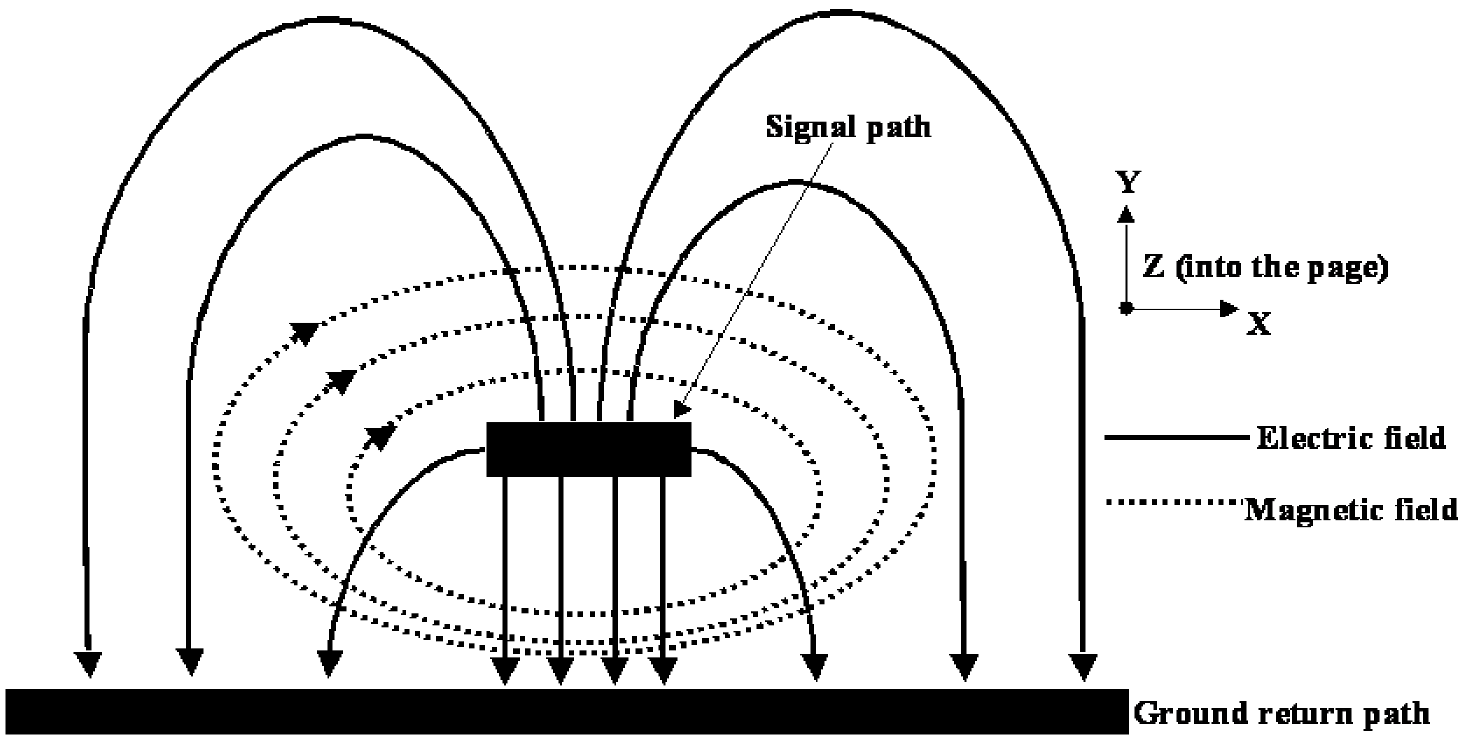
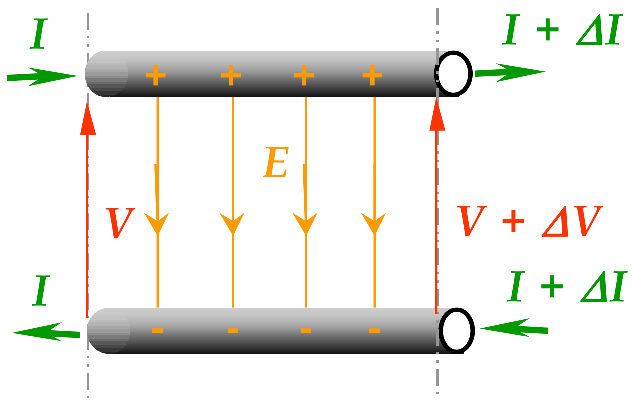
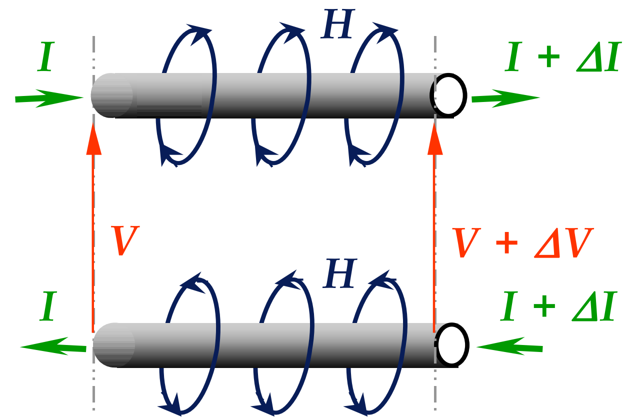
Distributed Systems

E & H Fields – Microstrip Case



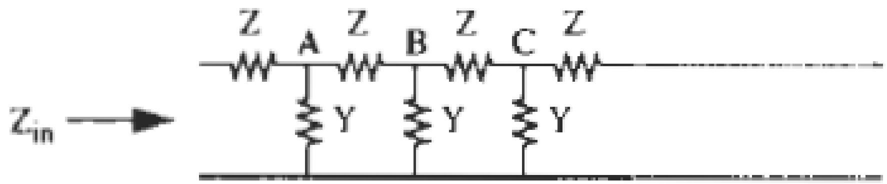
Presence of Electric and Magnetic Fields





- **Both Electric and Magnetic fields are present in the transmission lines**
- **Electric field is established by a potential difference between two conductors.**
  - Implies equivalent circuit model must contain capacitor.
- **Magnetic field induced by current flowing on the line**
  - Implies equivalent circuit model must contain inductor.

Ideal Transmission Line as Infinite Ladder Network



$$Z_{in} = Z + [(1/Y) \parallel Z_{in}],$$

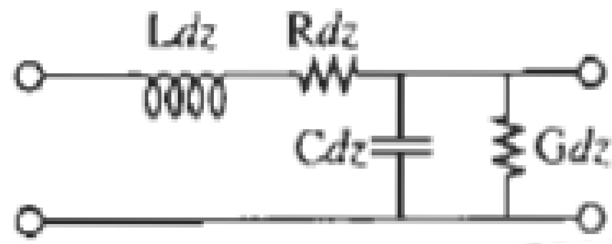
$$Z_{in} = Z + \frac{Z_{in}/Y}{1/Y + Z_{in}} \Longrightarrow (Z_{in} - Z)\left(\frac{1}{Y} + Z_{in}\right) = \frac{Z_{in}}{Y}$$

$$Z_{in} = \frac{Z \pm \sqrt{Z^2 + 4(Z/Y)}}{2} = \frac{Z}{2}\left[1 \pm \sqrt{1 + \frac{4}{ZY}}\right].$$

Assuming: $|ZY| \ll 1$ ➡ $Z_{in} \approx \sqrt{Z/Y}.$

In the case of a lossless line, $Z = sL$ and $Y = sC$, ➡ $Z_{in} \approx \sqrt{Z/Y} = \sqrt{sL/sC} = \sqrt{L/C}.$

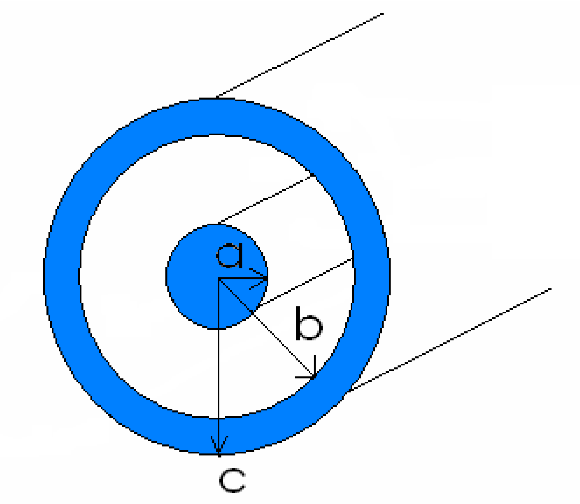
Lumped RLC Model of Infinitesimal Transmission Line



Lossy Transmission Line: $$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}.$$

Loss-less Transmission Line: $$Z_0 = \sqrt{L/C}.$$

Coax T-line Geometry



a = radius of inner conductor

b =inner radius of the outer conductor

c =outer radius of outer conductor

h: length of the cable

- Shunt capacitance per unit length, in farads per metre.

$$\left(\frac{C}{h}\right) = \frac{2\pi\epsilon}{\ln(\ \mathbf{b/a}\ )} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(\ \mathbf{b/a}\ )}$$

- Series inductance per unit length, in henrys per metre.

$$\left(\frac{L}{h}\right) = \frac{\mu}{2\pi}\ln(\ \mathbf{b/a}\ ) = \frac{\mu_0\mu_r}{2\pi}\ln(\ \mathbf{b/a}\ )$$

$$Z_0 = \sqrt{L/C} \quad \Rightarrow \quad Z_0 = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ \ln(\ \mathbf{b/a}\ ) \approx \frac{138\Omega}{\sqrt{\epsilon_r}}\log_{10}\frac{\mathbf{b}}{\mathbf{a}}$$

Coax T-line Example

- Exercise 1

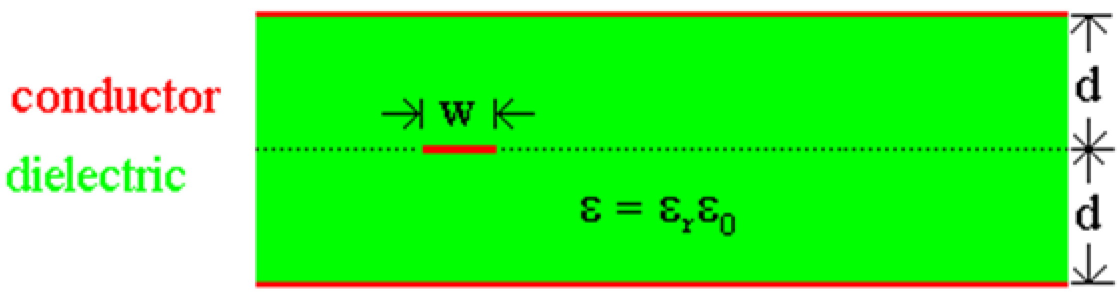
  Given: a coax T-line with a = 4 mm, b = 17.5 mm, and c = 20 mm. the dielectric has μ_r = 1, ε_r = 3.

  Z_0=?

$$Z_0 \approx \frac{138\Omega}{\sqrt{\epsilon_r}} \log_{10} \frac{\mathbf{b}}{\mathbf{a}} \quad \mathbf{=51}$$

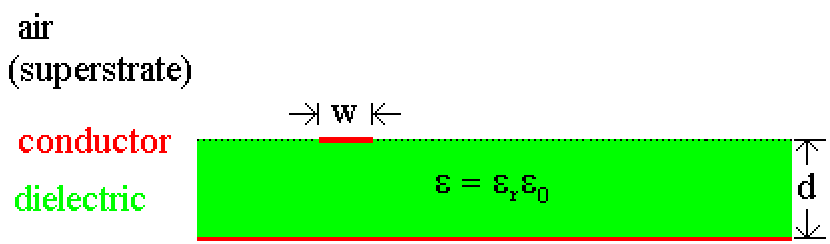
Striplines and Microstrip Lines

- Stripline
  - Single or double track strip of Cu imbedded in dielectric material sandwiched between conducting ground planes on top <u>and</u> bottom



Striplines and Microstrip Lines

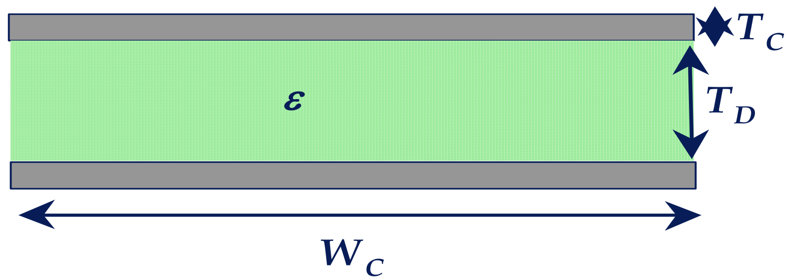
- Microstrip
  - Single or double track strip of Cu on top of a dielectric substrate material above a single conducting ground plane



- In practice, superstrate material may be other than air

Parallel Plate **Approximation**

- Assumptions
  - Uniform dielectric (ε) between conductors
  - $T_C << T_D$; $W_C >> T_D$
- T-line characteristics are function of:
  - Material electric and magnetic properties
  - Dielectric Thickness (T_D)
  - Width of conductor (W_C)
- Trade-off
  - $T_D \uparrow$; $C_0 \downarrow$, $L_0 \uparrow$, $Z_0 \uparrow$
  - $W_C \uparrow$; $C_0 \uparrow$, $L_0 \downarrow$, $Z_0 \downarrow$



$$C = \frac{\varepsilon * PlateArea}{d}$$ Base equation

$$C_0 = \varepsilon \cdot \frac{W_C}{T_D} \cdot \left(\frac{F}{m}\right) = 8.85 \cdot \varepsilon_r \cdot \frac{W_C}{T_D} \cdot \left(\frac{pF}{m}\right)$$

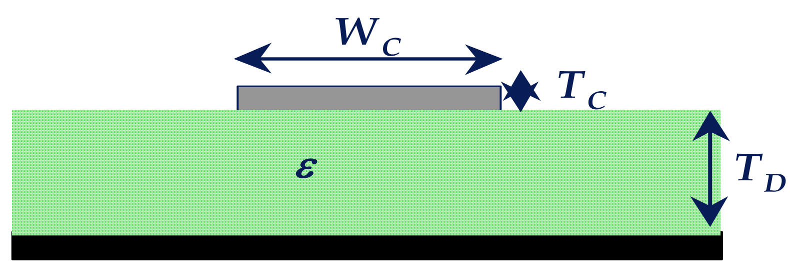
$$L_0 = \mu \cdot \frac{T_D}{W_C} \cdot \left(\frac{F}{m}\right) = 0.4 \cdot \pi \cdot \mu_r \cdot \frac{T_D}{W_C} \cdot \left(\frac{\mu H}{m}\right)$$

$$Z_0 = 377 \cdot \frac{T_D}{W_C} \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}} \cdot \Omega$$

To a first order, t-line capacitance and inductance can be approximated using the parallel plate approximation.

Improved Microstrip Formula

- Parallel Plate Assumptions +
  - Large ground plane with zero thickness



$$Z_0 \approx \frac{87}{\sqrt{\varepsilon_r + 1.41}} \ln\left(\frac{5.98 T_D}{0.8 W_C + T_C} \right)$$

Valid when:
$0.1 < W_C/T_D < 2.0$ and $1 < \varepsilon_r < 15$

- To accurately predict microstrip impedance, you must calculate the ***effective*** dielectric constant and modify the above formula by replacing ε_r with ε_e.

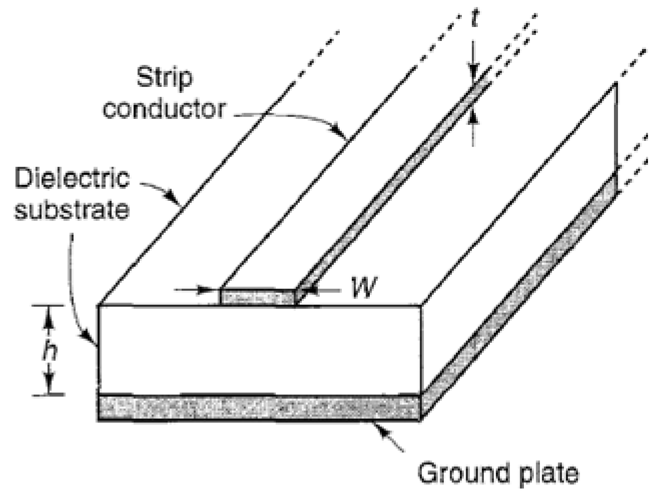
$$\varepsilon_e = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2\sqrt{1 + \frac{12 T_D}{W_C}}} + F - 0.217(\varepsilon_r - 1)\frac{T_C}{\sqrt{W_C T_D}}$$

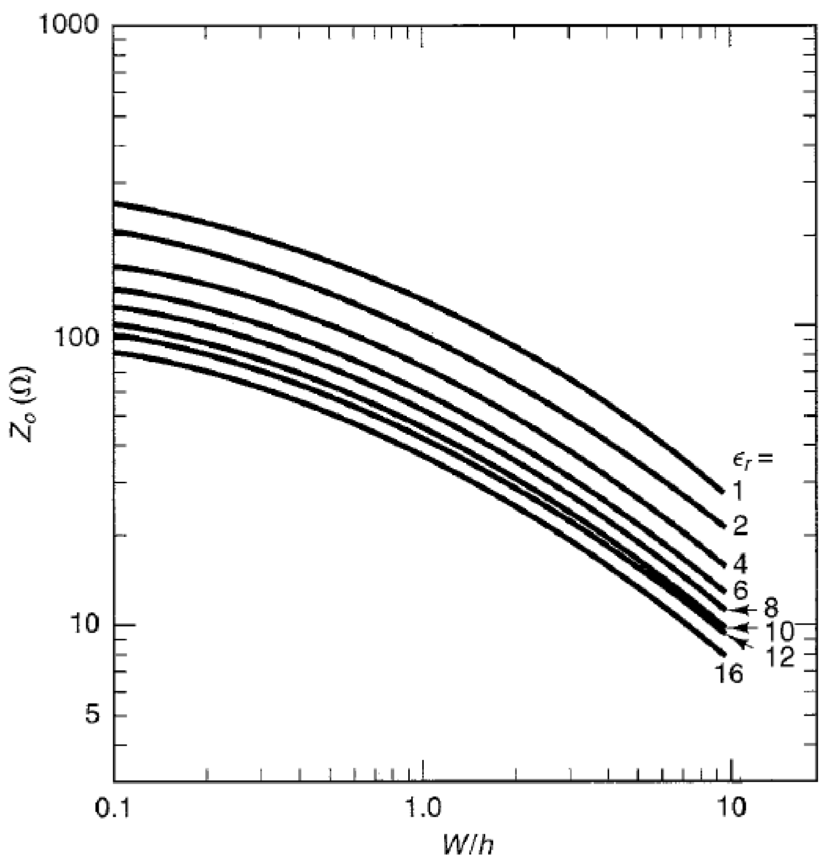
$$F = \begin{cases} 0.02(\varepsilon_r - 1)\left(1 - \frac{W_C}{T_D}\right)^2 & \text{for } \frac{W_C}{T_D} < 1 \\ 0 & \text{for } \frac{W_C}{T_D} > 1 \end{cases}$$

