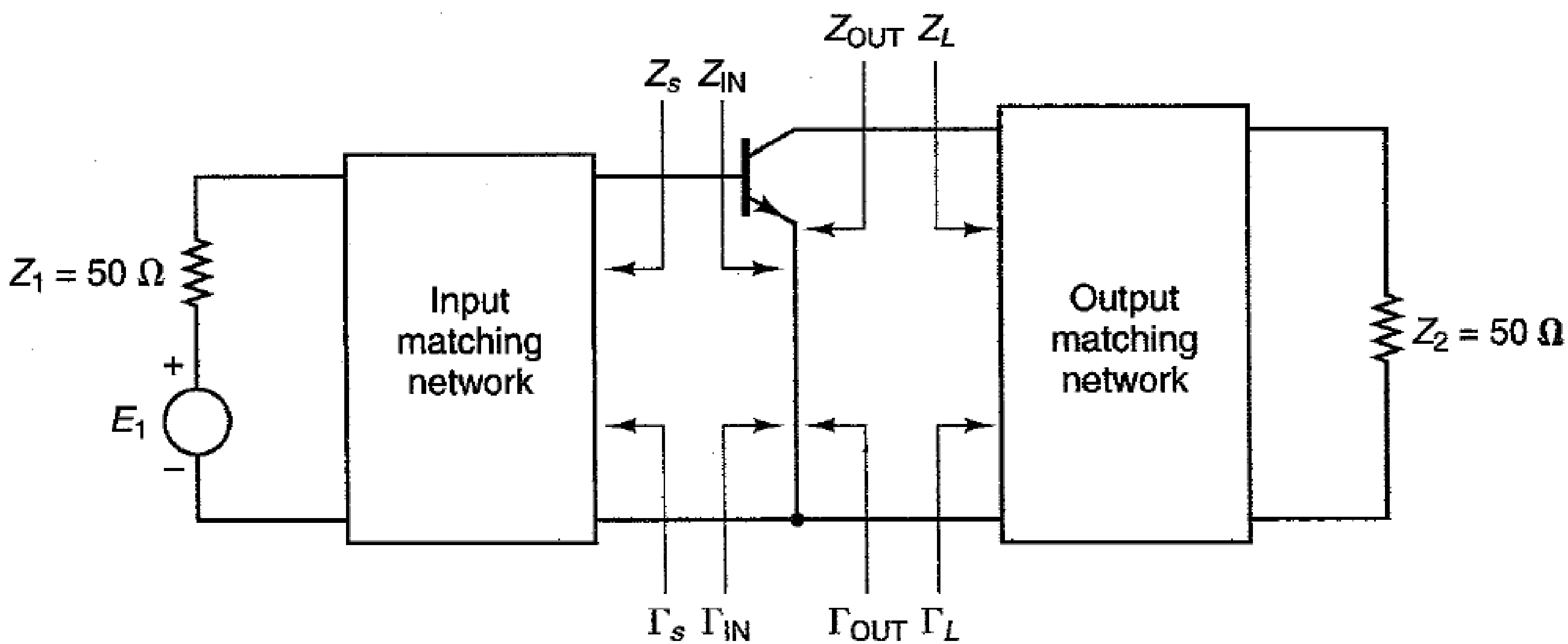
Microwave Amplifier Design

Microwave Amplifier Diagram



Gain Definitions

The transducer power gain G_T, the power gain G_p (also called the *operating power gain*), and the available power gain G_A are defined as follows:

$$G_T = \frac{P_L}{P_{AVS}} = \frac{\text{power delivered to the load}}{\text{power available from the source}}$$

$$G_p = \frac{P_L}{P_{IN}} = \frac{\text{power delivered to the load}}{\text{power input to the network}}$$

$$G_A = \frac{P_{AVN}}{P_{AVS}} = \frac{\text{power available from the network}}{\text{power available from the source}}$$

$$P_{IN} = \frac{1}{2}|a_1|^2 - \frac{1}{2}|b_1|^2$$

$$P_L = \frac{1}{2}|b_2|^2 - \frac{1}{2}|a_2|^2$$

$$P_{AVS} = P_{IN}\big|_{\Gamma_{IN} = \Gamma_s^*}$$

$$P_{AVN} = P_L\big|_{\Gamma_L = \Gamma_{OUT}^*}$$

Cont'd

After a few manipulations, we have:

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

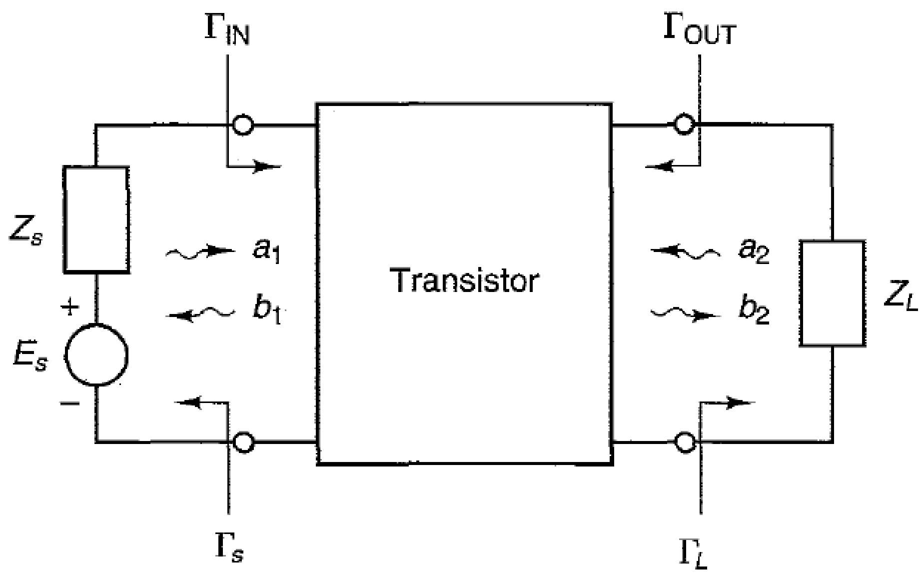
$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{IN}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{OUT}\Gamma_L|^2}$$

$$G_p = \frac{1}{1 - |\Gamma_{IN}|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$G_A = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1}{1 - |\Gamma_{OUT}|^2}$$

Source Mismatch Factor



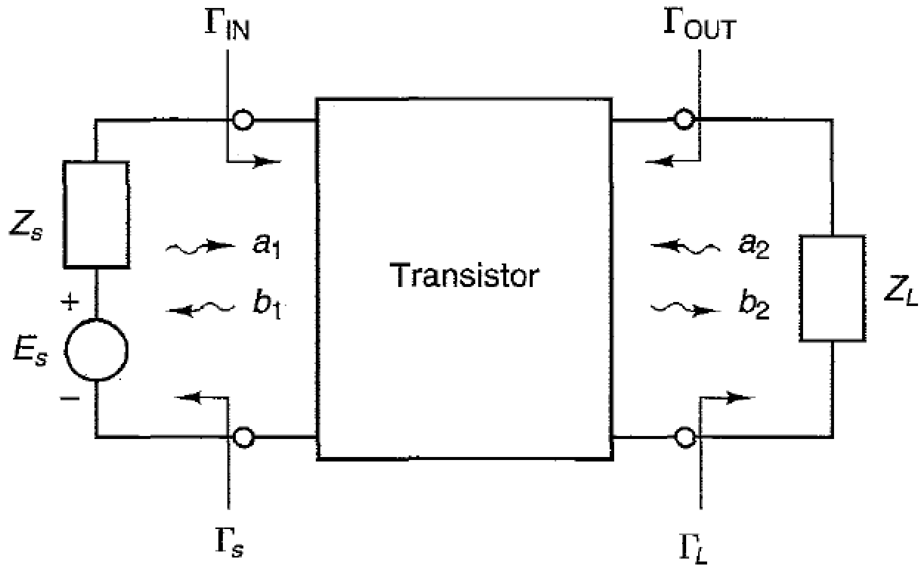
$$M_s \overset{\Delta}{=} \frac{P_{in}}{P_{AVS}} = \frac{G_T}{G_P}$$

$$M_s = \frac{(1 - |\Gamma_s|^2)(1 - |\Gamma_{IN}|^2)}{|1 - \Gamma_s \Gamma_{IN}|^2}$$

This factor is used to quantize what portion of P_{AVS} is delivered to the input of the transistor.
Observe that if $\Gamma_{IN} = \Gamma_s^*$ gives $M_s = 1$ and it follows that $P_{IN} = P_{AVS}$.
This fact is expressed in the form

$$P_{IN} = P_{AVS}\big|_{\Gamma_{IN} = \Gamma_s^*}$$

Load Mismatch Factor



$$M_L \triangleq \frac{P_L}{P_{AVN}} = \frac{G_T}{G_A}$$

$$M_L = \frac{(1 - |\Gamma_L|^2)(1 - |\Gamma_{OUT}|^2)}{|1 - \Gamma_{OUT}\Gamma_L|^2}$$

This factor is used to quantize what portion of P_{AVN} is delivered to the load.

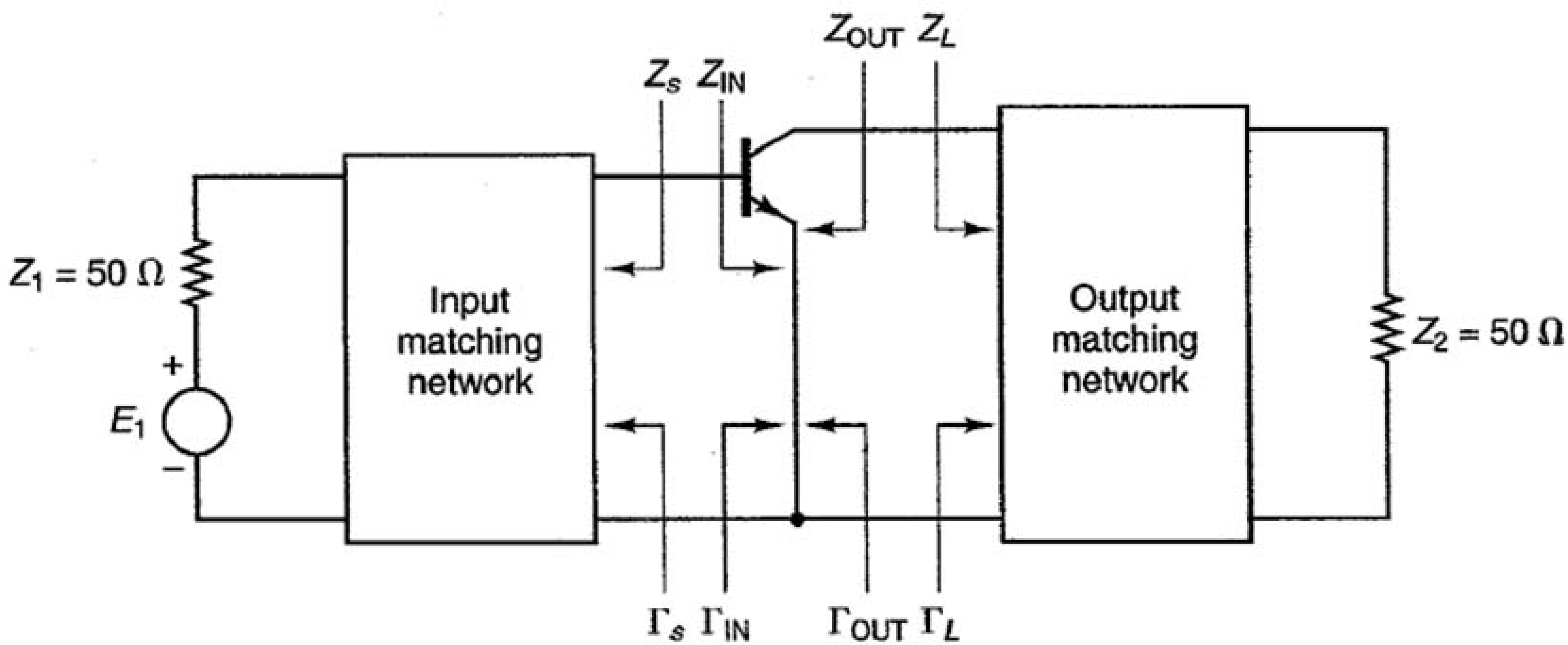
For $\Gamma_L = \Gamma_{OUT}^*$ gives $M_L = 1$ and it follows that $P_L = P_{AVN}$.

This fact is expressed in the form $P_L = P_{AVN}|_{\Gamma_L = \Gamma_{OUT}^*}$

Example#1

(a) The input and output matching networks in the following amplifier are designed to produce $\Gamma_s = 0.5\angle 120^\circ$ and $\Gamma_L = 0.4\angle 90^\circ$. Determine G_T, G_A, and G_p if the S parameters of the transistor are $S_{11} = 0.6\angle -160^\circ$ $S_{12} = 0.045\angle 16^\circ$ $S_{21} = 2.5\angle 30^\circ$ $S_{22} = 0.5\angle -90^\circ$

(b) Calculate P_{AVS}, P_{IN}, P_{AVN}, and P_L in Fig. 3.2.2 if $E_1 = 10\angle 0^\circ$, $Z_1 = 50\ \Omega$, and $Z_2 = 50\ \Omega$.



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$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \Rightarrow \Gamma_{IN} = 0.6\angle{-160^\circ} + \frac{0.045\angle{16^\circ}(2.5\angle{30^\circ})0.4\angle{90^\circ}}{1 - 0.5\angle{-90^\circ}(0.4\angle{90^\circ})} = 0.627\angle{-164.6^\circ}$$

$$\Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \Rightarrow \Gamma_{OUT} = 0.5\angle{-90^\circ} + \frac{0.045\angle{16^\circ}(2.5\angle{30^\circ})0.5\angle{120^\circ}}{1 - 0.6\angle{-160^\circ}(0.5\angle{120^\circ})} = 0.471\angle{-97.63^\circ}$$

$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{IN}\Gamma_s|^2}|S_{21}|^2\frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$\Downarrow$$

$$G_T = \frac{1 - (0.5)^2}{|1 - 0.627\angle{-164.6^\circ}(0.5\angle{120^\circ})|^2}(2.5)^2\frac{1 - (0.4)^2}{|1 - 0.5\angle{-90^\circ}(0.4\angle{90^\circ})|^2} = 9.43$$

(or 9.75 dB)

$$G_p = \frac{1}{1 - (0.627)^2}(2.5)^2\frac{1 - (0.4)^2}{|1 - 0.5\angle{-90^\circ}(0.4\angle{90^\circ})|^2} = 13.51 \quad \text{(or 11.31 dB)}$$

$$G_A = \frac{1 - (0.5)^2}{|1 - 0.6\angle{-160^\circ}(0.5\angle{120^\circ})|^2}(2.5)^2\frac{1}{1 - (0.471)^2} = 9.55 \quad \text{(or 9.8 dB)}$$

Cont'd

$$P_{AVS} = \frac{E_1^2}{8\,\mathrm{Re}[Z_1]} = \frac{10^2}{8(50)} = 0.25\ \mathrm{W}$$

$$P_L = G_T P_{AVS} = 9.43(0.25) = 2.358\ \mathrm{W}$$

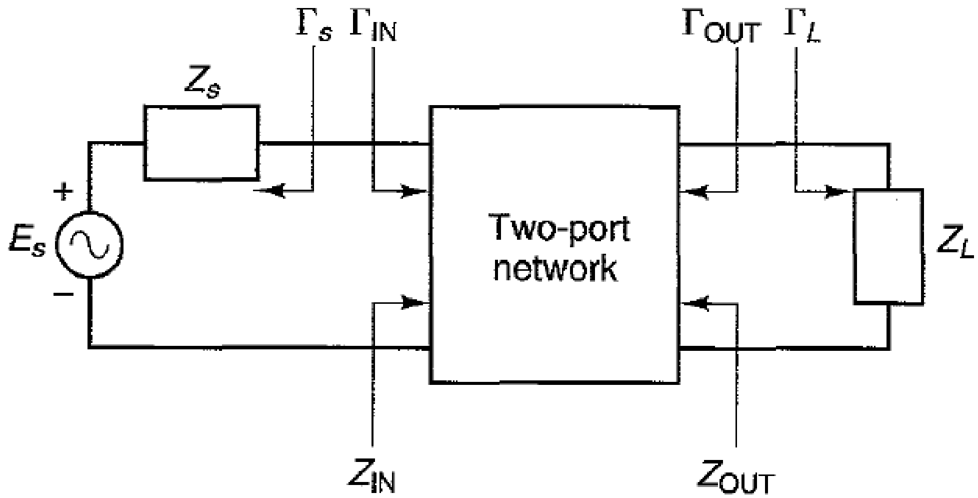
$P_L = G_P P_{IN}$ ➡ $2.358 = 13.51 \times P_{in}$ ➡ $P_{IN} = 0.1745\ \mathrm{W}$

$$P_{AVN} = G_A P_{AVS} = 9.55(0.25) = 2.39\ \mathrm{W}$$

$$M_s = \frac{[1-(0.5)^2]\,[1-(0.627)^2]}{|1-0.5\angle 120^\circ(0.627\angle -164.6^\circ)|^2} = 0.6983 \quad (\text{or } -1.56\ \mathrm{dB})$$

$$M_L = \frac{[1-(0.4)^2]\,[1-(0.471)^2]}{|1-0.471\angle -97.63^\circ(0.4\angle 90^\circ)|^2} = 0.9874 \quad (\text{or } -0.055\ \mathrm{dB})$$

Stability Consideration



In terms of reflection coefficients, the conditions for unconditional stability at a given frequency are

$$|\Gamma_s| < 1 \qquad |\Gamma_L| < 1$$

$$|\Gamma_{IN}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| < 1$$

$$|\Gamma_{OUT}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| < 1$$

Cont'd

$$\left|\Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|$$

$$\left|\Gamma_s - \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right|$$

Γ_L *values for* $|\Gamma_{IN}| = 1$ *(Output Stability Circle):*

$$r_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right| \qquad \text{(radius)}$$