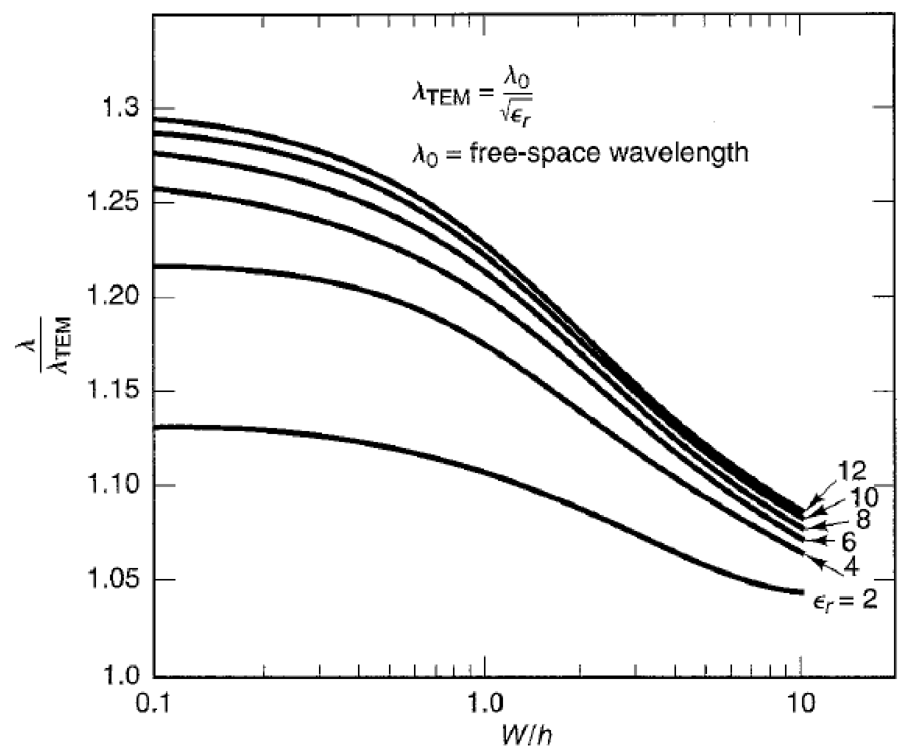
Cont'd

$$v_p = \frac{c}{\sqrt{\varepsilon_{ff}}}$$

$$\lambda = \frac{v_p}{f}$$





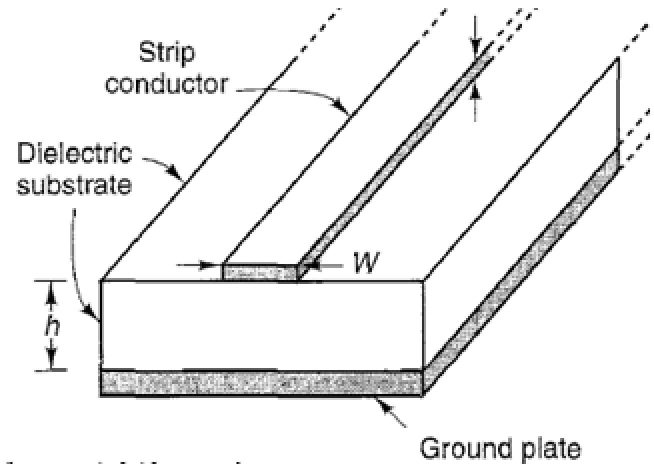


Example

A microstrip material with $\varepsilon_r = 10$ and $h = 1.016$ mm is used to build a transmission line. Determine the width for the microstrip transmission line to have a characteristic impedance of 50 Ω. Also determine the wavelength and the effective relative dielectric constant of the microstrip line.

Making use of graph of the previous slide:

$$Z_0 = 50\Omega \Rightarrow \frac{W}{h} = 1 \Rightarrow \quad W = h = 0.1016\left(\frac{1000}{2.54}\right) = 40 \text{ mils}$$



From previous slide , with $W/h = 1$ and $\varepsilon_r = 10$, it follows that the value of $\lambda/\lambda_{\text{TEM}}$ is approximately 1.23, or

$$\lambda = 1.23\lambda_{\text{TEM}} = 1.23\frac{\lambda_0}{\sqrt{\varepsilon_r}} = \frac{1.23}{\sqrt{10}}\lambda_0 = 0.389\lambda_0$$

$$\lambda = \frac{v_p}{f} = \frac{c}{f\sqrt{\varepsilon_{ff}}} = \frac{\lambda_0}{\sqrt{\varepsilon_{ff}}}$$

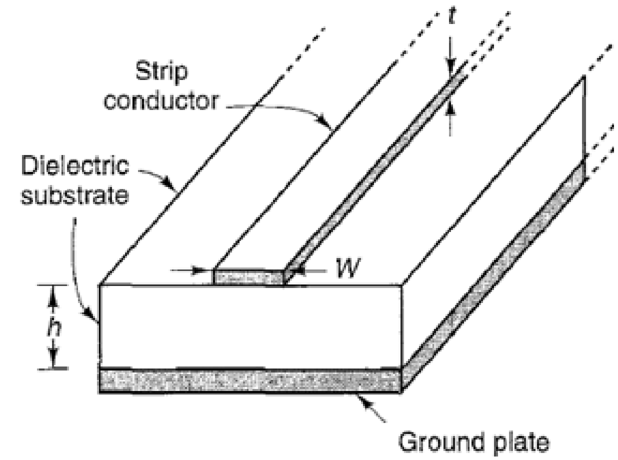
Hence , $\varepsilon_{ff} = \left(\frac{1}{0.389}\right)^2 = 6.61$

Microstrip Line with Z_0=50 ohm

The frequency below which dispersion may be neglected is given by

$$f\,(\text{GHz}) = 0.3\sqrt{\frac{Z_o}{h\sqrt{\varepsilon_r - 1}}}$$

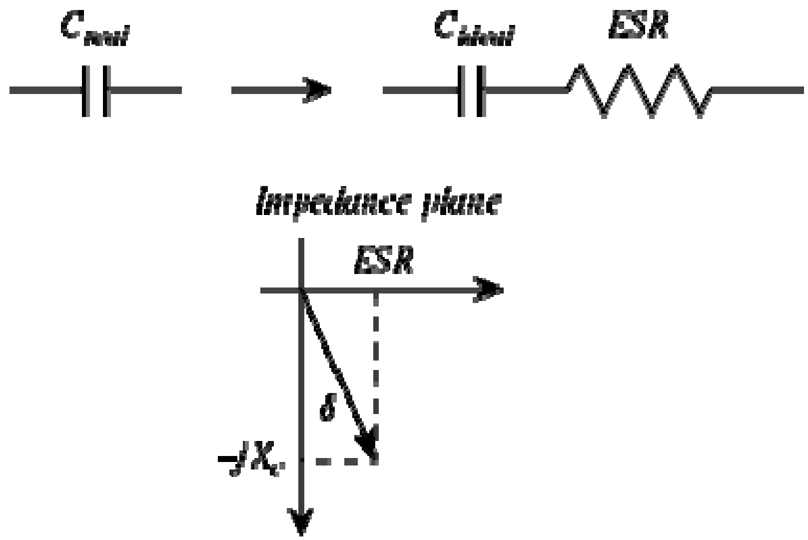
where h must be expressed in centimeters.



Width and ε_{ff} of microstrip lines for $Z_o = 50\ \Omega$ for various ε_r and h values.

	$\varepsilon_r = 2.23$	4.54	6	9.6	10	30
$h = 25$ mils	$W = 76.4$ mils	46.7	37.5	24.7	23.8	6.01
	$\varepsilon_{ff} = 1.91$	3.42	4.33	6.46	6.68	17.7
30	$W = 91.7$	56.1	45.0	29.7	28.5	7.21
	$\varepsilon_{ff} = 1.91$	3.42	4.33	6.46	6.68	17.7
40	$W = 122.2$	74.7	60.0	39.6	38.0	9.6
	$\varepsilon_{ff} = 1.91$	3.43	4.34	6.47	6.69	17.8
50	$W = 152.8$	93.4	75.0	49.4	47.6	12.0
	$\varepsilon_{ff} = 1.91$	3.43	4.34	6.48	6.71	17.8
100	$W = 305.6$	186.8	150.1	98.9	95.1	24.1
	$\varepsilon_{ff} = 1.91$	3.45	4.37	6.55	6.78	18.1

Loss tangent of Dielectrics



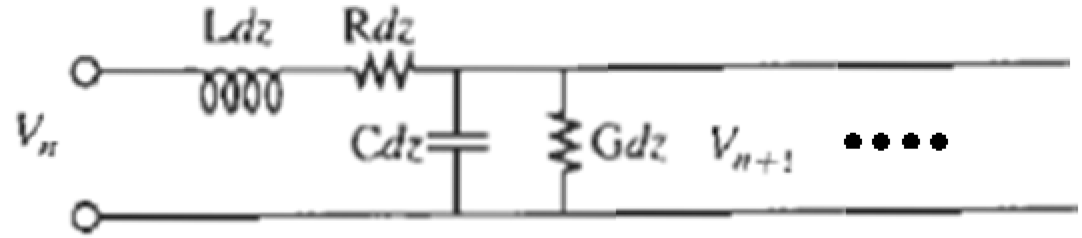
ESR: Equivalent Series Resistance

Loss tangent =tan(δ)

Dielectrics

Substance	Dielectric Constant (relative to air)	Dielectric Strength (V/mil)	Loss Tangent	Max Temp (°F)
Air	1.00054	30 - 70		
Alumina - 96% - 99.5%	10.0 9.6		0.0002 @ 1 GHz 0.0002 @ 100 MHz 0.0003 @ 10 GHz	
Aluminum Silicate	5.3 - 5.5			
Epoxy glass PCB	5.2	700		
FR-4 (G-10) - low resin - high resin	4.9 4.2		0.008 @ 100 MHz 0.008 @ 3 GHz	
Gallium Arsenide (GaAs)	13.1		0.0016 @ 10 GHz	
Glass	4 - 10			
Glass (Corning 7059)	5.75		0.0036 @ 10 GHz	
RT/Duroid 5880 (go to Rogers)	2.20		0.0009@ 10 GHz	
Silicon	11.7 - 12.9	100 - 700	0.005 @ 1 GHz 0.015 @ 10 GHz	300
Teflon® (PTFE)	2.0 - 2.1	1000	0.00028 @ 3 GHz	480
Vacuum (free space)	1.00000			

Propagation Constant



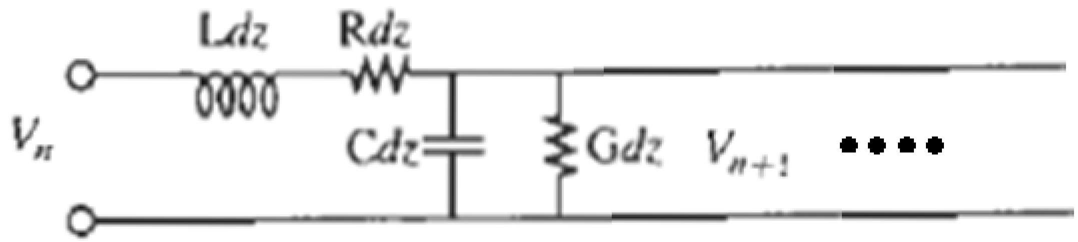
$$V_{n+1} = V_n \left\{ \frac{Z_0 \parallel (1/Y\,dz)}{Z\,dz + [Z_0 \parallel (1/Y\,dz)]} \right\}. \qquad \frac{V_{n+1}}{V_n} = \frac{Z_0}{Z_0 Z Y (dz)^2 + Z_0 + Z\,dz}$$

$$\frac{V_{n+1}}{V_n} \approx \frac{Z_0}{Z_0 + Z\,dz} = \frac{1}{1 + (Z/Z_0)\,dz} \approx 1 - \frac{Z}{Z_0}\,dz = 1 - \sqrt{ZY}\,dz.$$

$$V_{n+1} = V_n(1 - \sqrt{ZY}\,dz) \Longrightarrow \frac{V_{n+1} - V_n}{dz} = -\sqrt{ZY}\,V_n.$$

$$\frac{dV}{dz} = -\sqrt{ZY}\,V. \quad \Longrightarrow \quad V(z) = V_0 e^{-\sqrt{ZY}\,z}.$$

Propagation Constant



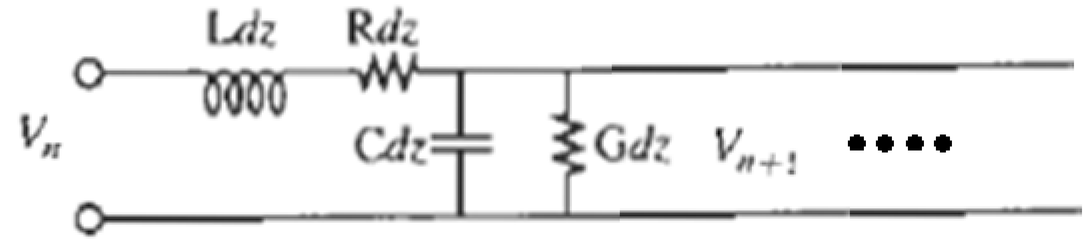
$$V(z) = V_0 e^{-\sqrt{ZY}z}.$$

$$\gamma = \sqrt{ZY} = \sqrt{(R + j\omega L)(G + j\omega C)}.$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta.$$

$$V(z) = V_0 e^{-\gamma z} = V_0 e^{-(\alpha + j\beta)z} = V_0 e^{-\alpha z} e^{-j\beta z}.$$

Propagation Constant



$$V(z) = V_0 e^{-\gamma z} = V_0 e^{-(\alpha + j\beta)z} = V_0 e^{-\alpha z} e^{-j\beta z},$$

$$\alpha \approx \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}. \qquad \beta = \mathrm{Im}[\gamma] \approx \omega\sqrt{LC}.$$

Delay of a Lossless Line

$$V(z) = V_0 e^{-\gamma z} = V_0 e^{-(\alpha + j\beta)z} = V_0 e^{-\alpha z} e^{-j\beta z}.$$

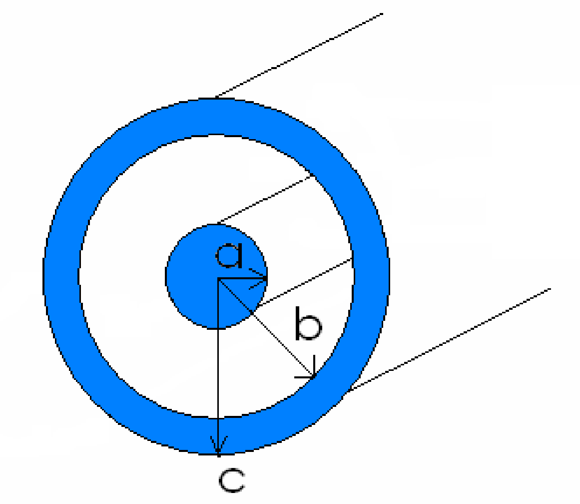
Loss-less Transmission Line: $\alpha = \mathrm{Re}[\gamma] = 0$

$$\beta = \mathrm{Im}[\gamma] = \omega\sqrt{LC}.$$

Hence, a lossless line doesn't attenuate (no big surprise). Since the attenuation is the same (zero) at all frequencies, a lossless line *has no bandwidth limit*. In addition, the propagation constant has an imaginary part that is exactly proportional to frequency. Since the delay of a system is simply (minus) the derivative of phase with frequency, the delay of a lossless line is a constant, independent of frequency:

$$T_{\mathrm{delay}} = -\frac{\partial}{\partial\omega}\Phi(\omega) = -\frac{\partial}{\partial\omega}(-\beta z) = \sqrt{LC}\,z.$$

Delay of a Lossless Coax T-Line



a = radius of inner conductor

b = inner radius of the outer conductor

c = outer radius of outer conductor

$$\left(\frac{C}{h}\right) = \frac{2\pi\epsilon}{\ln(\ b/a\ )} = \frac{2\pi\epsilon_0\epsilon_r}{\ln(\ b/a\ )}$$

$$\left(\frac{L}{h}\right) = \frac{\mu}{2\pi}\ln(\ b/a\ ) = \frac{\mu_0\mu_r}{2\pi}\ln(\ b/a\ )$$

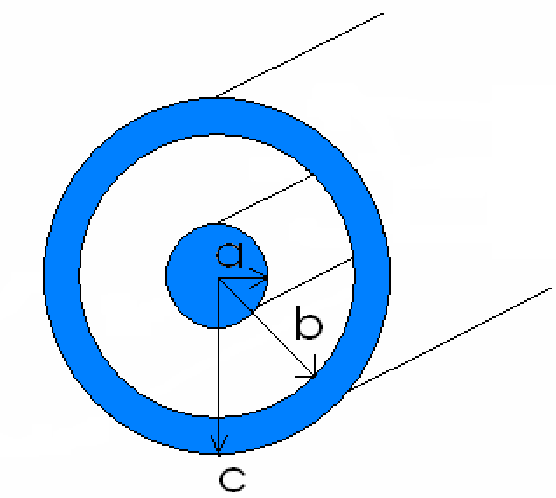
From Field and Waves: $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \quad v_p = \frac{1}{\sqrt{\mu\varepsilon}} \quad n = \sqrt{\mu_r\varepsilon_r} \quad v_p = \frac{c}{n}$

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_r\varepsilon_r} \qquad \beta = 2\pi f\frac{1}{c}\sqrt{\mu_r\varepsilon_r} = \frac{2\pi}{\lambda}$$