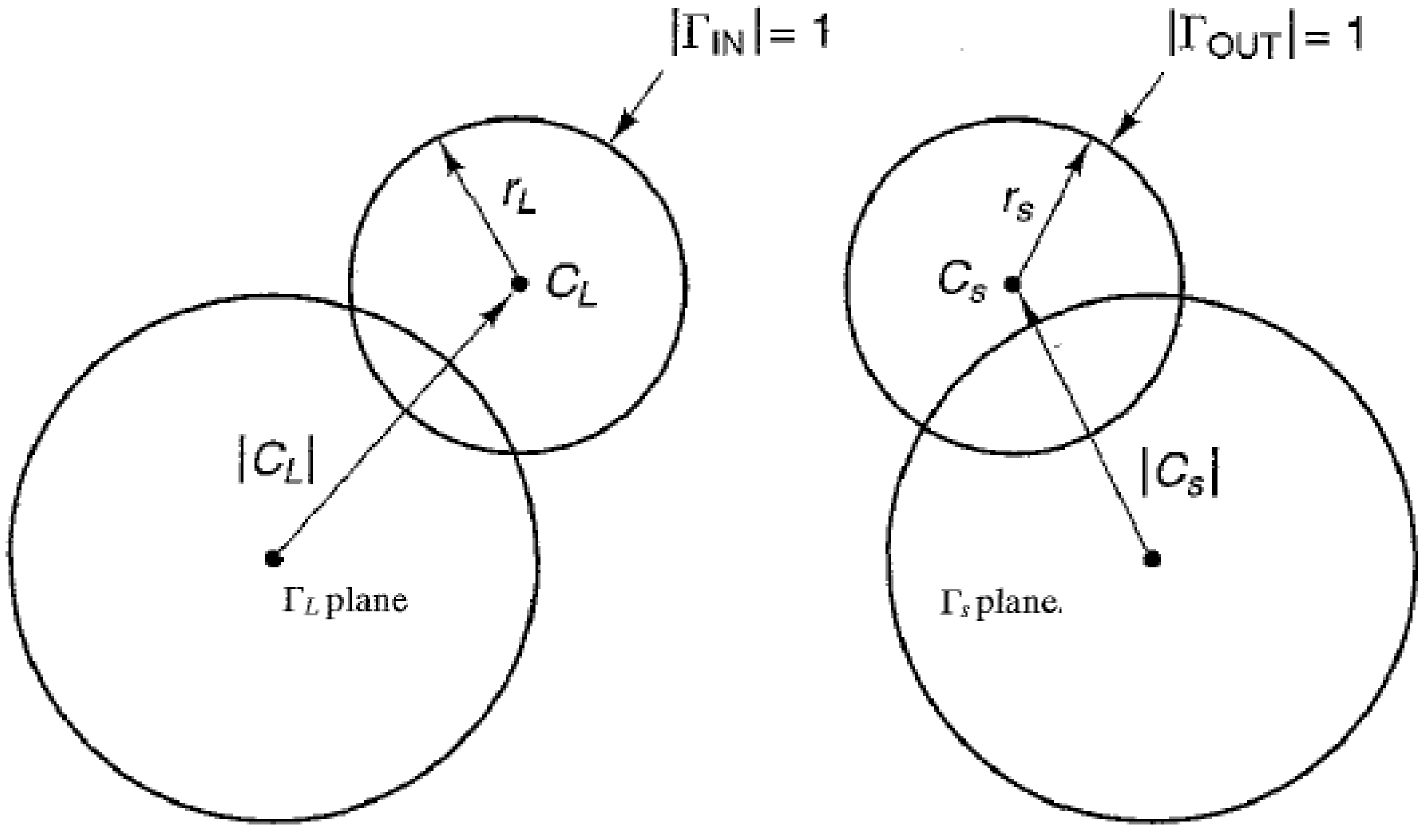
$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2} \qquad \text{(center)}$$

Γ_s *values for* $|\Gamma_{OUT}| = 1$ *(Input Stability Circle):*

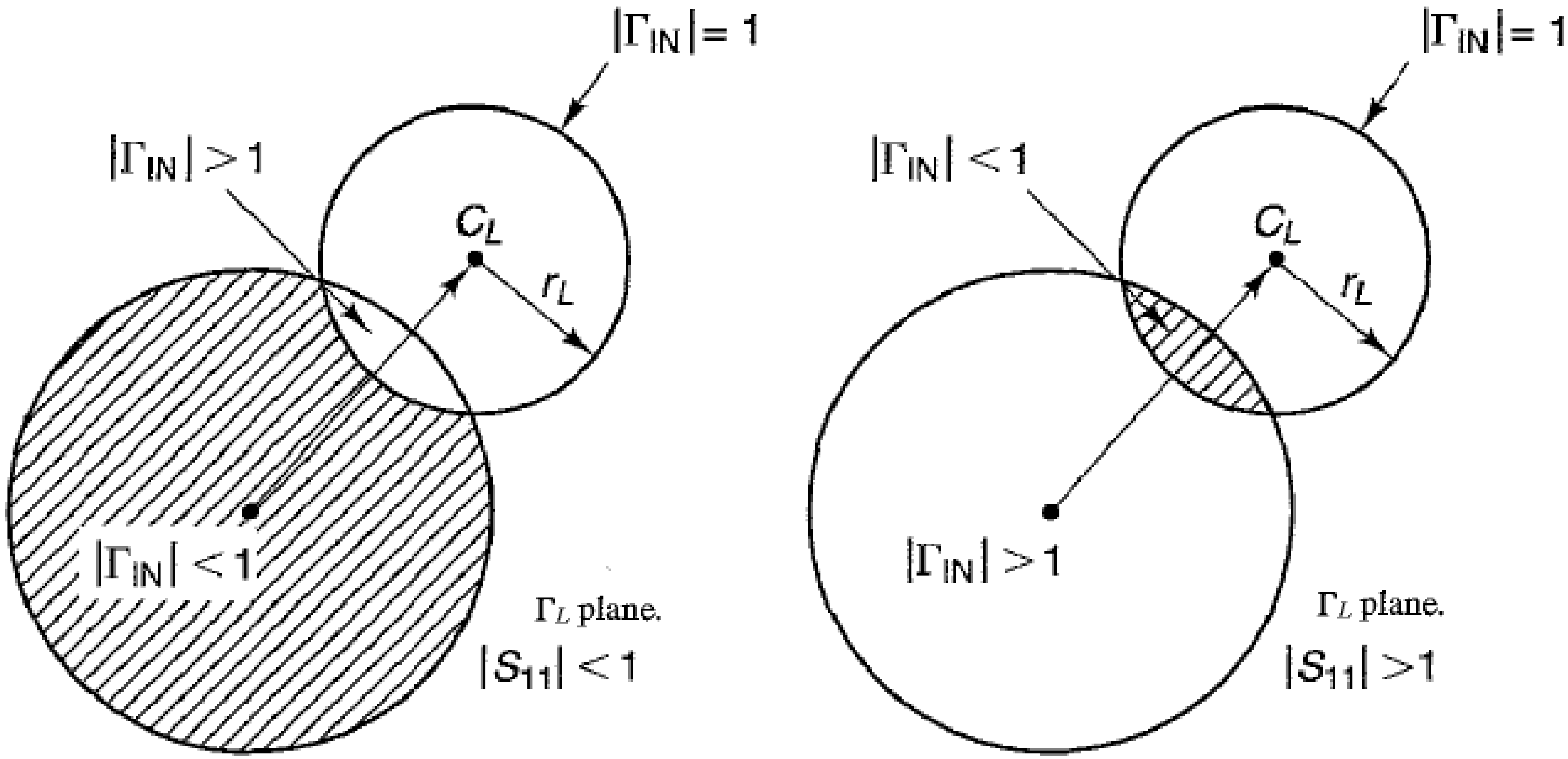
$$r_s = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right| \qquad \text{(radius)}$$

$$C_s = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \qquad \text{(center)}$$

Stability Circle in the Smith Chart



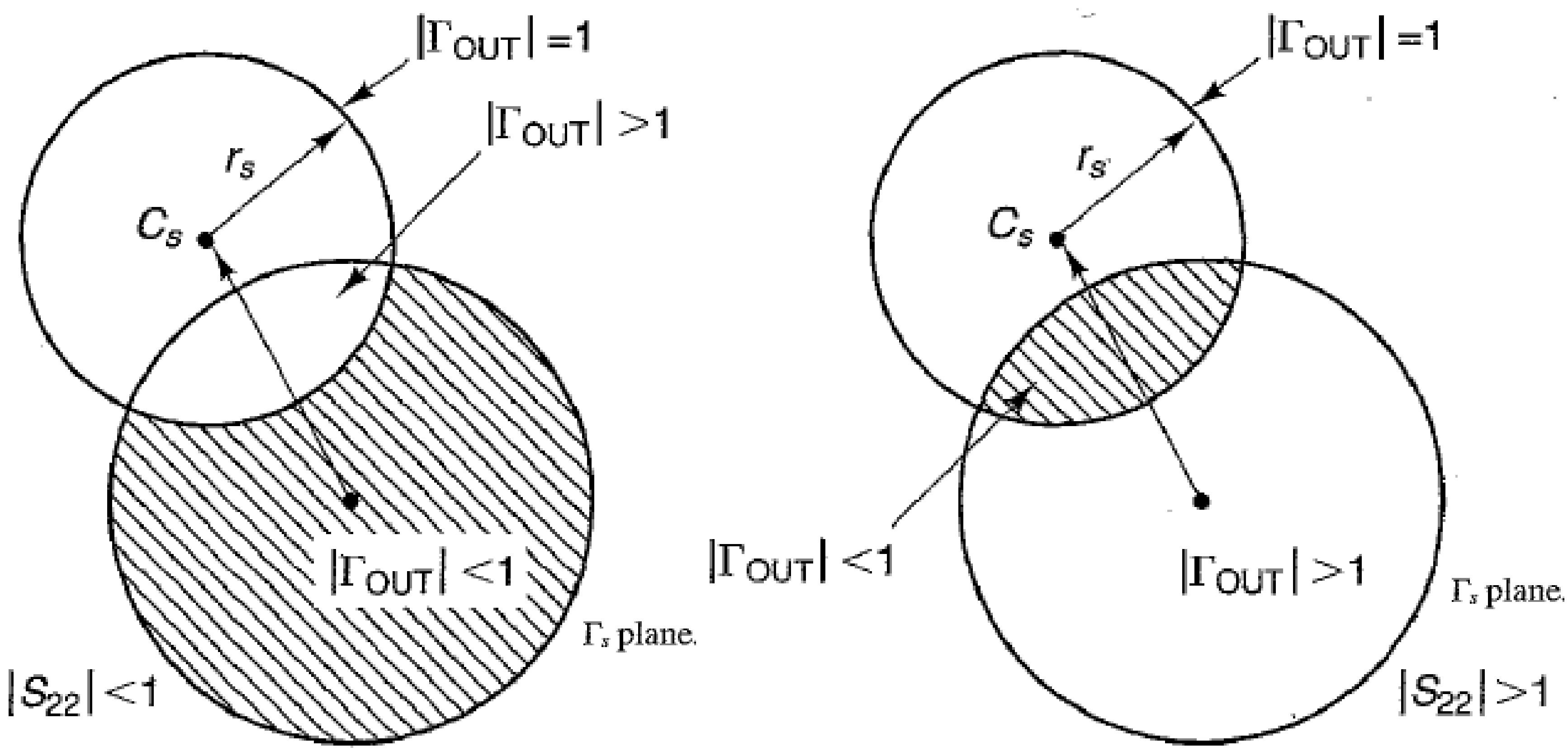
Stable and Unstable Regions in Γ_L Plane



$|\Gamma_{IN}| = |S_{11}|$ for $\Gamma_L = 0$

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

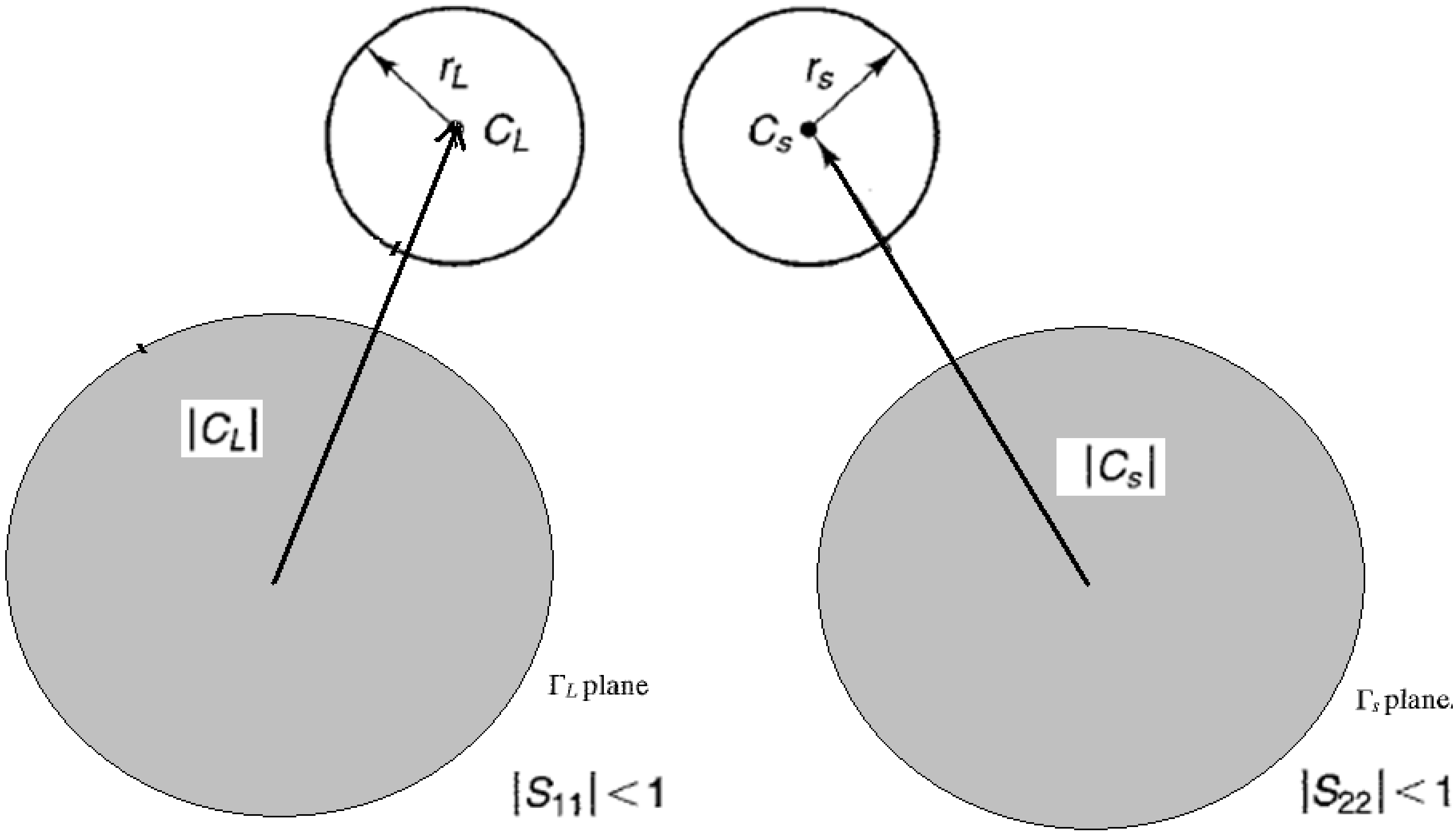
Stable and Unstable Regions in Γ_s Plane



$|\Gamma_{OUT}| = |S_{22}|$ for $\Gamma_s = 0$

$$\Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

Conditions for Unconditional Stability



If either $|S_{11}| > 1$ or $|S_{22}| > 1$, the network cannot be unconditionally stable because the termination $\Gamma_L = 0$ or $\Gamma_s = 0$ [see (3.3.3) and (3.3.4)] will produce $|\Gamma_{IN}| > 1$ or $|\Gamma_{OUT}| > 1$.

Example#2

The S parameters of a BJT at $V_{CE} = 15$ V and $I_C = 15$ mA at $f = 500$ MHz and 1 GHz, are as follows:

f (GHz)	S_{11}	S_{12}	S_{21}	S_{22}
0.5	$0.761\angle{-151^\circ}$	$0.025\angle{31^\circ}$	$11.84\angle{102^\circ}$	$0.429\angle{-35^\circ}$
1	$0.770\angle{-166^\circ}$	$0.029\angle{35^\circ}$	$6.11\angle{89^\circ}$	$0.365\angle{-34^\circ}$

For which values of Zs and ZL, the amplifier is stable.

For f=500 MHz, we have:

$$C_s = \frac{(0.761\angle{-151^\circ} - 0.221\angle{-123^\circ}(0.429\angle{35^\circ}))^*}{(0.761)^2 - (0.221)^2} = 1.36\angle{157.6^\circ}$$

$$r_s = \left|\frac{0.025\angle{31^\circ}(11.84\angle{102^\circ})}{(0.761)^2 - (0.221)^2}\right| = 0.558$$

$C_L = 2.8\angle{57.86^\circ}$ $\quad r_L = 2.18$

For f=1 GHz, we have: $C_s = 1.28\angle{169^\circ}$ and $r_s = 0.315$

$C_L = 2.62\angle{51.3^\circ}$ and $r_L = 1.71$

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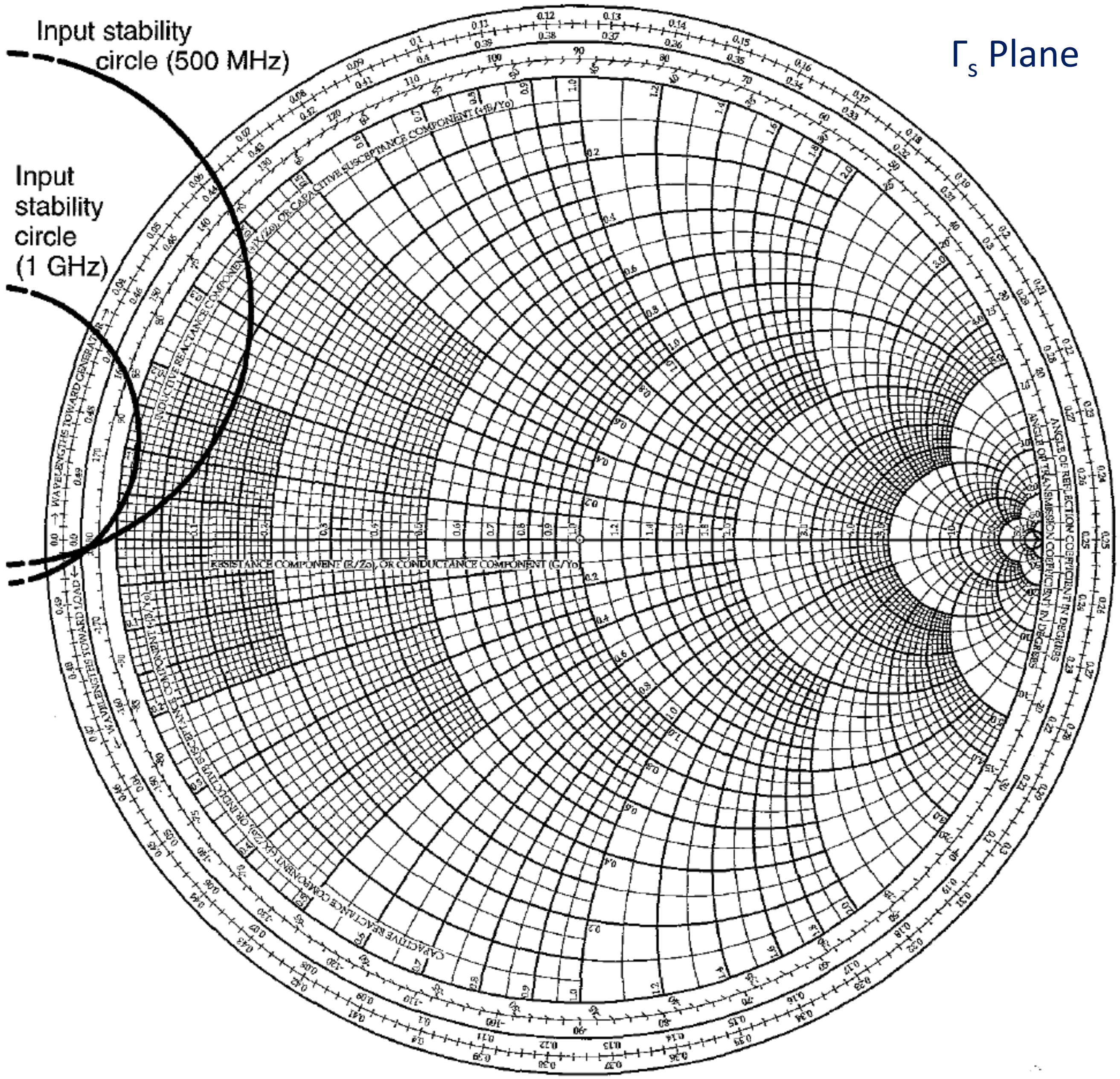