Loss-less Transmission Line: $V(z) = V_0 e^{-j\beta z} = V_0 e^{-j\frac{2\pi}{\lambda}z}$

$$T_{Delay} = \frac{z}{v_p}$$

Example



a = radius of inner conductor

b =inner radius of the outer conductor

c =outer radius of outer conductor

Assume a loss-less Transmission Line with following specifications:
z=10cm

$$\mu_r = 1, \quad \varepsilon_r = 3$$

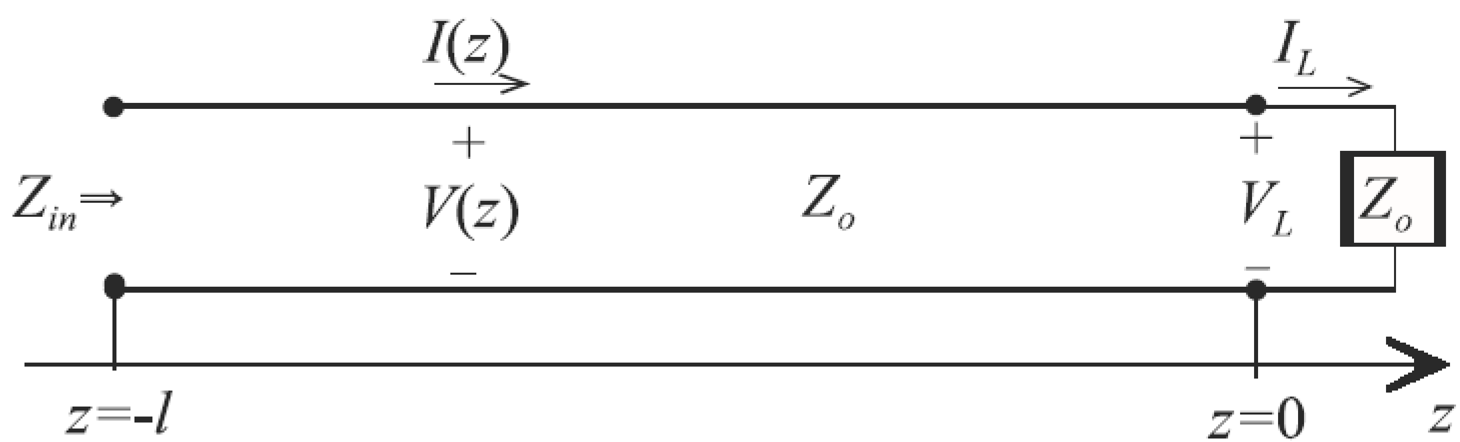
What is the delay?

$$n = \sqrt{\mu_r \varepsilon_r} = 1.7, \quad v = \frac{c}{n} = 176\frac{m}{\mu s}$$

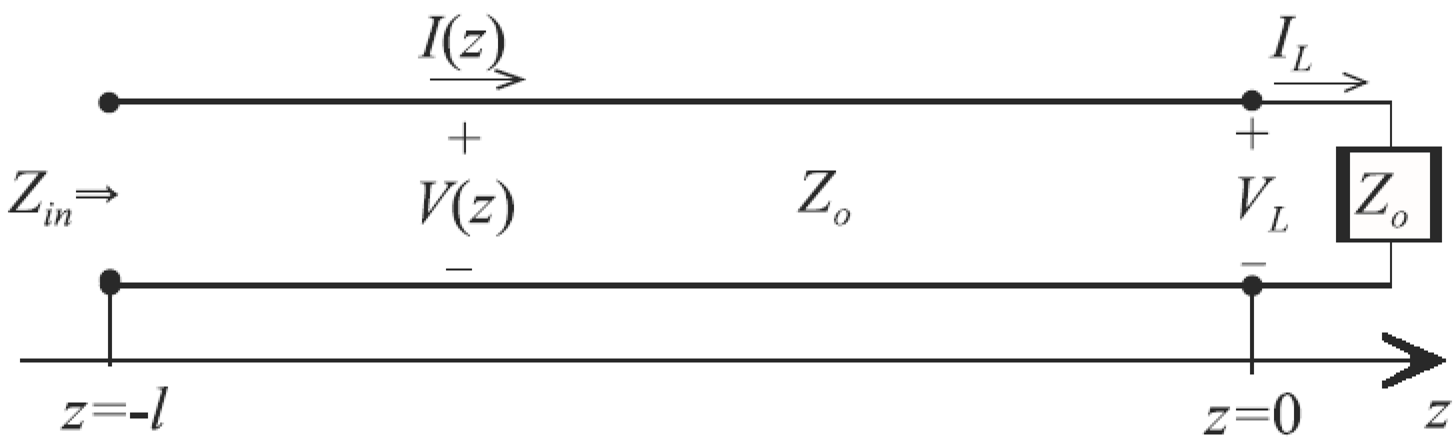
$$T_{delay} = \frac{0.1m}{176\frac{m}{\mu s}} = 0.57ns$$

Transmission Line with Matched Termination

The driving-point impedance of an infinitely long line is simply Z_0. Suppose we cut the line somewhere, discard the infinitely long remainder, and replace it with a single lumped impedance of value Z_0. The driving-point impedance must remain Z_0: there's no way for the measurement apparatus to distinguish the lumped impedance from the line it replaces. Hence, a signal applied to the line simply travels down the finite segment of line, eventually gets to the resistor, heats it up, and contributes to global warming.



Transmission Line with Matched Termination



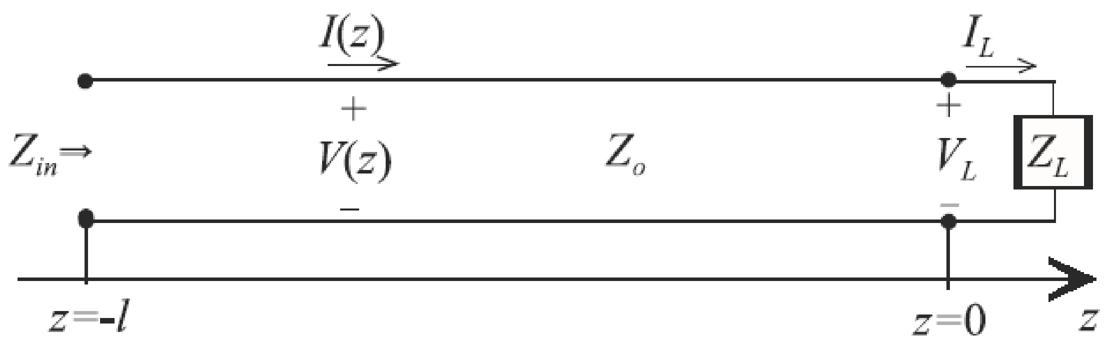
$$\Gamma = 0$$

$$Z_{in} = Z_o \qquad \text{(independent of line length)}$$

$$V(z) = V_o^+ e^{-j\beta z} \qquad |V(z)| = |V_o^+|$$

$$I(z) = \frac{V_o^+}{Z_o} e^{-j\beta z} \qquad |I(z)| = \frac{|V_o^+|}{Z_o}$$

Transmission Line with Arbitrary Termination



$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{+\gamma z}$$

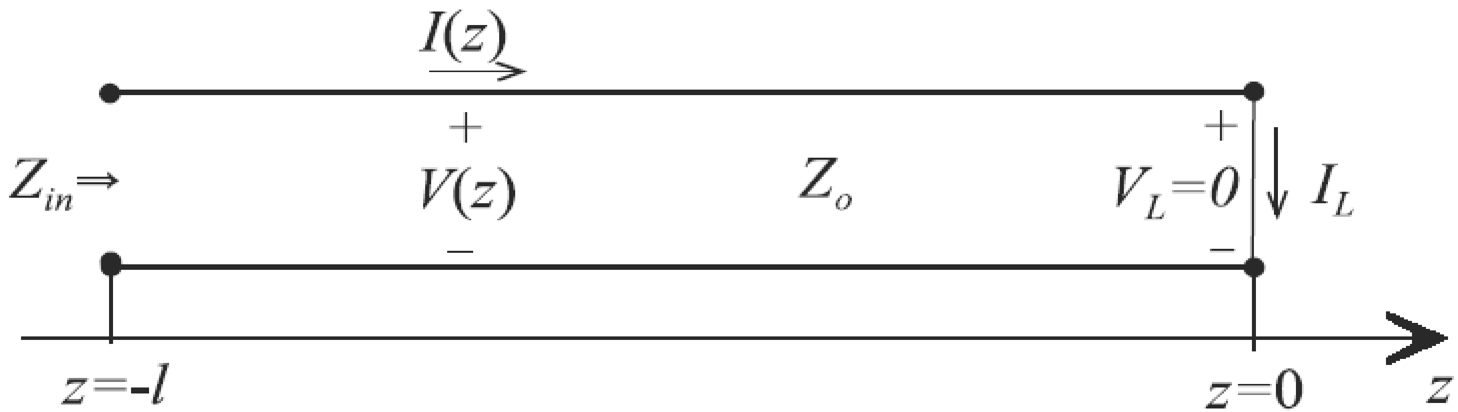
$$\Gamma(z) = \frac{V_0^- e^{+\gamma z}}{V_0^+ e^{-\gamma z}} = \Gamma(z=0) e^{+2\gamma z} = \Gamma_L e^{+2\gamma z}$$

$$\frac{Z(z)}{Z_0} = \frac{\frac{Z_L}{Z_0} - \tanh \gamma z}{1 - \frac{Z_L}{Z_0} \tanh \gamma z} \qquad \Gamma(z) = \Gamma_L e^{2\gamma z}$$

Loss-less Transmission Line:

$$\frac{Z(z)}{Z_0} = \frac{\frac{Z_L}{Z_0} - j tan(\beta z)}{1 - j\frac{Z_L}{Z_0} tan(\beta z)} \qquad \Gamma(z) = \Gamma_L e^{j2\beta z}$$

Short-Circuited Transmission Line



$$\Gamma(Z) = -e^{j2\beta Z} \rightarrow \Gamma(Z=0) = -1$$

$$\forall\, Z \leq 0 \rightarrow |\Gamma(Z)| = 1$$

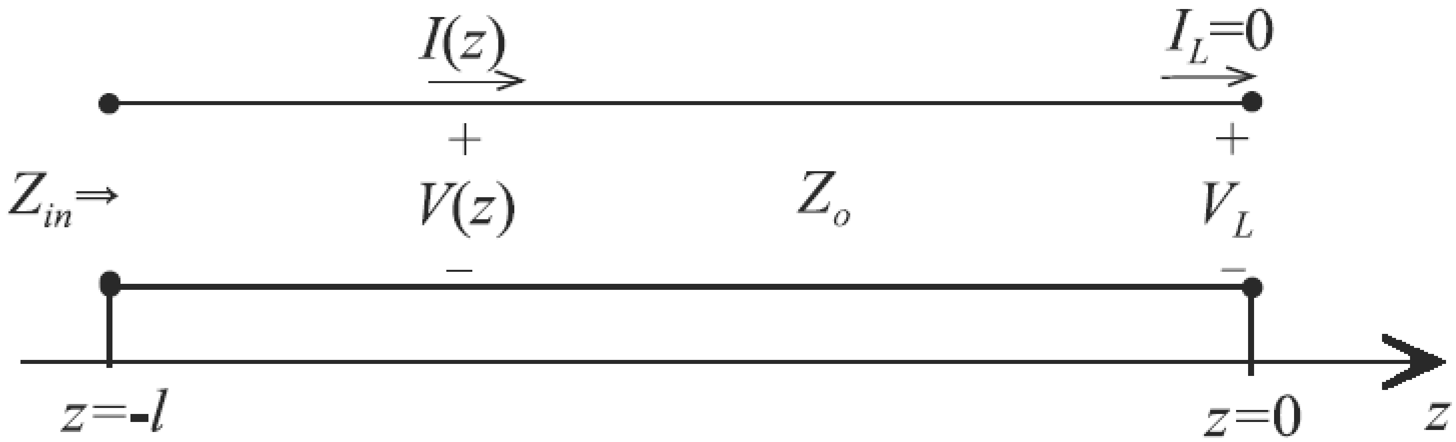
$Z_{in} = jZ_o \tan\beta l$ (purely reactive, dependent on line length)

The input impedance of a short-circuited lossless transmission line is purely reactive and can take on any value of capacitive or inductive reactance depending on the line length.

$l = n\dfrac{\lambda}{2}$ $\quad |Z_{in}| = 0 \quad n = 0, 1, 2, \ldots$

$l = (2n-1)\dfrac{\lambda}{4}$ $\quad |Z_{in}| = \infty \quad n = 1, 2, \ldots$

Open-Circuited Transmission Line



$$\Gamma = -1$$

$$Z_{in} = -jZ_o \cot \beta l$$ **(purely reactive, dependent on line length)**

The input impedance of a short-circuited lossless transmission line is purely reactive and can take on any value of capacitive or inductive reactance depending on the line length.

$$l = n\frac{\lambda}{2} \qquad |Z_{in}| = \infty \qquad n = 0, 1, 2, \ldots$$

$$l = (2n-1)\frac{\lambda}{4} \qquad |Z_{in}| = 0 \qquad n = 1, 2, \ldots$$

Example#1

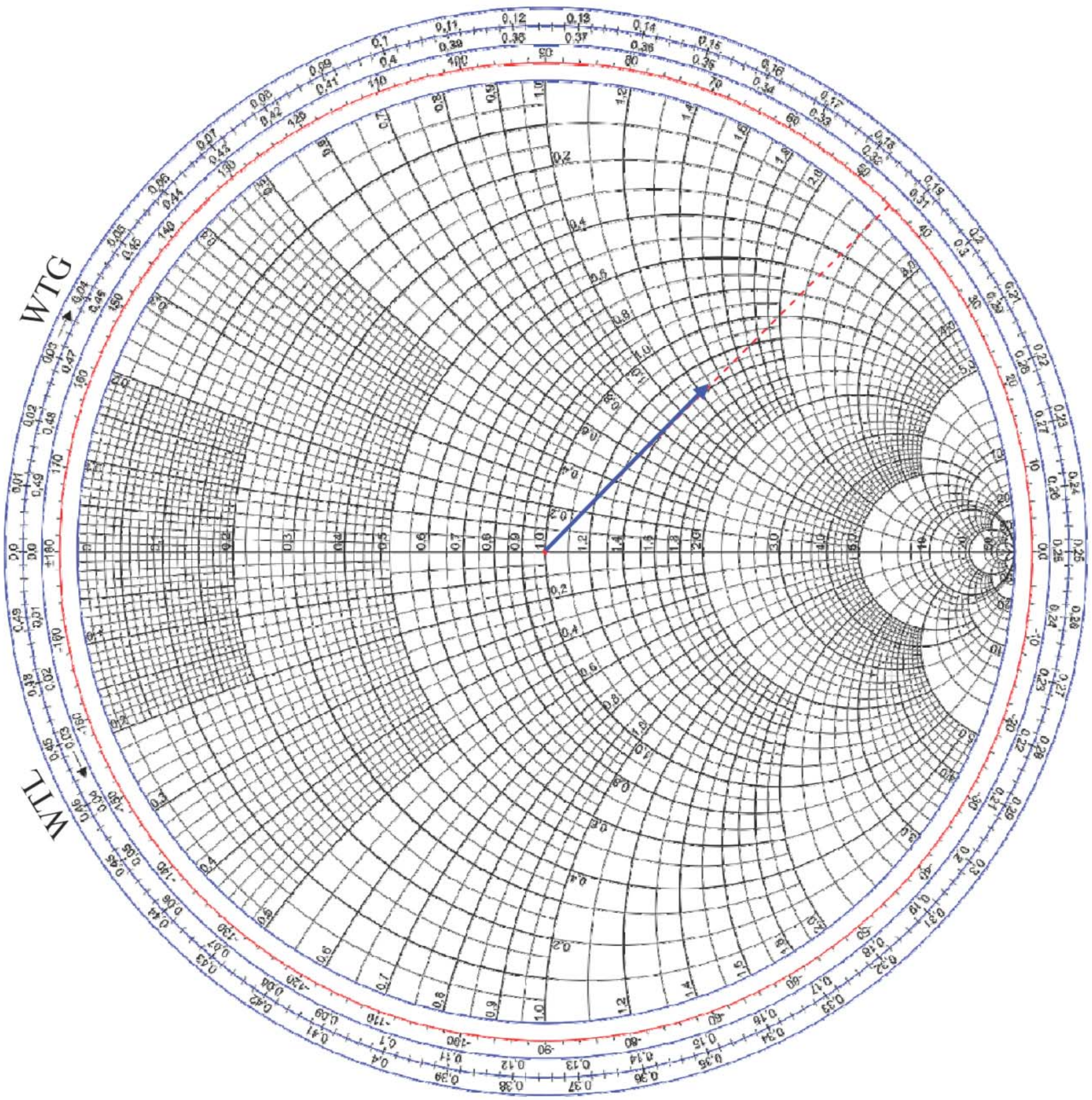
Given:

$$\Gamma_L = 0.5\angle 45°$$

$$Z_o = 50\Omega$$

What is Z_L?

$$Z_L = 50\Omega(1.39 + j1.35)$$
$$= 69.5\Omega + j67.5\Omega$$



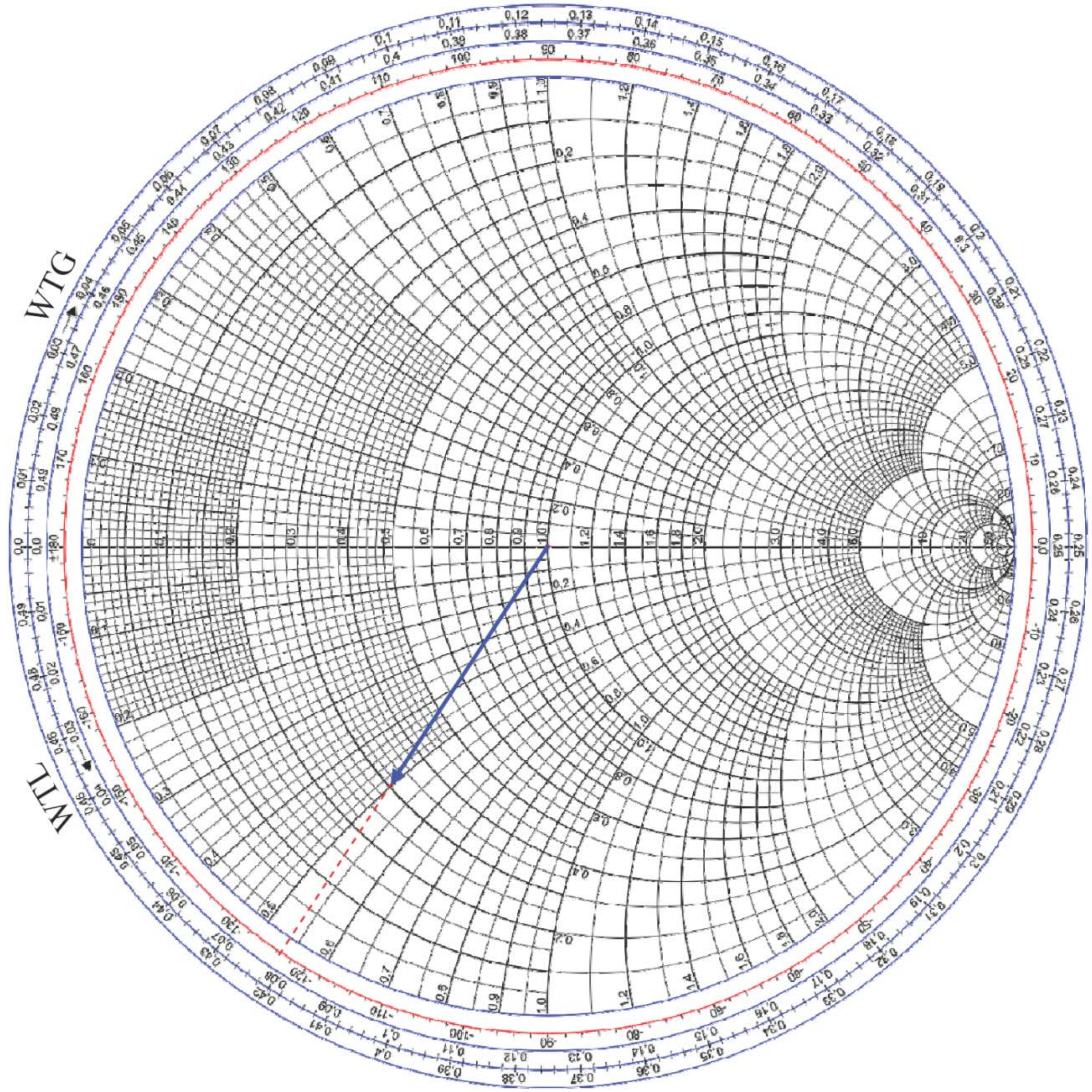
Example#2

Given:

$$Z_L = 15\Omega - j25\Omega$$
$$Z_o = 50\Omega$$

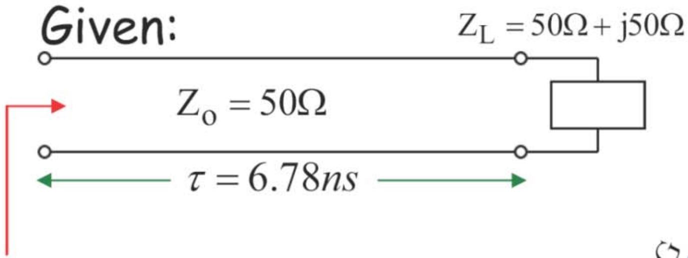
What is Γ_L?

$$z_L = \frac{15\Omega - j25\Omega}{50\Omega}$$
$$= 0.3 - j0.5$$
$$\Gamma_L = 0.6\angle -123°$$



Example#3

Given:



What is Z_{in} at 50 MHz?

$$z_L = \frac{50\Omega + j50\Omega}{50\Omega}$$

$$= 1.0 + j1.0$$

$$\Gamma_L = 0.445\angle 64°$$

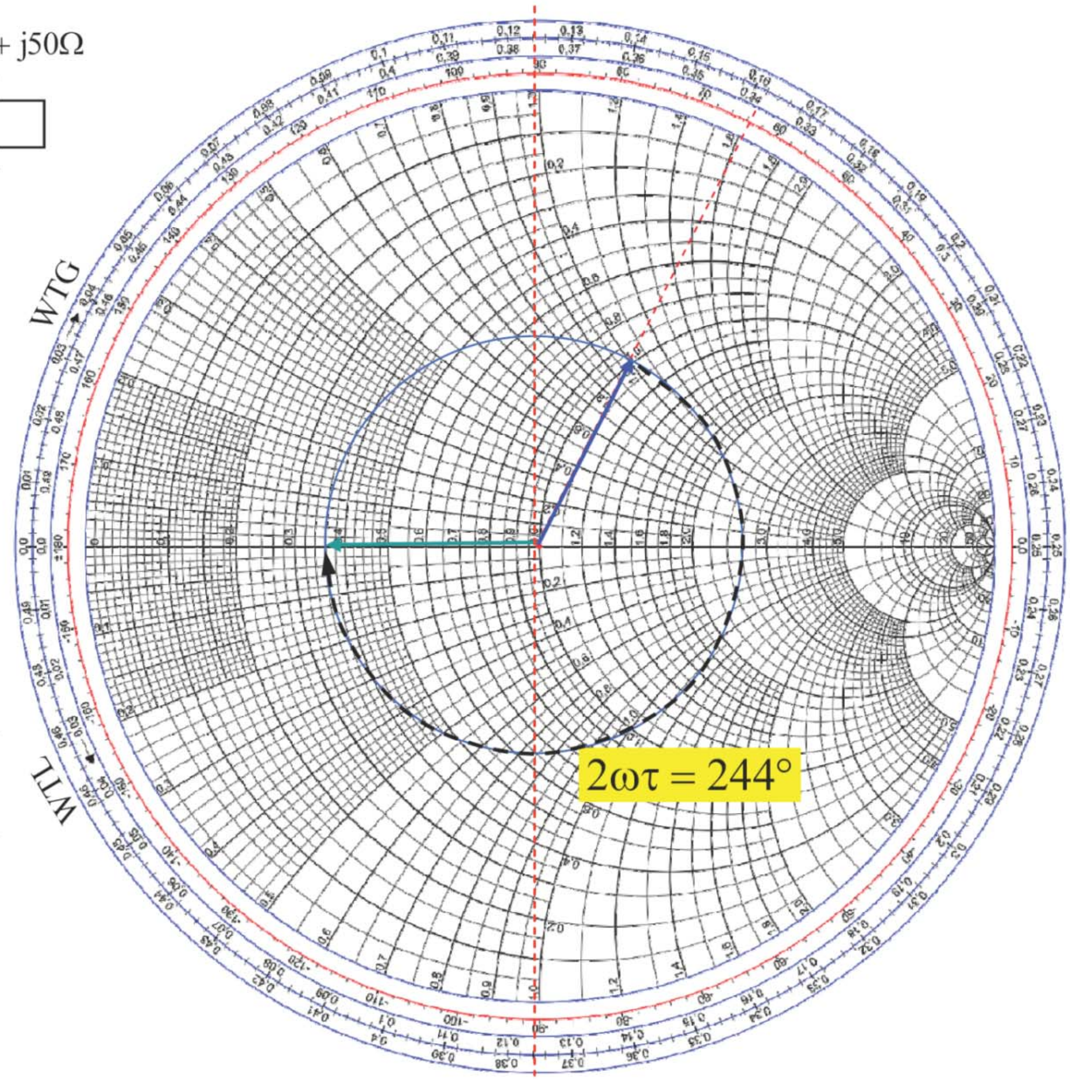
$$\Gamma_{in} = \Gamma_L e^{-j2\beta l} = \Gamma_L e^{-j4\pi l/\lambda} = \Gamma_L e^{-j2\omega\tau}$$

$$l = f\lambda\tau = 50 \cdot 10^6 \cdot 6.78 \cdot 10^{-9}\,\lambda = 0.339\lambda$$

$$\vartheta_{in} = 180°$$

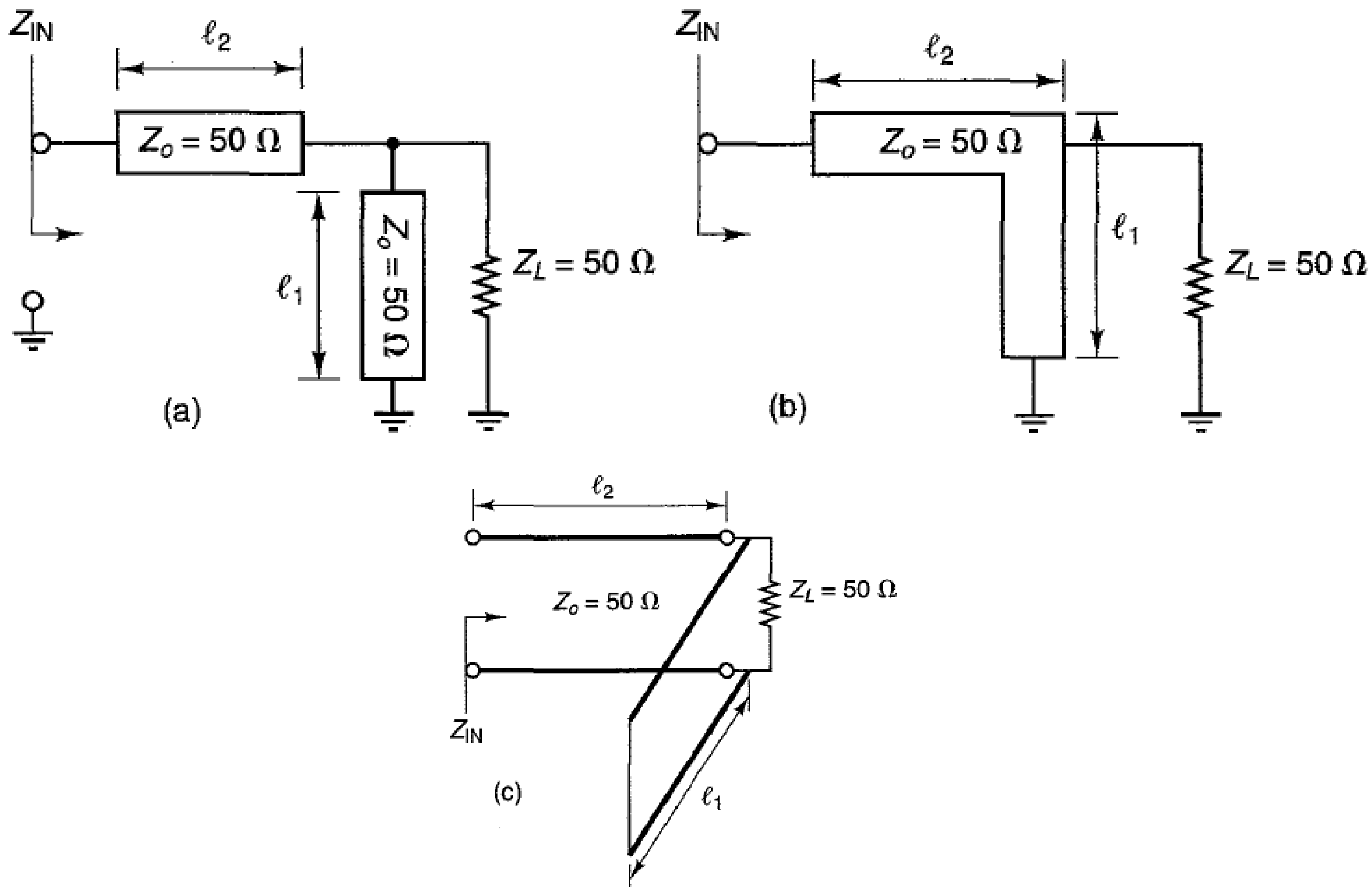
$$\Gamma_{in} = 0.445\angle 180°$$

$$Z_{in} = 50\Omega(0.38 + j0.0) = 19\Omega$$



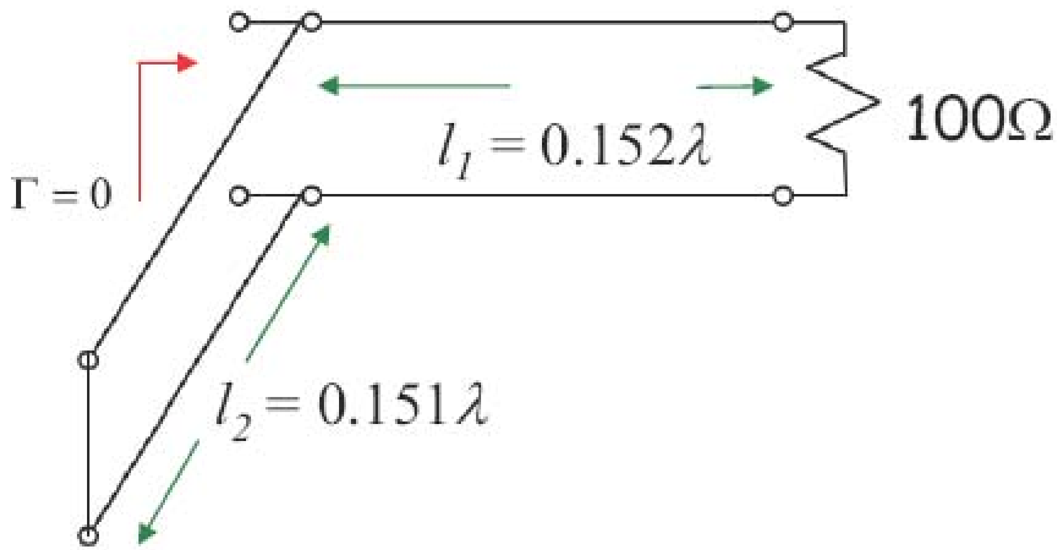
Matching Networks With Microstrip Lines

A microstrip line can be used as a series transmission line, as an open-circuited stub, or as a short-circuited stub. In fact, a series microstrip line together with a short- or open-circuited shunt stub can transform a 50-Ω resistor into any value of impedance.



Example#4 (Single Stub Tuner)

Prove that Γ=0



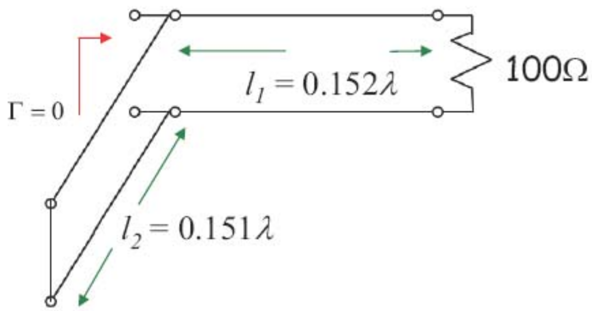
$$\frac{Z(z)}{Z_0} = \frac{\frac{Z_L}{Z_0} - jtan(\beta z)}{1 - j\frac{Z_L}{Z_0}tan(\beta z)} \qquad \Gamma(z) = \Gamma_L e^{j2\beta z}$$

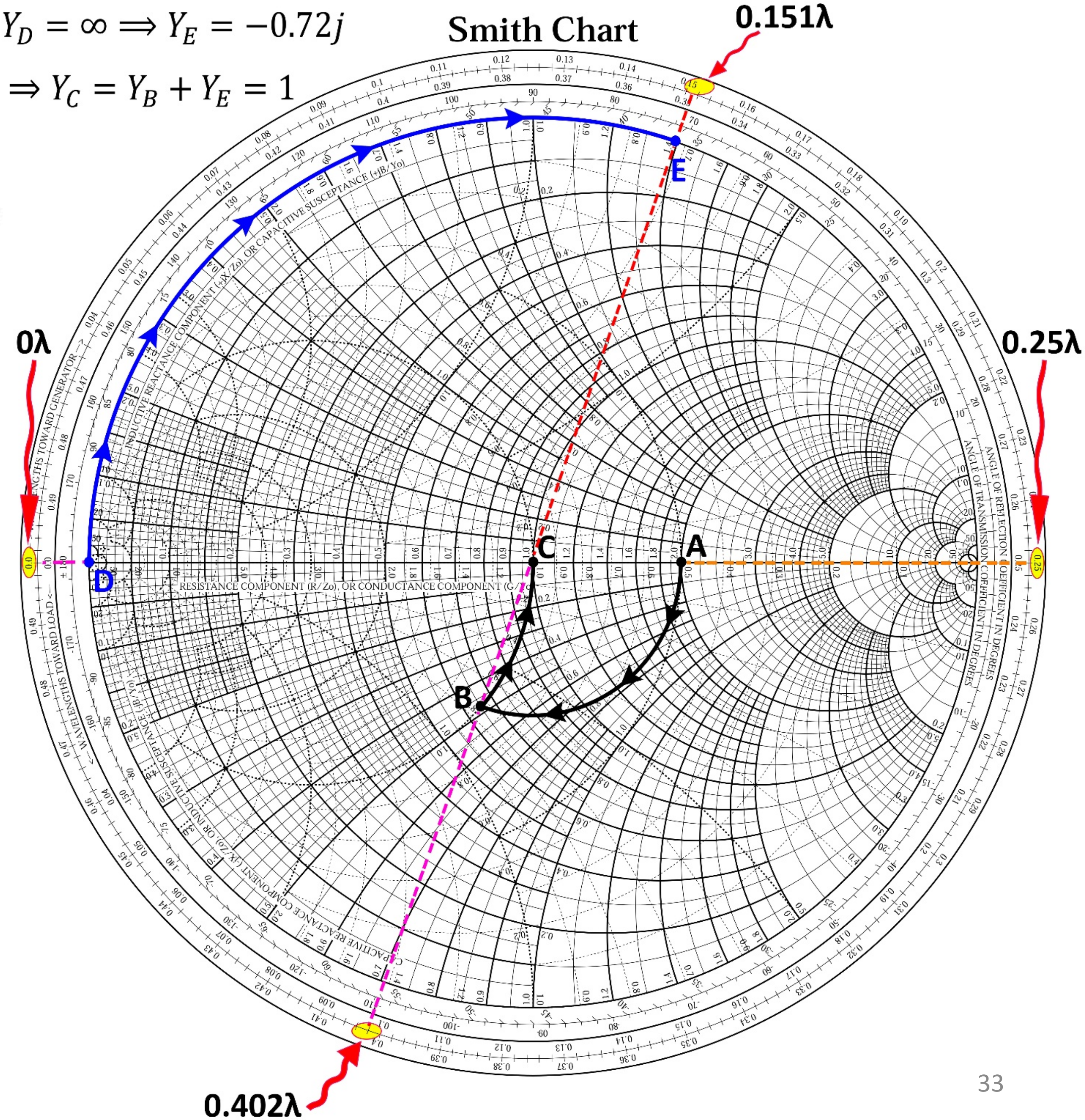
Cont'd

$$Z_L = 100\Omega \xrightarrow{Normalize} Z_A = \widehat{Z_L} = 2\ ,\ \ 0.25\lambda + 0.152\lambda = 0.402\lambda\ \ \Longrightarrow Y_B = 1 + 0.72j$$

$$Y_D = \infty \Longrightarrow Y_E = -0.72j$$

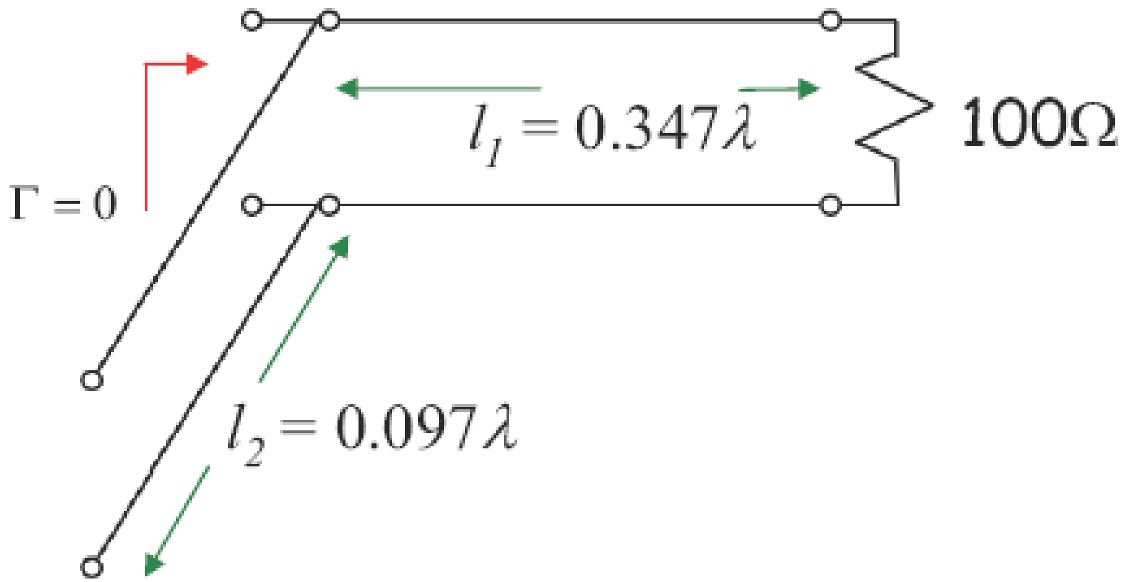
$$\Rightarrow Y_C = Y_B + Y_E = 1$$





Example#5 (Single Stub Tuner)