Input stability
circle (500 MHz)
Input
stability
circle
(1 GHz)
Γ_s Plane

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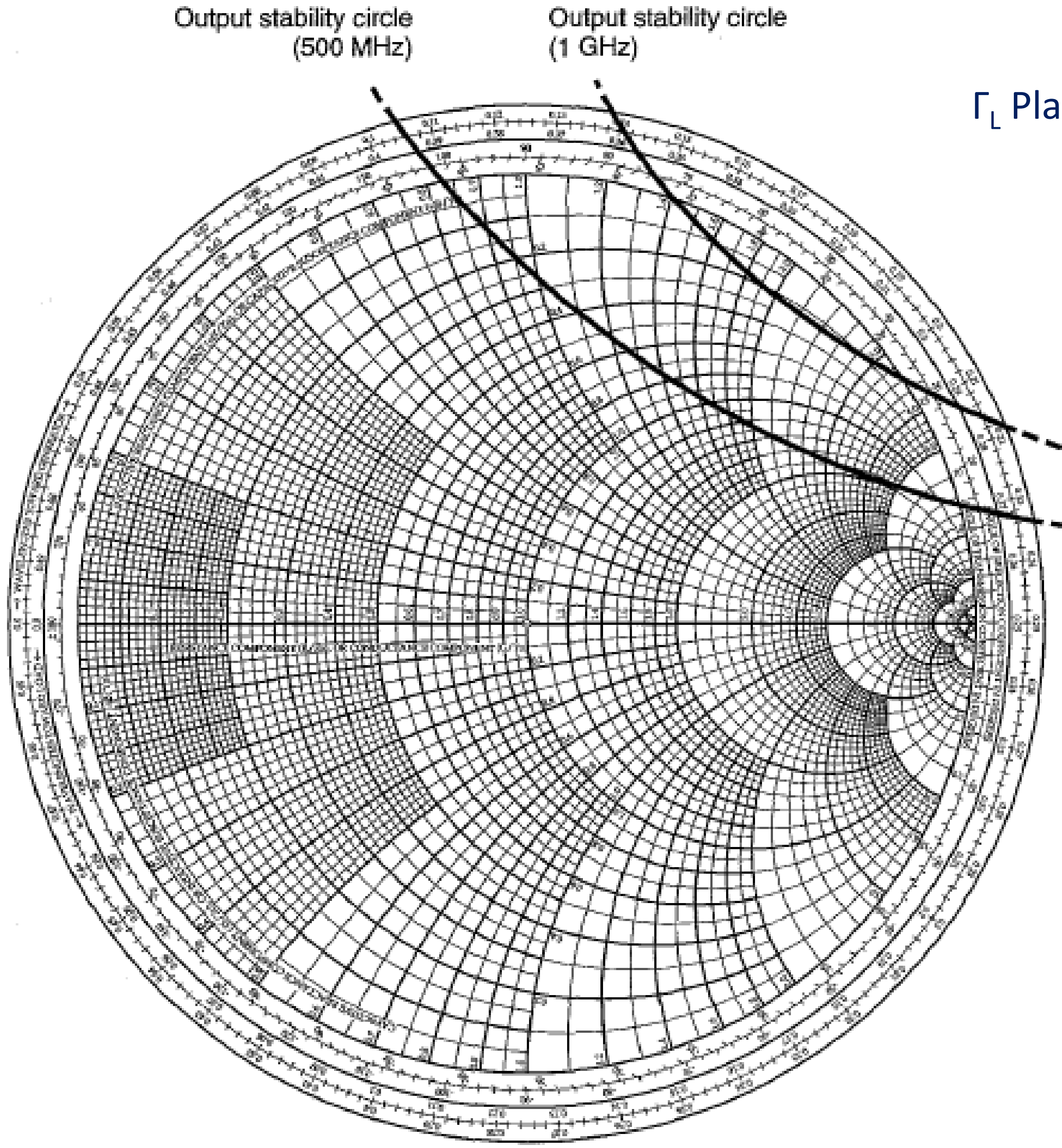
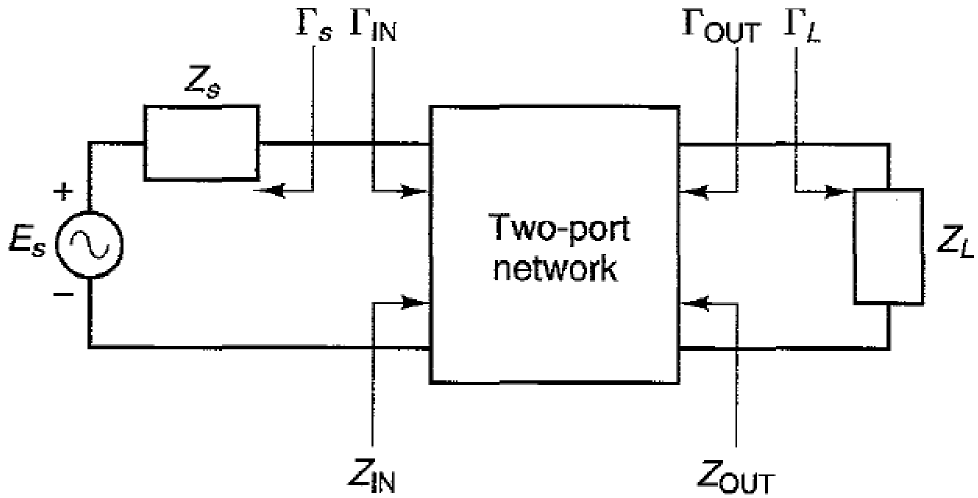


Γ_L Plane

Stability Consideration



A straightforward but somewhat lengthy manipulations result in the following necessary and sufficient conditions for unconditional stability:

$$K > 1 \text{ and } |\Delta| < 1$$

Stern's stability factor: $$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

Example#3

The S parameters of a BJT at $V_{CE} = 15$ V and $I_C = 15$ mA at $f = 500$ MHz, 1 GHz, 2 GHz, and 4 GHz are as follows:

f (GHz)	S_{11}	S_{12}	S_{21}	S_{22}
0.5	$0.761\angle{-151^\circ}$	$0.025\angle{31^\circ}$	$11.84\angle{102^\circ}$	$0.429\angle{-35^\circ}$
1	$0.770\angle{-166^\circ}$	$0.029\angle{35^\circ}$	$6.11\angle{89^\circ}$	$0.365\angle{-34^\circ}$
2	$0.760\angle{-174^\circ}$	$0.040\angle{44^\circ}$	$3.06\angle{74^\circ}$	$0.364\angle{-43^\circ}$
4	$0.756\angle{-179^\circ}$	$0.064\angle{48^\circ}$	$1.53\angle{53^\circ}$	$0.423\angle{-66^\circ}$

Determine the stability.

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$f = 500$ MHz → $\Delta = S_{11}S_{22} - S_{12}S_{21}$ $\quad \Delta = 0.221\angle{-123^\circ}$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} \quad K = 0.482$$

↓

the transistor is potentially unstable.

$f = 1$ GHz → $K = 0.857$ and $\Delta = 0.173\angle{-162.9^\circ}$

the transistor is potentially unstable.

$f = 2$ GHz → $K = 1.31$ and $\Delta = 0.174\angle{160^\circ}$

the transistor is unconditionally stable

$f = 4$ GHz → $K = 1.535$ and $\Delta = 0.226\angle{121^\circ}$

the transistor is unconditionally stable

Example#4

The S parameters of a transistor at $f = 800$ MHz are

$$S_{11} = 0.65\angle -95^\circ$$

$$S_{12} = 0.035\angle 40^\circ$$

$$S_{21} = 5\angle 115^\circ$$

$$S_{22} = 0.8\angle -35^\circ$$

Determine the stability and show how resistive loading can stabilize the transistor.

$$K = 0.547 \qquad \Delta = 0.504\angle 249.6^\circ$$

Since $K < 1$, the transistor is potentially unstable at $f = 800$ MHz.

The input and output stability circles are calculated:

$$C_s = 1.79\angle 122^\circ \qquad C_L = 1.3\angle 48^\circ$$

$$r_s = 1.04 \qquad r_L = 0.45$$

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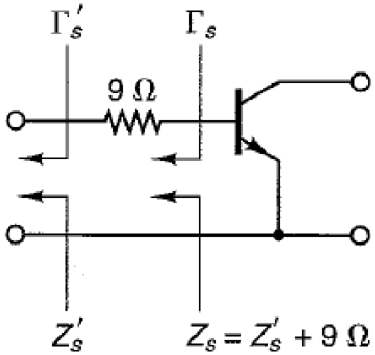
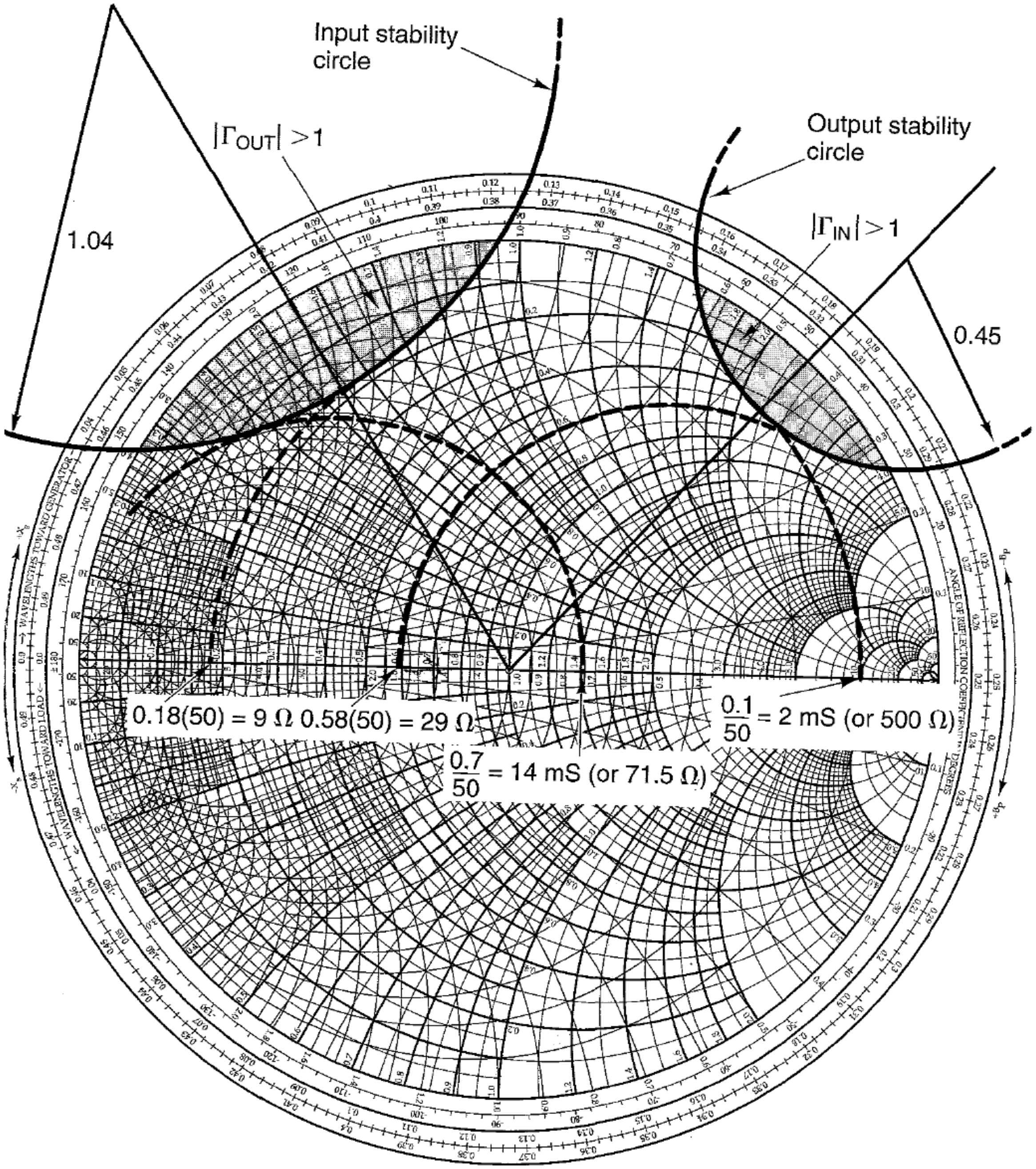


Γs'
Γs
9 Ω
Zs'
Zs = Zs' + 9 Ω



Input stability circle
|ΓOUT| > 1
Output stability circle
|ΓIN| > 1
1.04
0.45
0.18(50) = 9 Ω
0.58(50) = 29 Ω
0.7/50 = 14 mS (or 71.5 Ω)
0.1/50 = 2 mS (or 500 Ω)

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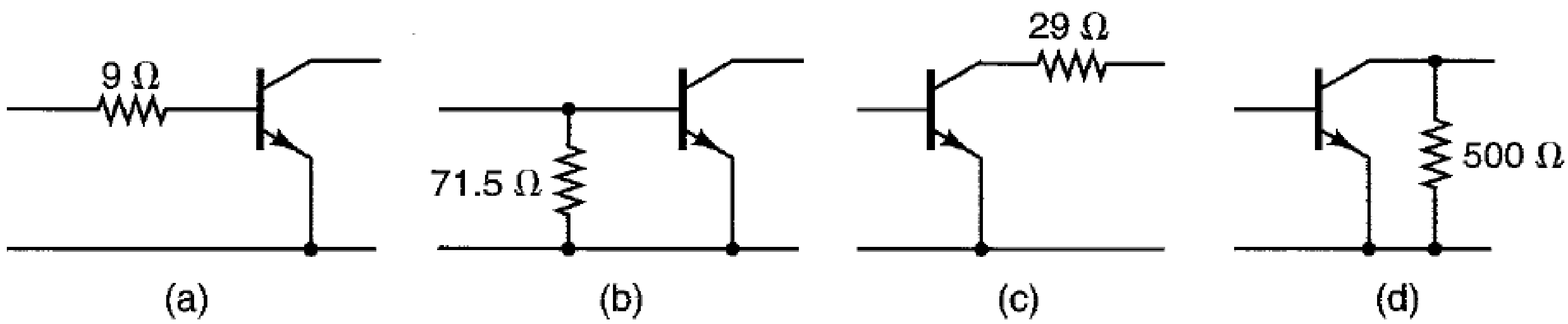
It can be seen that a series resistor with the input of approximately 9 Ω assures stability at the input.

The series addition of a 9-Ω resistor produces an impedance Z_s equal to $Z_s' + 9\ \Omega$. For any passive termination Z_s', the real part of Z_s will be greater than 9 Ω. Therefore, its associated reflection coefficient Γ_s will always be in the stable region.

Also, a shunt resistor with the input of approximately 0.7/50 = 14 mS (or 71.5 Ω) produces stability at the input.

Looking at the output stability circle, it follows that either a series resistor of approximately 29 Ω or a shunt resistor of approximately 500 Ω at the output produces stability at the output.

Improving Stability



$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

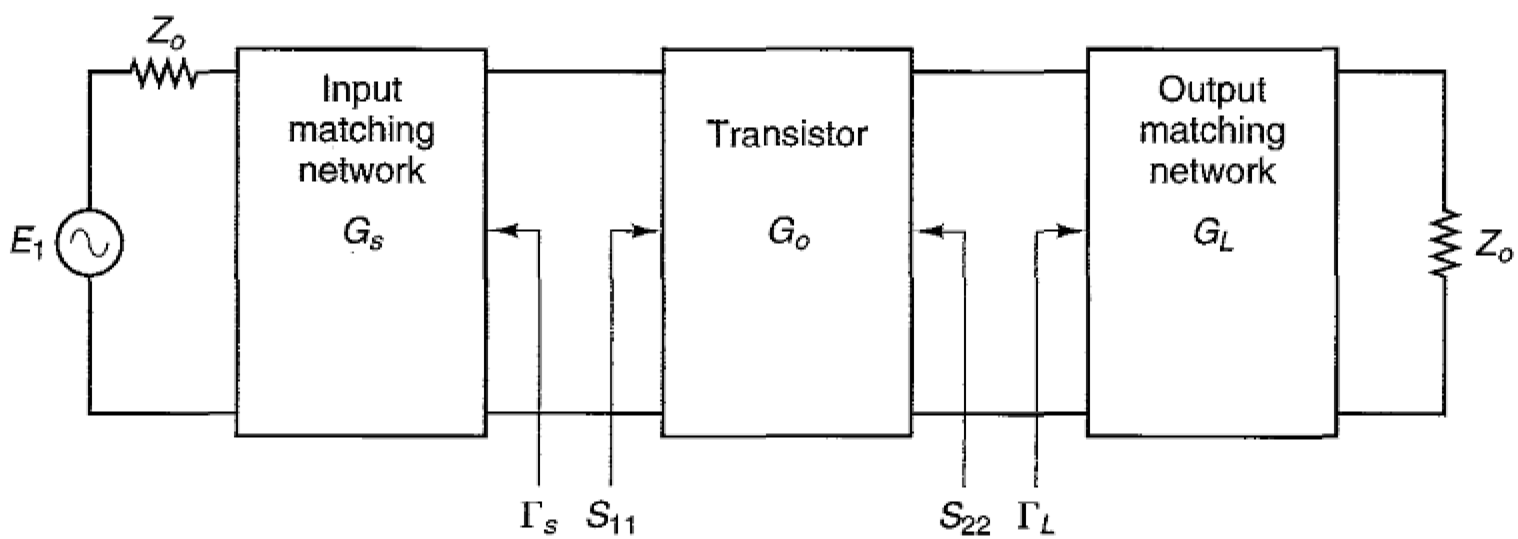
Unilateral Transistor: $S_{12}=0$

A two-port network is unilateral when $S_{12} = 0$.

In a unilateral case we have:

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11} \qquad \Gamma_{OUT} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = S_{22}$$

$$G_T = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{IN}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2} \Rightarrow G_{TU} = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$



Unilateral transducer power gain block diagram.

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$$G_{TU} = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

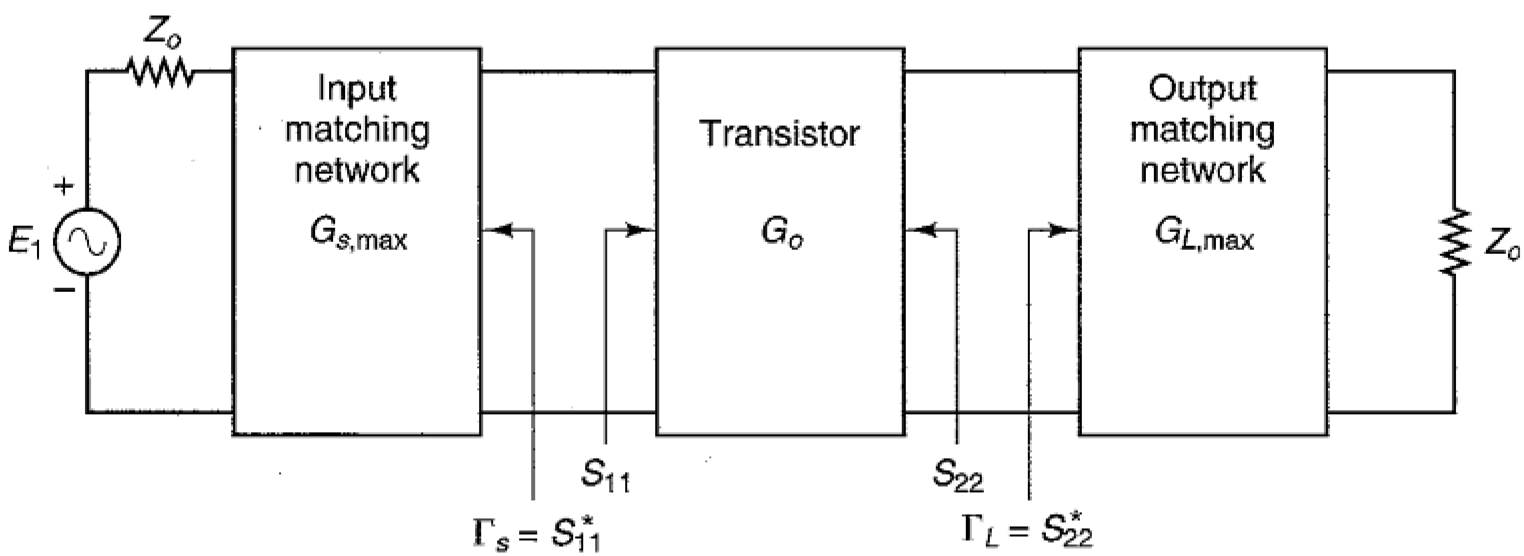
$$G_{TU} = G_s G_o G_L$$

$$G_s = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2}$$

$$G_o = |S_{21}|^2$$

$$G_L = \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

Maximum Achievable Gain (Unilateral Case: S_{12}=0)