Prove that Γ=0

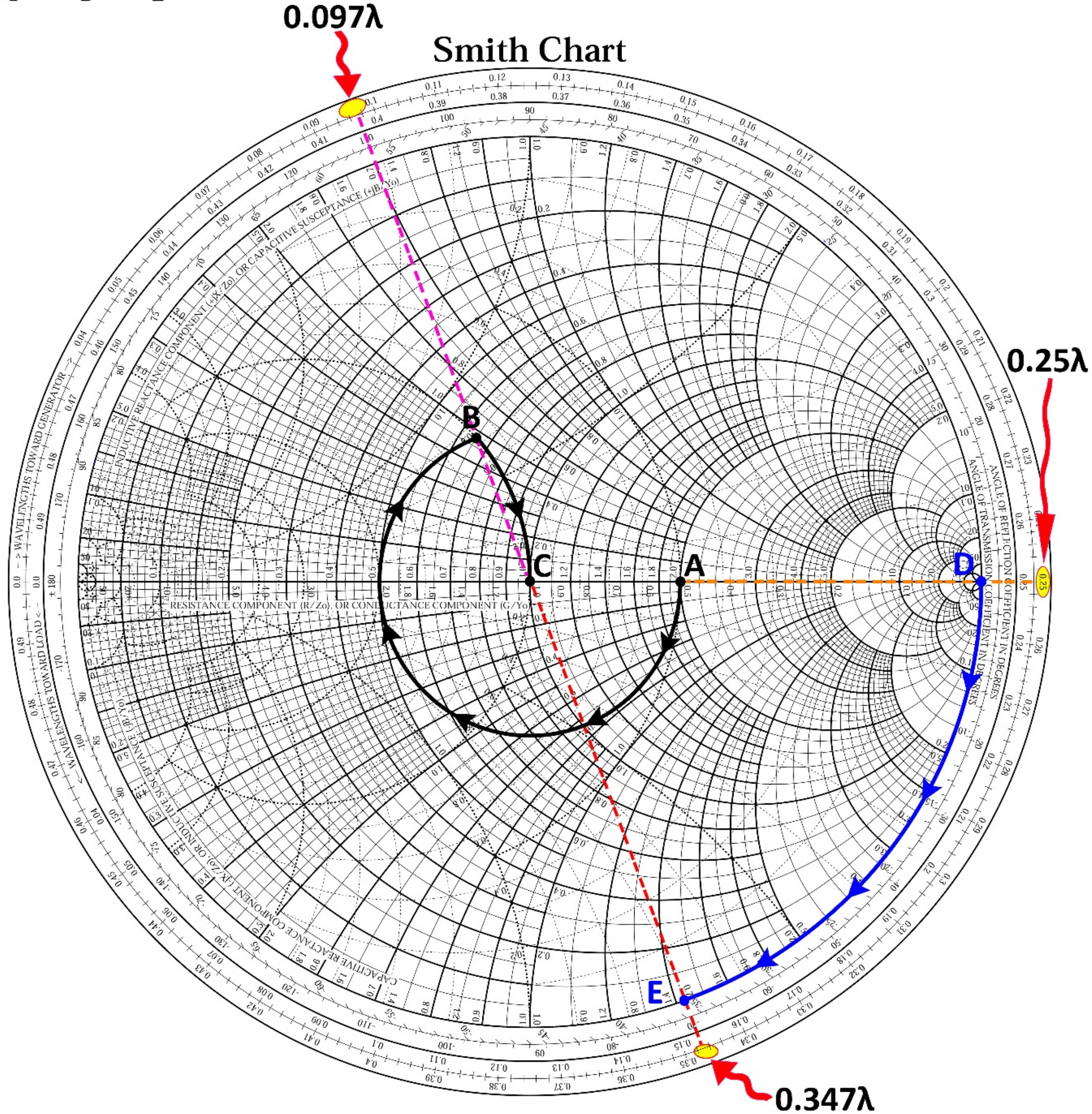


$$\frac{Z(z)}{Z_0} = \frac{\frac{Z_L}{Z_0} - jtan(\beta z)}{1 - j\frac{Z_L}{Z_0}tan(\beta z)}$$

$$Z_L = 100 \xrightarrow{Normalize} Z_A = \widehat{Z_L} = 2 \quad , \quad \begin{cases} Load \to 0.25\lambda + 0.347\lambda = 0.5\lambda + 0.097\lambda \\ Open\ Stub \to 0.25\lambda + 0.097\lambda = 0.347\lambda \end{cases}$$

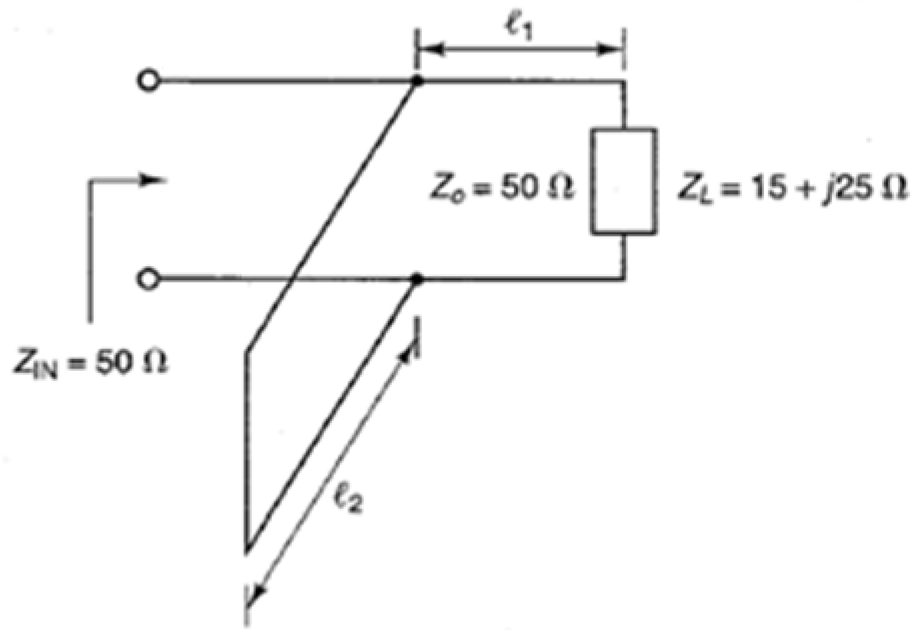
$$\Rightarrow Y_B = 1 - 0.72j \quad , \quad Y_D = 0 \Rightarrow Y_E = 0.72j$$

$$Y_C = Y_B + Y_E = 1$$



Example#6

Find L1 and L2 so that Z_L is matched to 50 ohm at the input terminal.

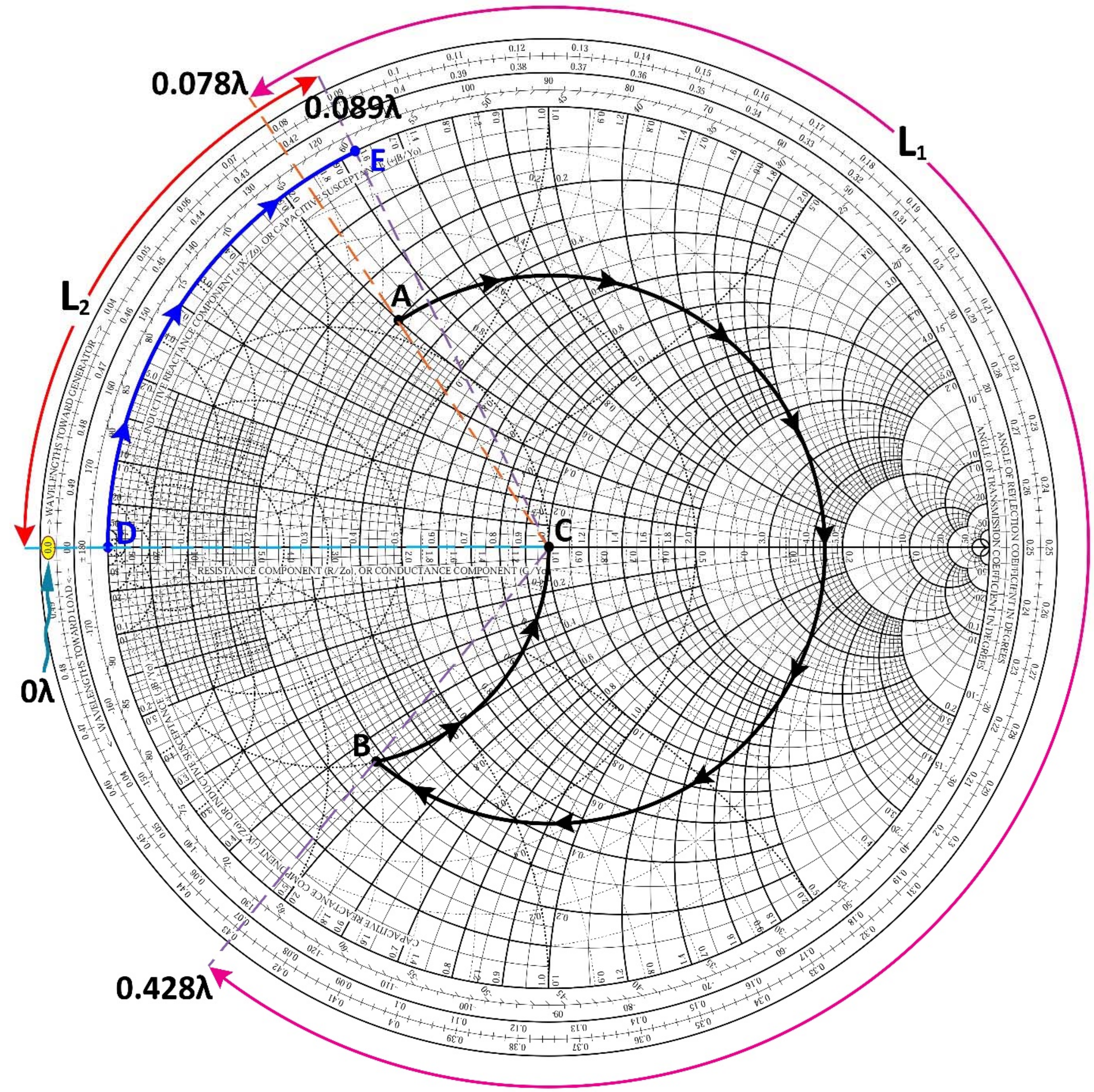


Matching of an arbitrary load to 50ohm

$$Z_L = 15 + 25j \Rightarrow \widehat{Z_L} = 0.3 + 0.5j \xrightarrow{L_1 = 0.428\lambda - 0.078\lambda = 0.35\lambda} Y_B = 1 + 1.6j$$

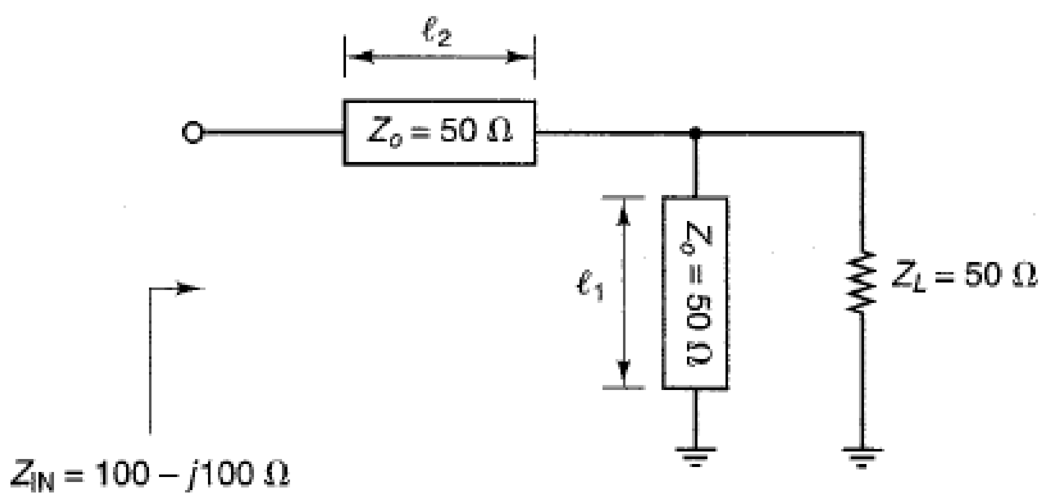
$$Y_D = \infty \xrightarrow{L_2 = 0.089\lambda} Y_E = -1.6j \longrightarrow Y_C = Y_B + Y_E = 1$$

Smith Chart



Example#7

Find L1 and L2 so that Z_L is matched to 100-j100 at the input terminal.



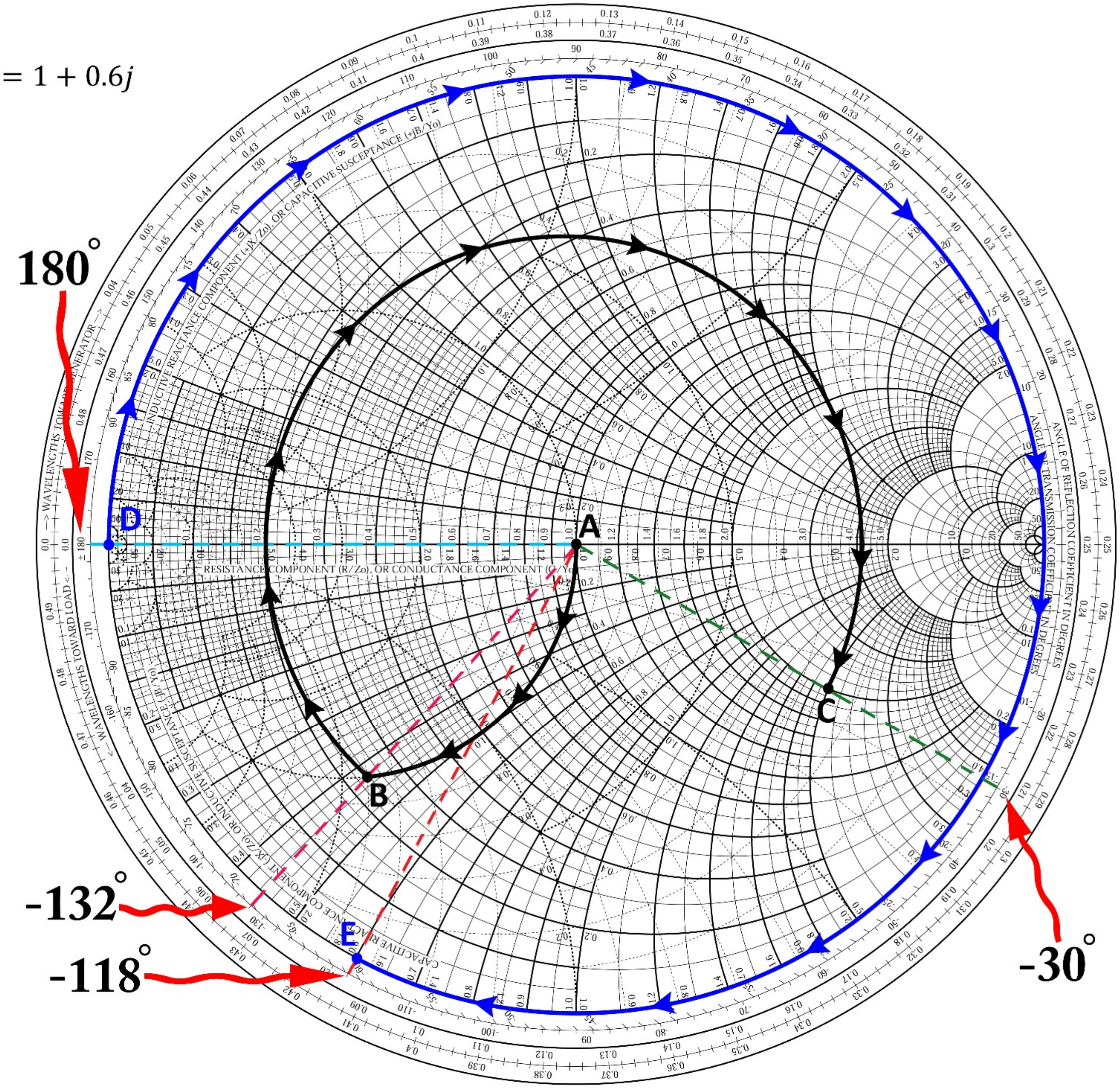
Matching of a 50ohm load to an arbitrary impedance