Maximum unilateral transducer power gain block diagram.

$$\Gamma_s = S_{11}^*$$

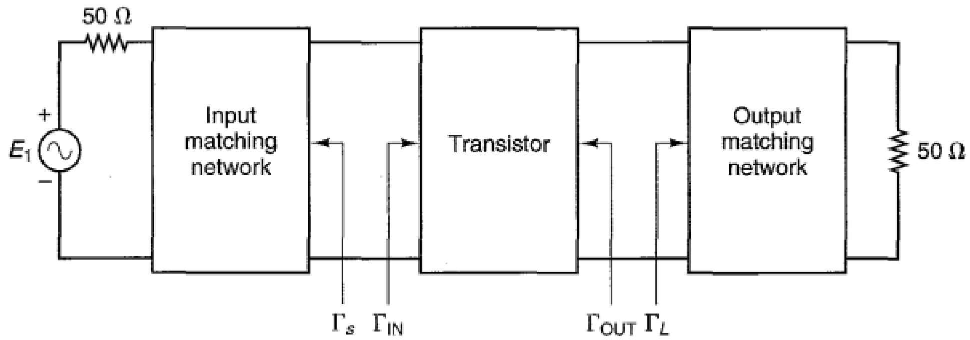
$$\Gamma_L = S_{22}^*$$

$$G_{TU,\max} = G_{s,\max} G_o G_{L,\max}$$

$$= \frac{1}{1 - |S_{11}|^2} |S_{21}|^2 \frac{1}{1 - |S_{22}|^2}$$

$$G_{TU,\max} = G_{pU,\max} = G_{AU,\max}.$$

Bilateral Transistor($S_{12} \neq 0$)



Simultaneous conjugate match exists when $\Gamma_s = \Gamma_{IN}^*$ and $\Gamma_L = \Gamma_{OUT}^*$.

It is proven maximum gain is obtained when simultaneous conjugate match exists.

$$\left.\begin{aligned}\Gamma_s &= \Gamma_{IN}^* \\ \Gamma_L &= \Gamma_{OUT}^*\end{aligned}\right\} \Rightarrow \quad \begin{aligned}\Gamma_s^* &= S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \\ \Gamma_L^* &= S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}\end{aligned}$$

Solving above equations, Γ_s and Γ_L are obtained:

$$\Gamma_{Ms} = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

$$\Gamma_{ML} = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

$$B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \qquad C_1 = S_{11} - \Delta S_{22}^* \qquad \Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \qquad C_2 = S_{22} - \Delta S_{11}^*$$

Maximum Achievable Gain (Bilateral Case: $S_{12} \neq 0$)

The maximum transducer power gain, under simultaneous conjugate match conditions, is obtained with $\Gamma_s = \Gamma_{IN}^* = \Gamma_{Ms}$ and $\Gamma_L = \Gamma_{OUT}^* = \Gamma_{ML}$.

$$G_{T,\max} = \frac{1}{1 - |\Gamma_{Ms}|^2} |S_{21}|^2 \frac{1 - |\Gamma_{ML}|^2}{|1 - S_{22}\Gamma_{ML}|^2}$$

Hint: under simultaneous conjugate match conditions we have

$$G_T = G_p = G_A = G_{T,\max} = G_{p,\max} = G_{A,\max}.$$

It is proven that: $$G_{T,\max} = \frac{|S_{21}|}{|S_{12}|}\left(K - \sqrt{K^2 - 1}\right)$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

The maximum stable gain is defined as the value of G_{Tmax} when K=1. Namely,

$$G_{MSG} = \frac{|S_{21}|}{|S_{12}|}$$

Example

Design a microwave amplifier using a GaAs FET to operate $f = 6$ GHz with maximum transducer power gain. The transistor S parameters at the linear bias point, $V_{DS} = 4$ V and $I_{DS} = 0.5 I_{DSS}$, are

$$S_{11} = 0.641\angle{-171.3^\circ} \qquad S_{12} = 0.057\angle{16.3^\circ}$$
$$S_{21} = 2.058\angle{28.5^\circ} \qquad S_{22} = 0.572\angle{-95.7^\circ}$$

$K = 1.504$

$\Delta = 0.3014\angle{109.88^\circ}$

Since $K > 1$ and $|\Delta| < 1$, the GaAs FET is unconditionally stable.

$$G_{T,\max} = \frac{2.058}{0.057}\left(1.504 - \sqrt{(1.504)^2 - 1}\right) = 13.74 \quad \text{or} \quad 11.38 \text{ dB}$$

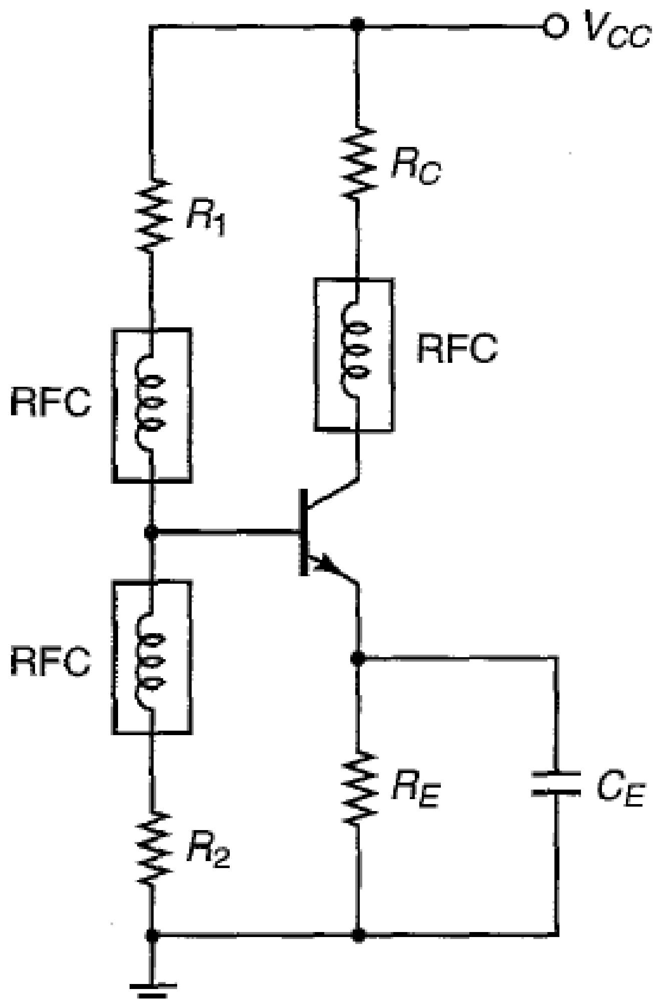
$B_1 = 0.9928$ $\qquad C_1 = 0.4786\angle{-177.3^\circ}$

$B_2 = 0.8255$ $\qquad C_2 = 0.3911\angle{-103.9^\circ}$

➡ $\Gamma_{Ms} = 0.762\angle{177.3^\circ} \qquad \Gamma_{ML} = 0.718\angle{103.9^\circ}$

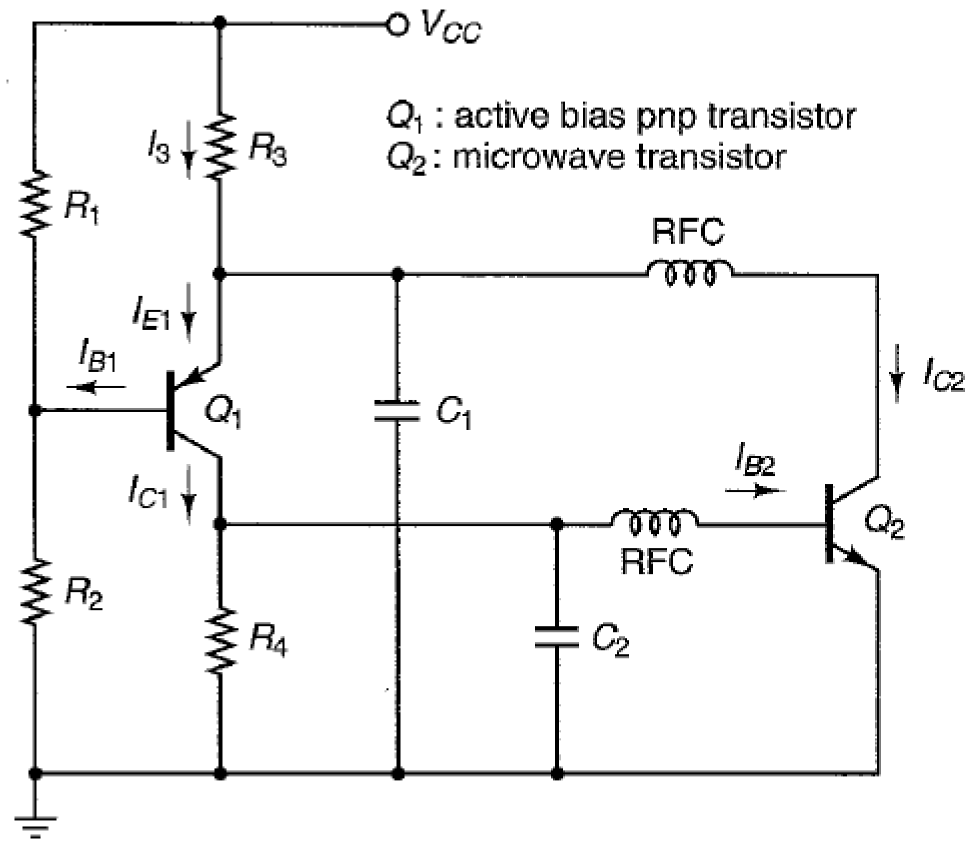
➡ $Y_{Ms} = \dfrac{7.2 - j1.23}{50} = (144 - j24.6) \times 10^{-3}$ S $\qquad Y_{ML} = \dfrac{0.414 - j1.19}{50} = (8.28 - j23.8) \times 10^{-3}$ S