$Z_{in} = 100 - 100j \implies \widehat{Z_a} = 2 - 2j \quad , \quad Y_{in}(-L_1) = +1.7j \Leftrightarrow \acute{\Gamma}(-L_1) = \acute{\Gamma}_{\mathrm{L}} \times e^{j2\beta z} = \acute{\Gamma}_{\mathrm{L}} \times e^{-j\frac{4\pi}{\lambda}L_1} = \acute{\Gamma}_{\mathrm{L}} \times e^{-j\acute{\theta}}$

$\acute{\theta} = -118^\circ - 180^\circ = 298^\circ \implies \frac{4\pi}{\lambda}L_1 = \frac{298^\circ}{360^\circ} \times 2\pi \implies \boxed{L_1 = 0.41\lambda}\ ,\ \frac{4\pi}{\lambda}L_2 = \frac{258^\circ}{360^\circ} \times 2\pi \implies \boxed{L_2 = 0.36\lambda}$

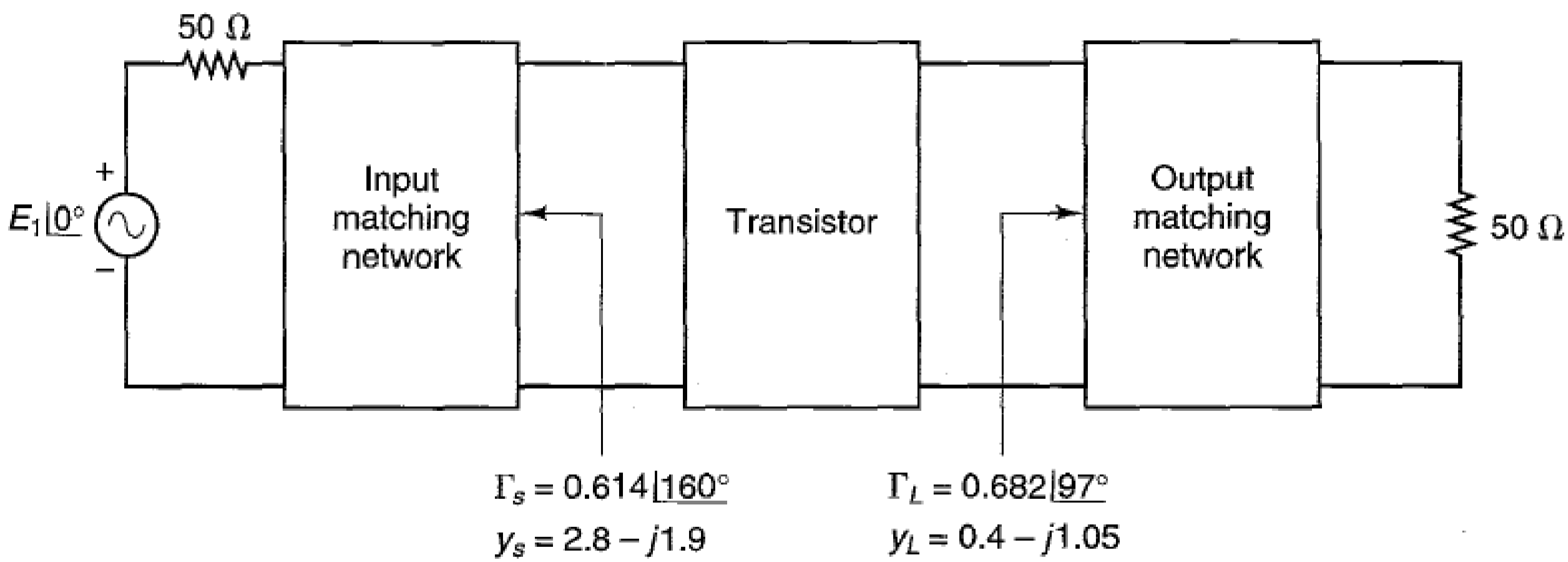
$$\begin{cases} Y_A = 1 \\ Y_D = \infty \xrightarrow{T.L_1} Y_E = 0.6j \end{cases} \implies Y_B = Y_A + Y_E = 1 + 0.6j$$

$$Y_B = 1 + 0.6j \xrightarrow{T.L_2} Z_C = 2 - 2j$$



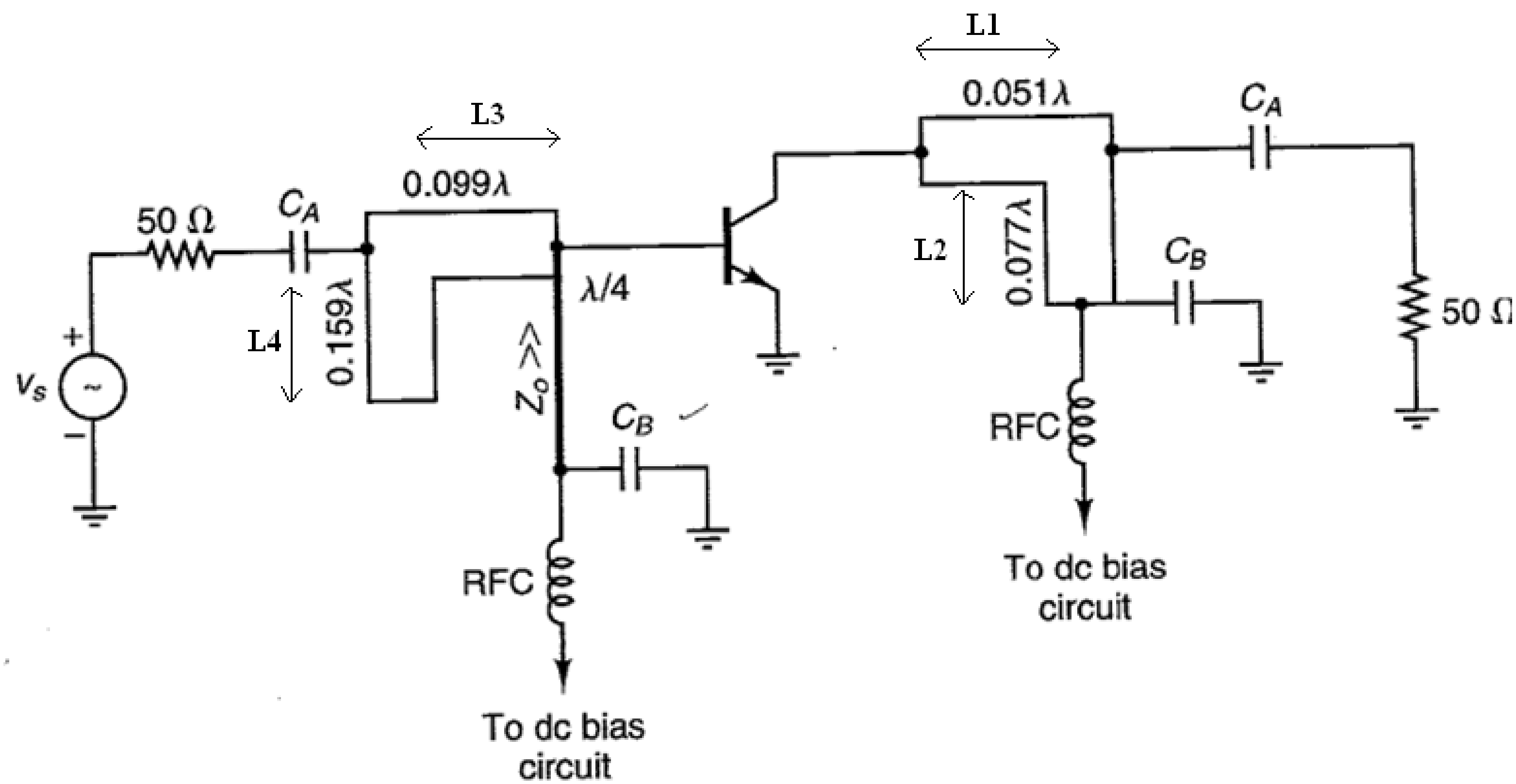
Example#8

Design two microstrip matching networks for the amplifier shown bellow whose reflection coefficients for a good match, in a 50-Ω system, are $\Gamma_s = 0.614\angle 160^\circ$ and $\Gamma_L = 0.682\angle 97^\circ$.



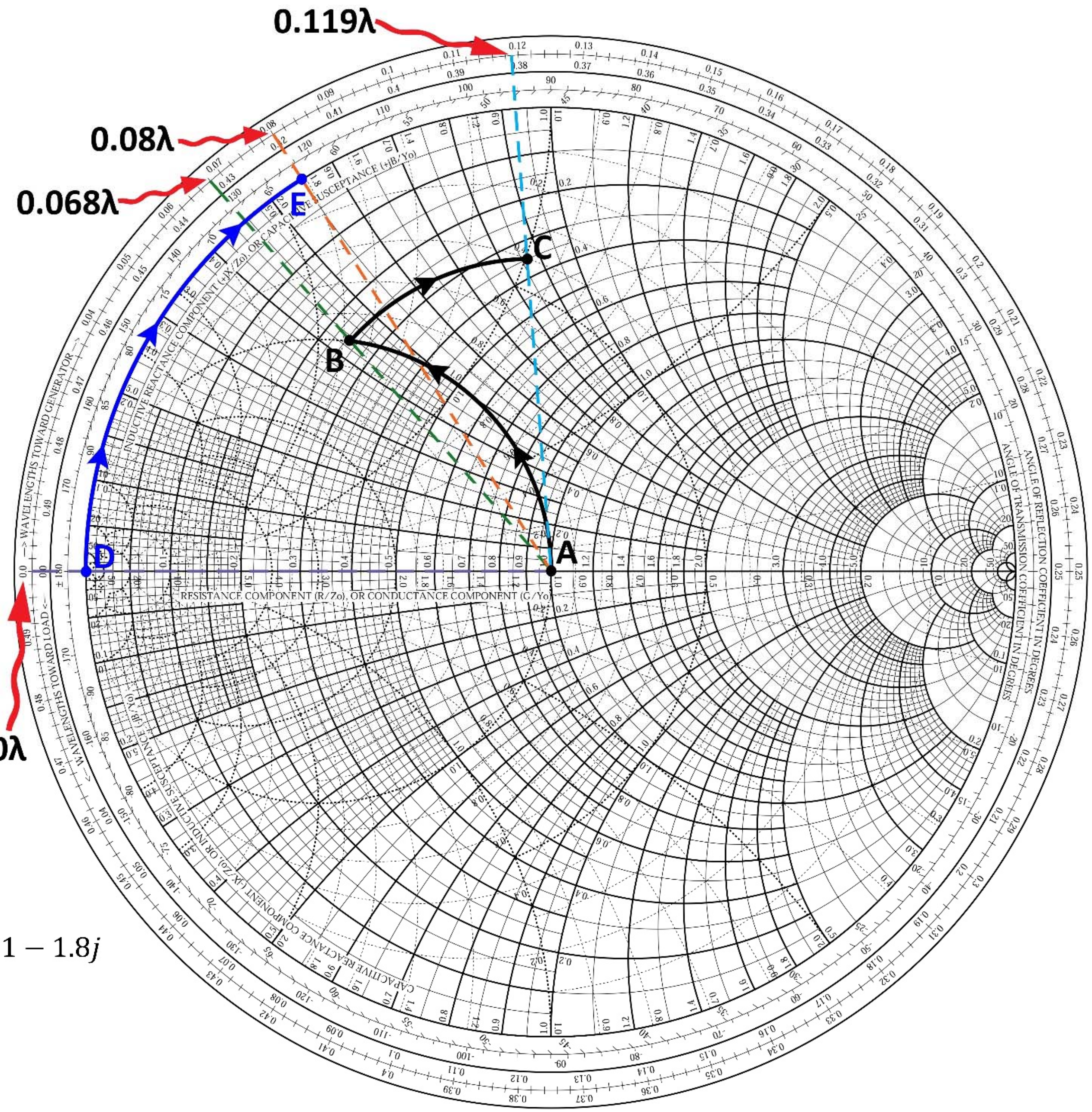
Amplifier block diagram.

Cont'd



The characteristic impedance of the microstrip line is 50 Ω

محاسبات در خروجی

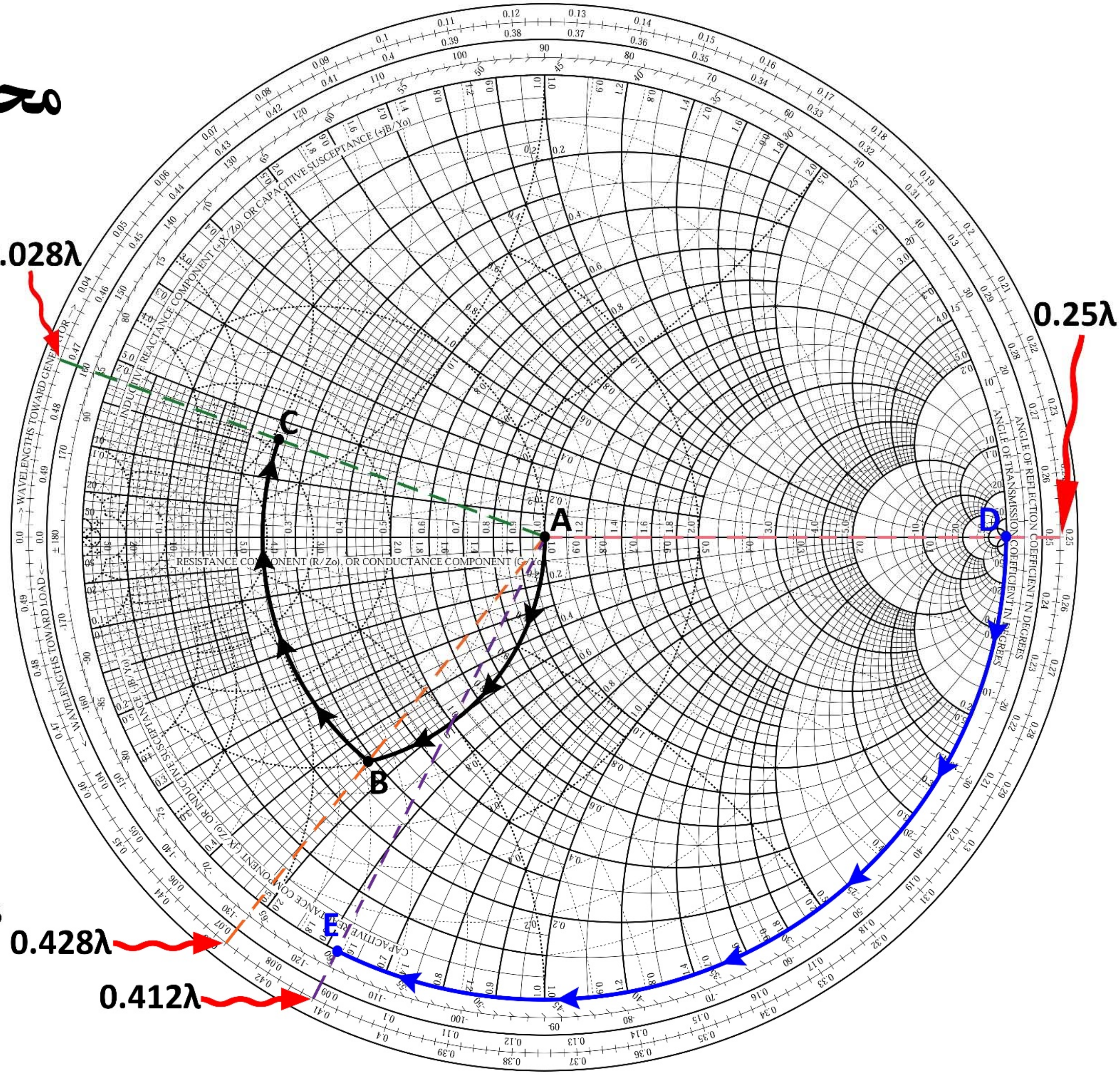


$$\begin{cases} Y_D = \infty \xrightarrow{T.L_2} Y_E = -1.8j \\ Y_A = 1 \end{cases} \Rightarrow Y_B = Y_A + Y_E = 1 - 1.8j$$

$$Y_B \xrightarrow{T.L_1} Y_C = 0.4 - 1.05j$$

$$\Rightarrow \begin{cases} \boxed{L_2 = 0.08\lambda} \\ \boxed{L_2 = 0.119\lambda - 0.068\lambda = 0.051\lambda} \end{cases}$$