BJT Bias Network



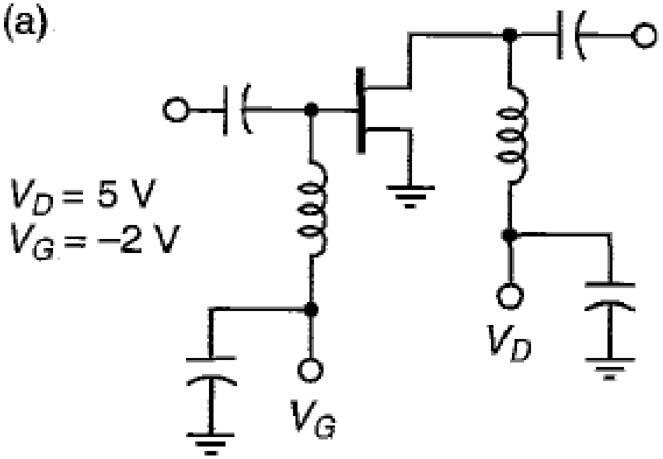
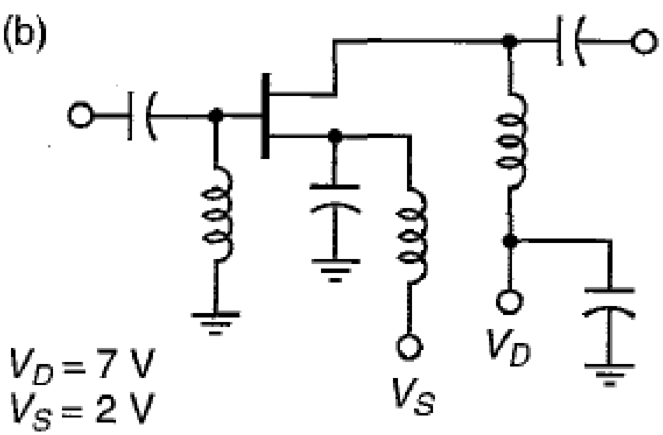
A dc bias network with a bypassed emitter resistor.

BJT Bias Network

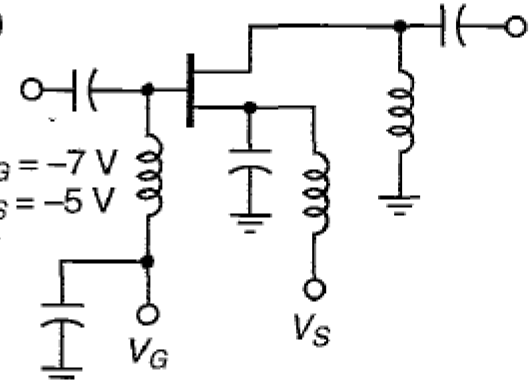
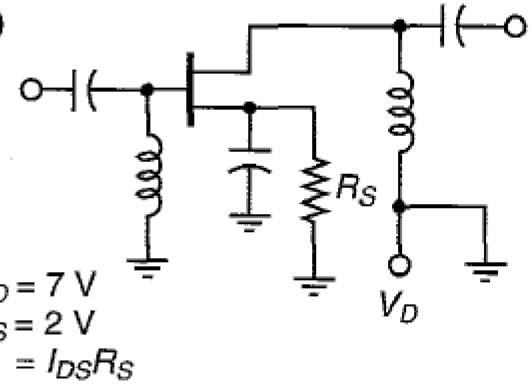
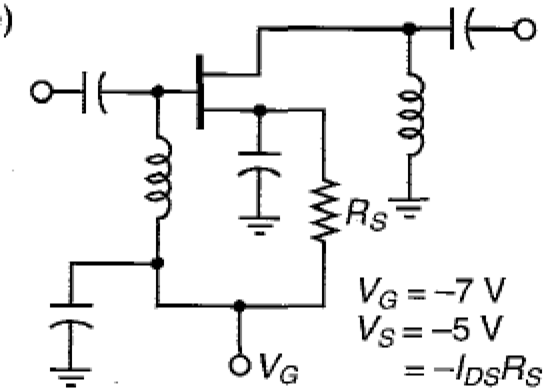


Active bias network for a BJT.

GaAs FET Bias Network

Figure	How	Amplifier characteristics	Power supply used
(a) 	Apply V_G, then V_D	Low noise High gain High power High efficiency	Bipolar, Minimum source inductance
(b) 	Apply V_S, then V_D	[same as (a)]	Positive supply

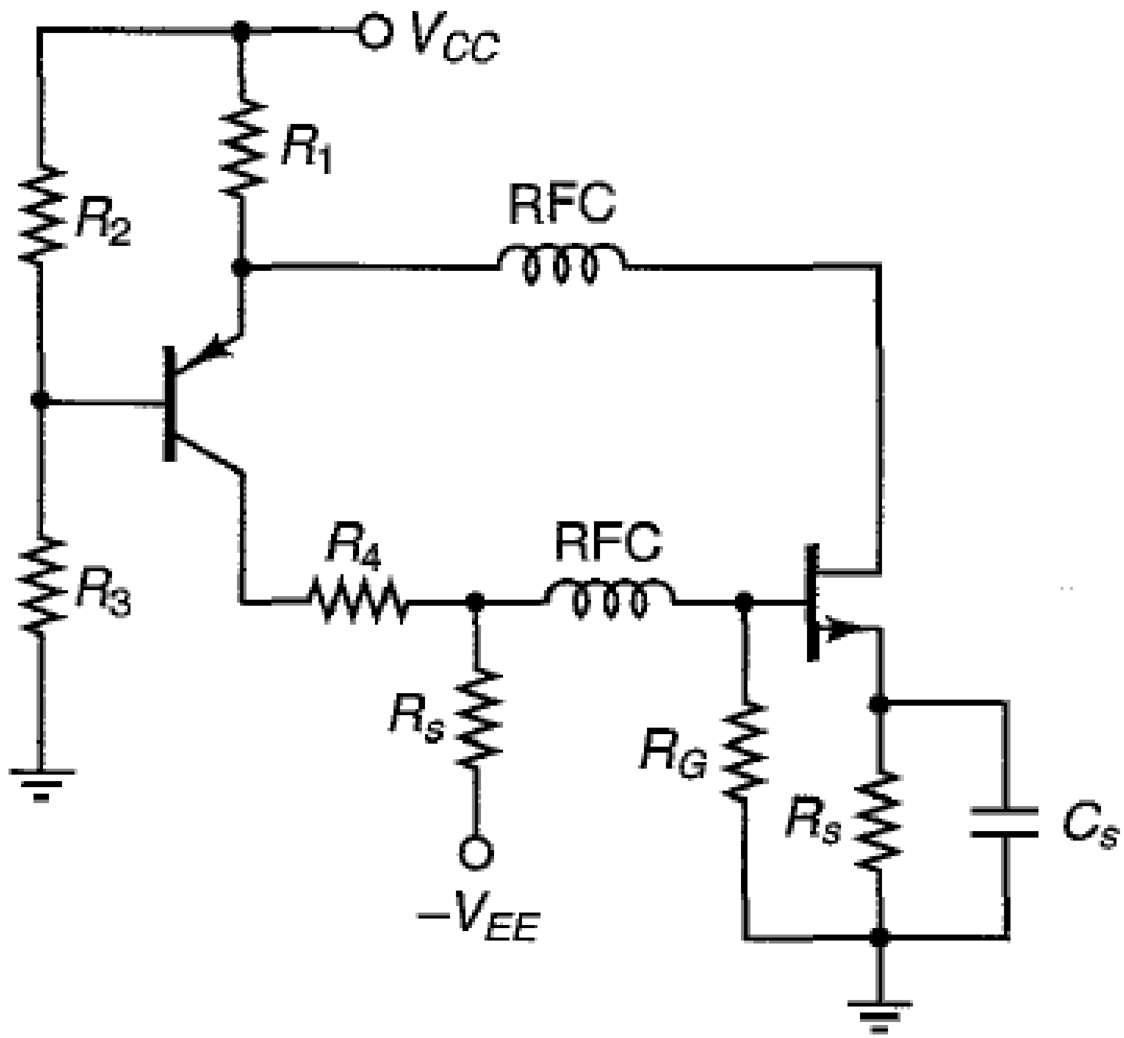
GaAs FET Bias Network

(c) 	Apply V_S, then V_G	[same as (a)]	Negative supply
(d) 	Apply V_D	Low noise High gain High power Lower efficiency Gain easily adjusted by varying R_S	Unipolar, incorporating R_S automatic transient protection
(e) 	Apply V_G	[same as (d)]	Negative unipolar, incorporating R_S

GaAs FET Bias Network

Therefore, the proper turn-on sequence is: first apply a negative bias to the gate (i.e., $V_G < 0$) and then apply the drain voltage ($V_D > 0$). One method to accomplish the previous turn-on procedure is to turn both sources at the same time and to include a long RC time constant network in the V_D supply and a short RC time constant network in the negative supply V_G.

GaAs FET Bias Network



Active bias for a common- source GaAs FET.