محاسبات در ورودی

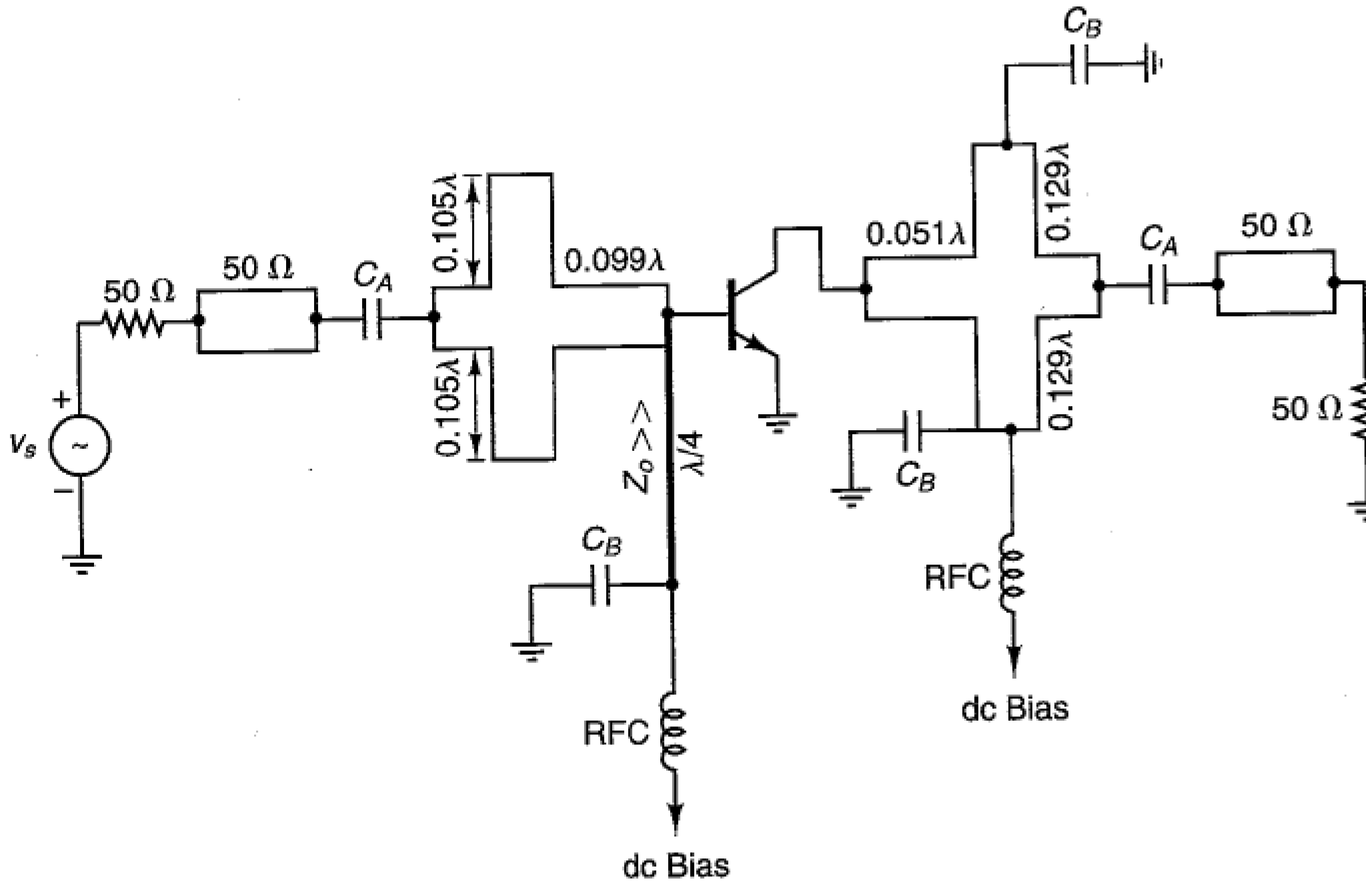


$$\begin{cases} Y_D = 0 \xrightarrow{T.L_4} Y_E = 1.6j \\ Y_A = \end{cases} \Longrightarrow Y_B = 1 + 1.6$$

$$Y_B \xrightarrow{T.L_3} Y_C = 2.8 - 1.9j$$

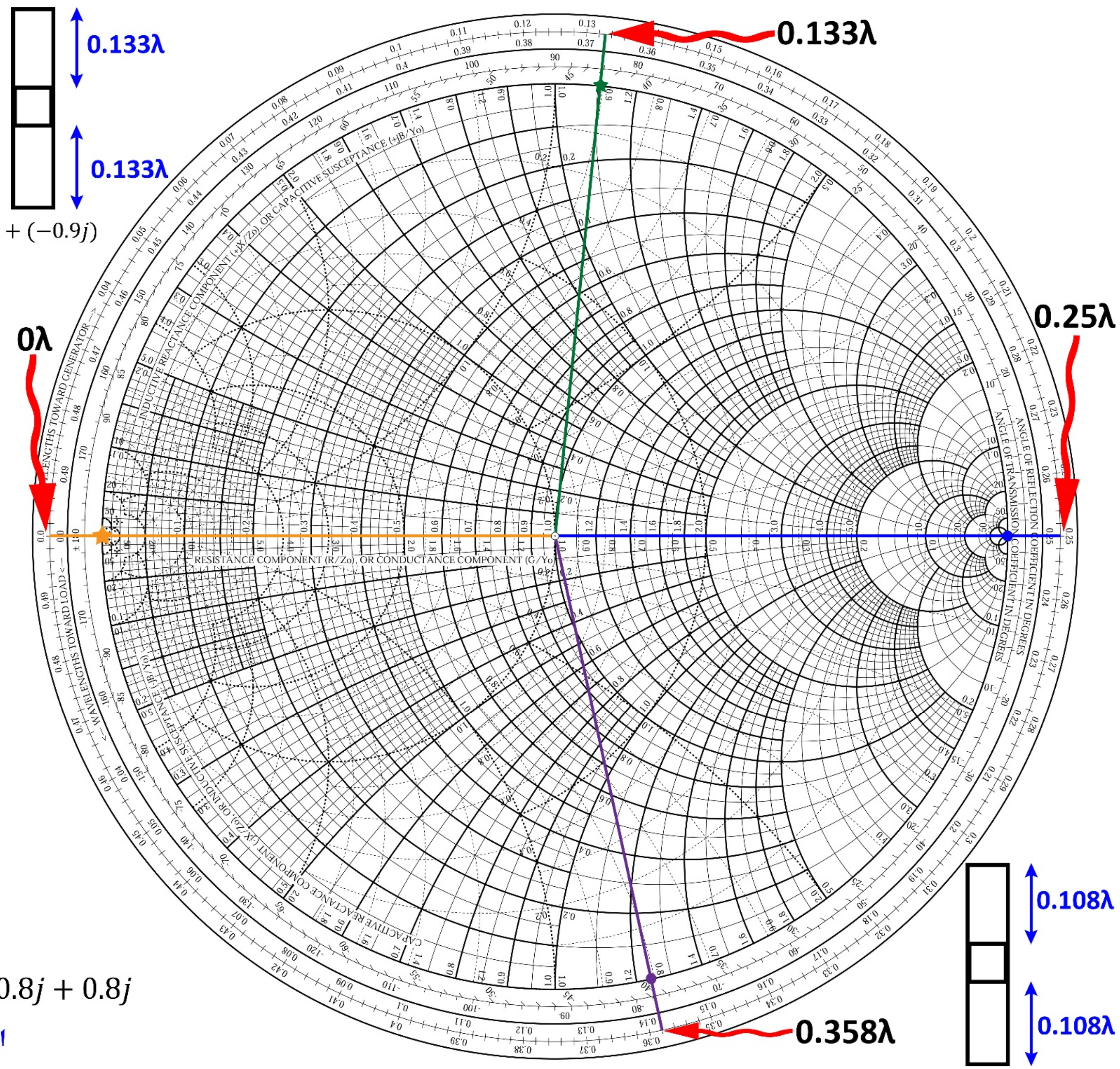
$$\Longrightarrow \begin{cases} \boxed{L_3 = 0.1\lambda} \\ \boxed{L_4 = 0.412\lambda - 0.25\lambda = 0.162\lambda} \end{cases}$$

Cont'd



Complete amplifier schematic using balanced shunt stubs. The characteristic impedance of the microstrip lines is 50 Ω.

بالانس کردن خطوط در خروجی



$-1.8j = (-0.9j) + (-0.9j)$: سمت خروجی

ادمیتانس مورد نیاز

$+1.6j = 0.8j + 0.8j$: سمت ورودی

ادمیتانس مورد نیاز

Cont'd

If we use RT/Duroid® with $\varepsilon_r = 2.23$ and $h = 0.7874$ mm to build the amplifier, we find that a characteristic impedance of 50 Ω is obtained with $W = 2.42$ mm and $\varepsilon_{ff} = 1.91$. The microstrip wave length in the 50-Ω Duroid microstrip line is $\lambda = \lambda_0/\sqrt{1.91} = 0.7236\lambda_0$, where $\lambda_0 =$ 30 cm at $f = 1$ GHz. For a characteristic impedance of 100 Ω in the $\lambda/4$ line, the width must be $W = 0.7$ mm. The line lengths are

$$0.105\lambda = 2.28 \text{ cm}$$

$$0.099\lambda = 2.15 \text{ cm}$$

$$0.051\lambda = 1.10 \text{ cm}$$

$$0.129\lambda = 2.80 \text{ cm}$$

$$\lambda/4 = 5.43 \text{ cm}$$

h=31mil
See slide 14 → $W = 91.7$ mil, $\varepsilon_{ff} = 1.91$ → W=2.3mm

Cont'd

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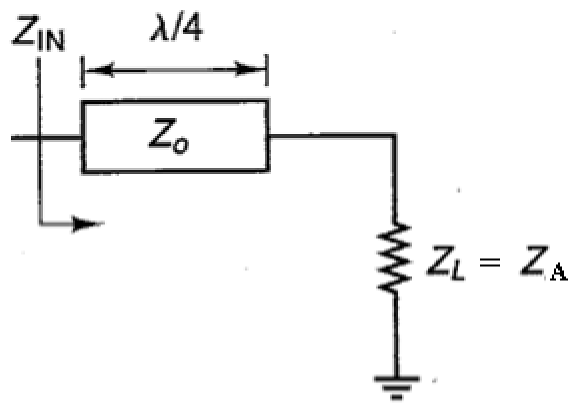
h=31mil
See slide 14 → $W = 91.7$ mil, $\varepsilon_{ff} = 1.91$ → W=2.3mm

λ/4 T.L. Properties



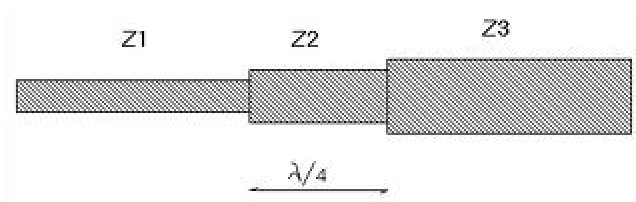
$$\frac{Z(z)}{Z_0} = \frac{\frac{Z_L}{Z_0} - j tan(\beta z)}{1 - j\frac{Z_L}{Z_0} tan(\beta z)}$$

$$Z_{in}(L = \frac{\lambda}{4}) = Z_0 \left[\frac{Z_A + jZ_0 \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right)}{Z_0 + jZ_A \tan\left(\frac{2\pi}{\lambda}\frac{\lambda}{4}\right)} \right]$$

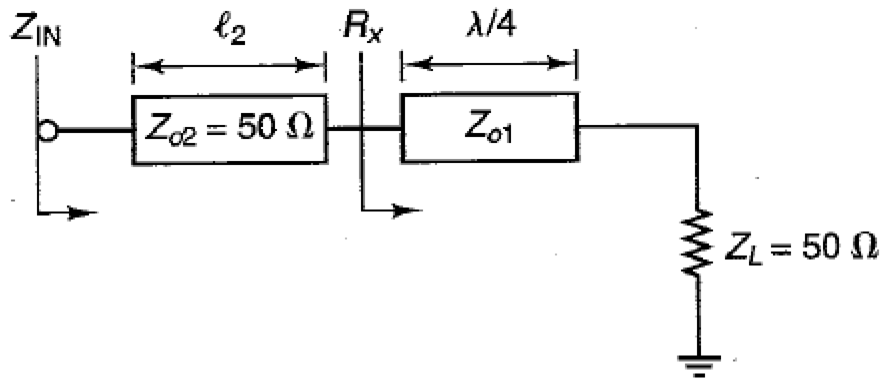
∞ ∞

$$Z_{in}(L = \frac{\lambda}{4}) = \frac{Z_0^2}{Z_A}$$

Therefore the quarter wave T.L. acts as an impedance transformer.



Example#9



We need Zin=33+j50. Find L_2 and Z_{o1}?

$$\bar{Z}_{in} = 0.66 + j1$$

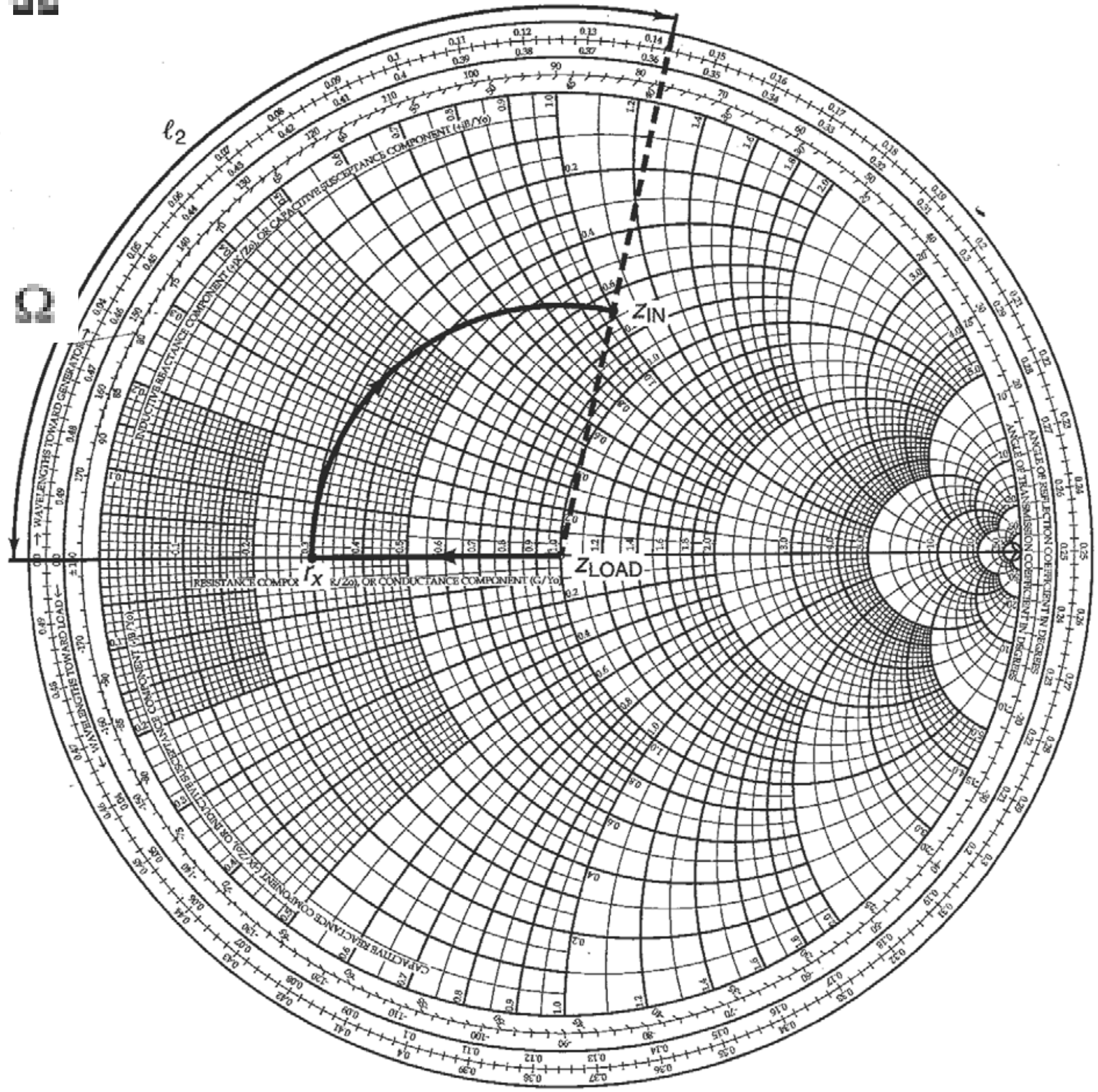
Cont'd

$R_x = 50(0.3) = 15\,\Omega$

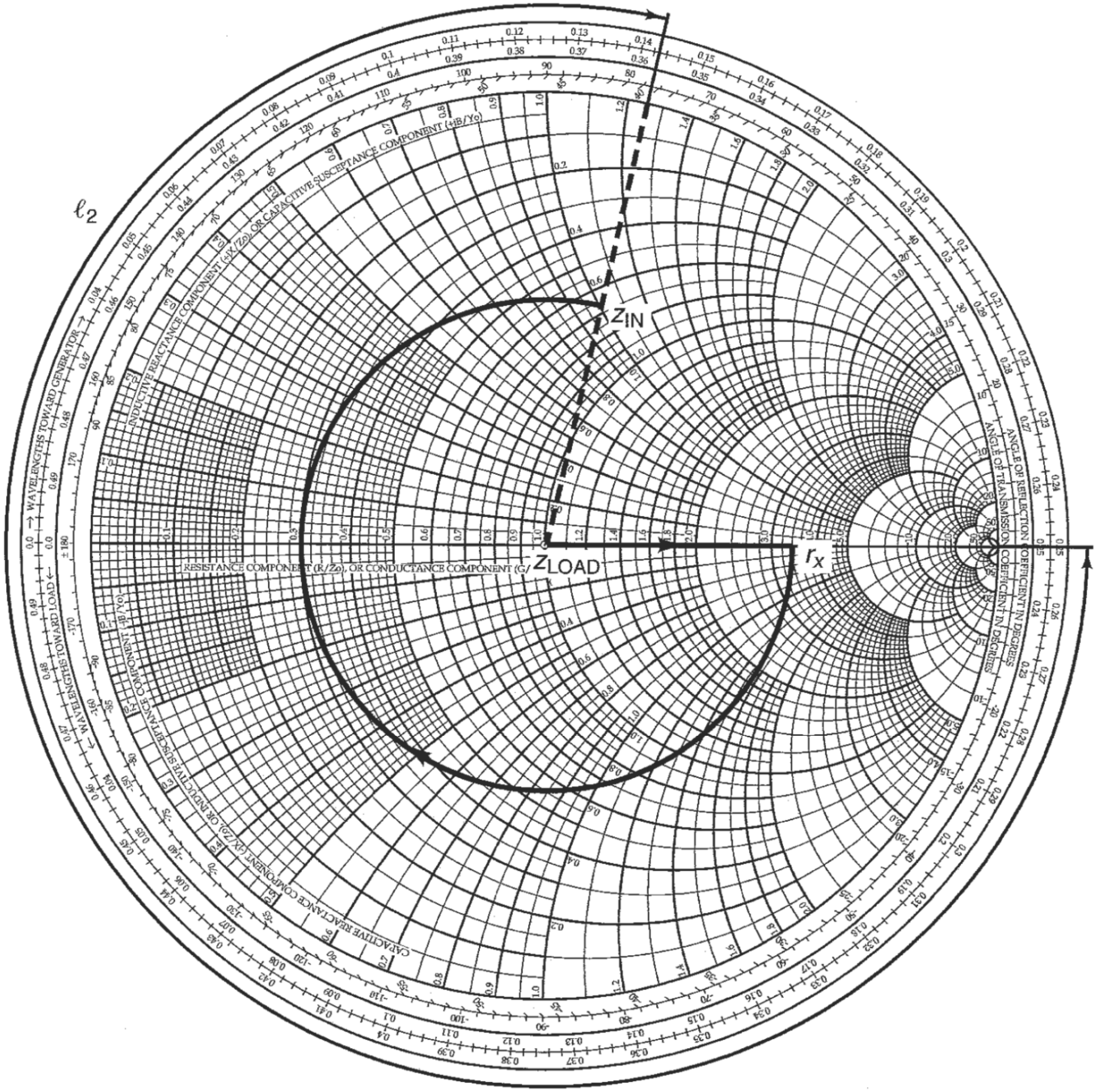
$$R_x = \frac{Z_{o1}^2}{Z_L}$$

$Z_{o1} = \sqrt{50(15)} = 27.4\,\Omega$

$l_2 = 0.143\lambda$



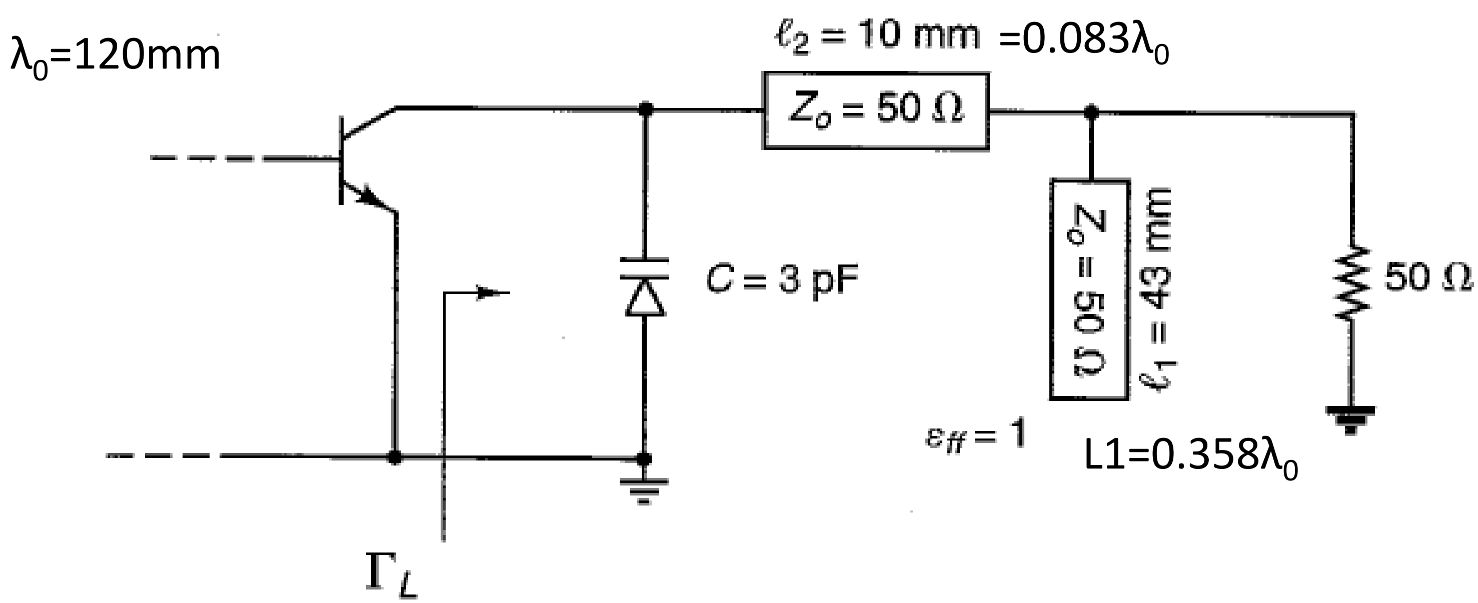
Alternative Solution



Example#10

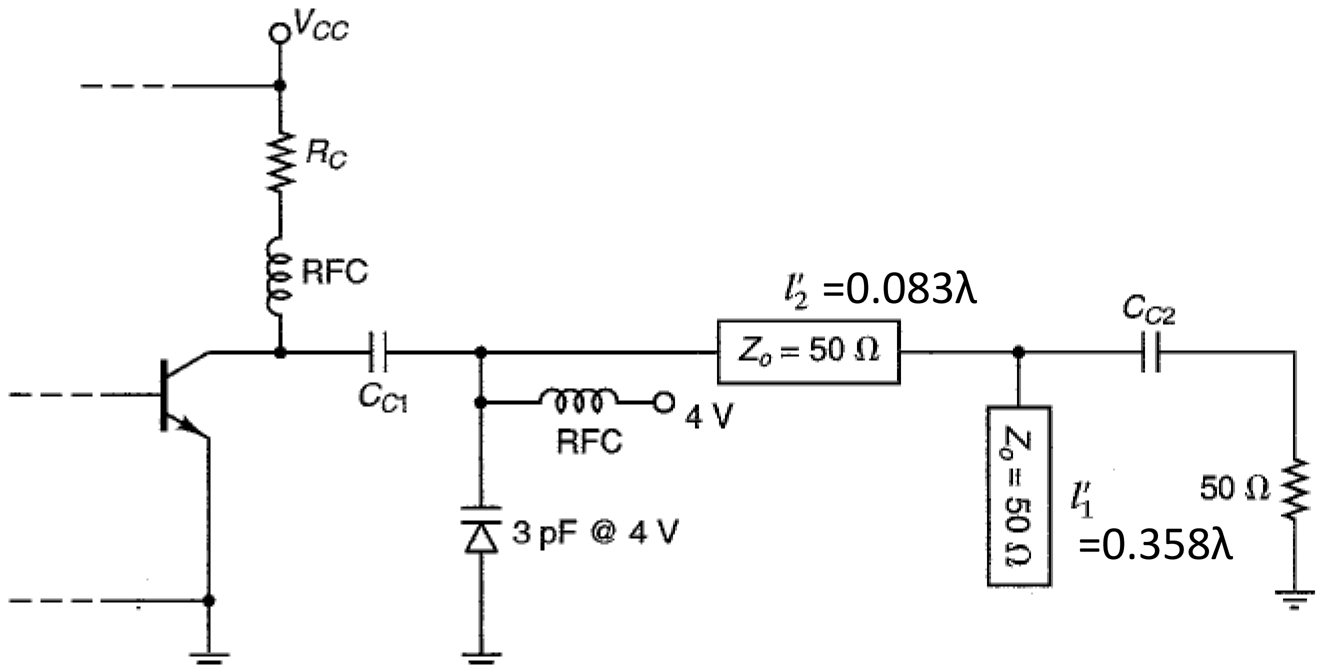
(a) An oscillator is designed at 2.5 GHz using the output matching topology shown in bellow. The length of the microstrips is shown for $\varepsilon_{ff} = 1$ (i.e., for $v = c = 3 \times 10^{10}$ cm/s). The matching network uses a varactor diode as a voltage-variable capacitor for the control of the oscillator frequency. Determine the value of the load reflection coefficient.

(b) Specify the width, height, and length of the microstrip lines if they are constructed using an alumina substrate ($\varepsilon_r = 9.6$).



Utilizing both lumped and distributed components for matching

Cont'd



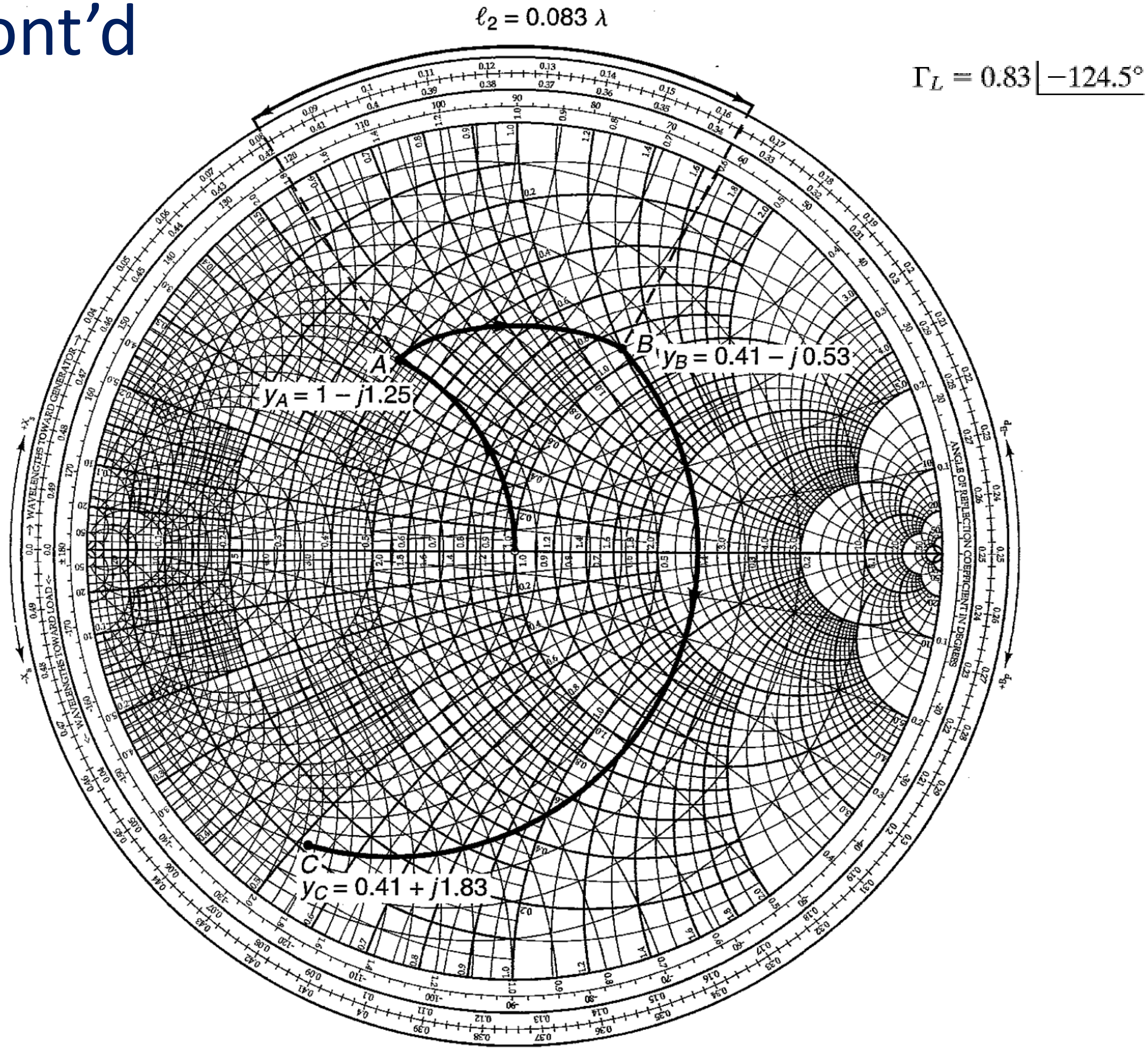
For Alumina substrate, we have:
(See slide 14)

h=40mil, W=40mil, ε_{eff}=6.46

$$l_1' = \frac{l_1}{\sqrt{\varepsilon_{ff}}} = \frac{43}{\sqrt{6.46}} = 16.9\text{ mm}$$

$$l_2' = \frac{l_2}{\sqrt{\varepsilon_{ff}}} = \frac{10}{\sqrt{6.46}} = 3.93\text{ mm}$$

Cont'd



$\ell_2 = 0.083\ \lambda$
$\Gamma_L = 0.83\angle -124.5°$
A
$y_A = 1 - j1.25$
B
$y_B = 0.41 - j0.53$
C
$y_C = 0.41 + j1.83$

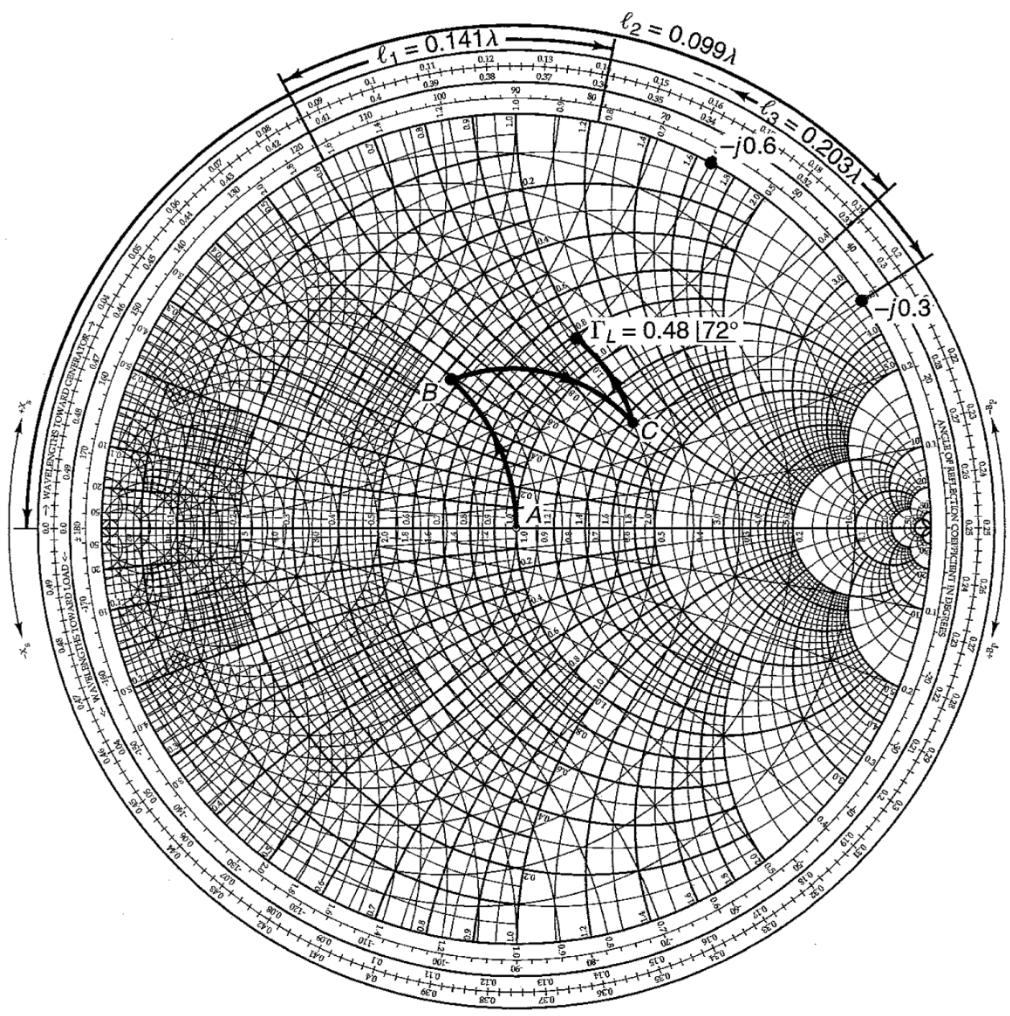
Example#11

Design a three-element microstrip matching network to transform a 50-Ω termination to a load reflection coefficient given by $\Gamma_L = 0.48\angle 72^\circ$.

$y_B = 1 - j0.82$

$y_C = 0.5 - j0.3$

$Y_L = 0.5 - j0.6$



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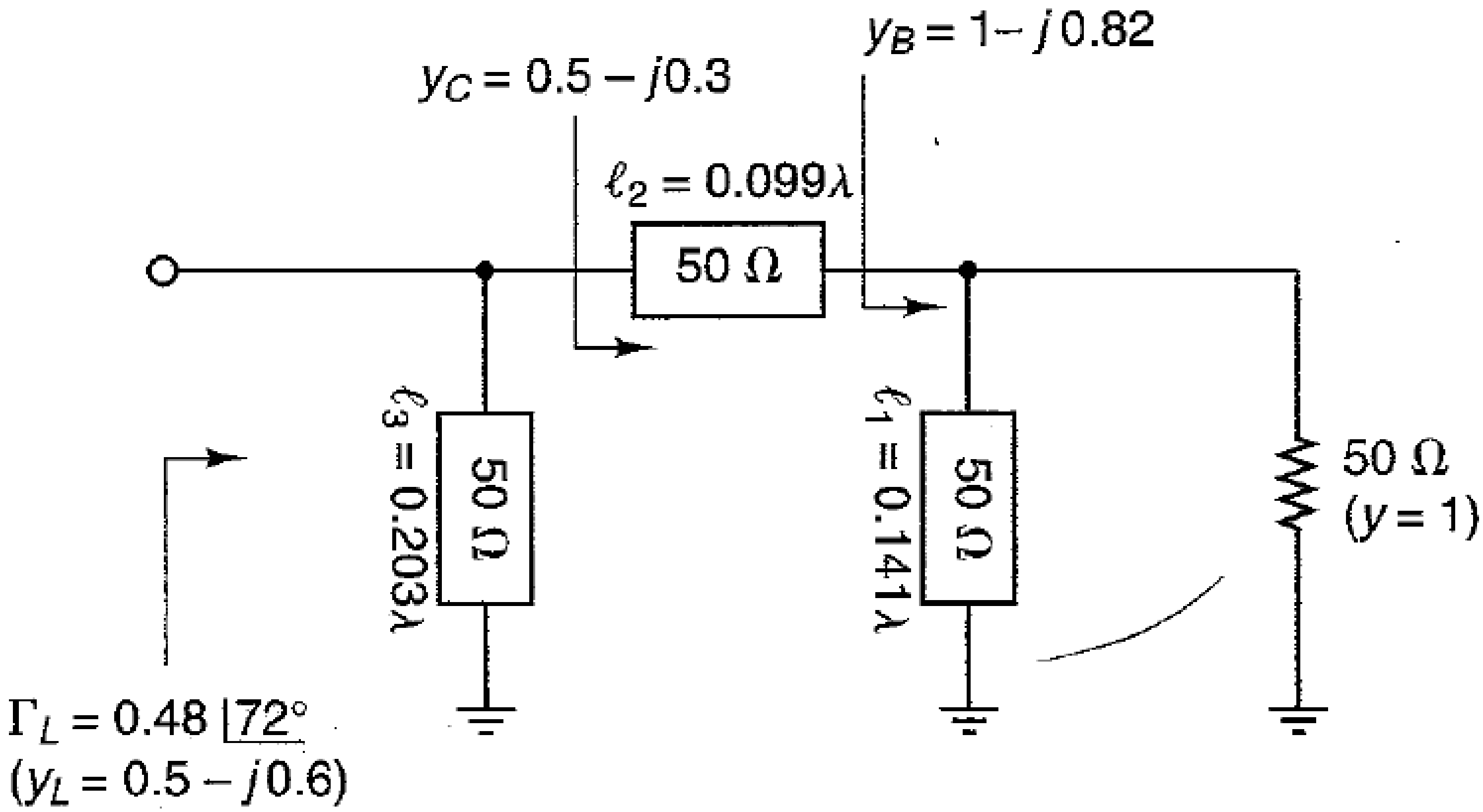

yB = 1 − j0.82
yC = 0.5 − j0.3
ℓ2 = 0.099λ
50 Ω
ℓ3 = 0.203λ
50 Ω
ℓ1 = 0.141λ
50 Ω
50 Ω
(y = 1)
ΓL = 0.48 ∠72°
(yL = 0.5 − j0.6)